Advanced Tools from Modern Cryptography

Lecture 4

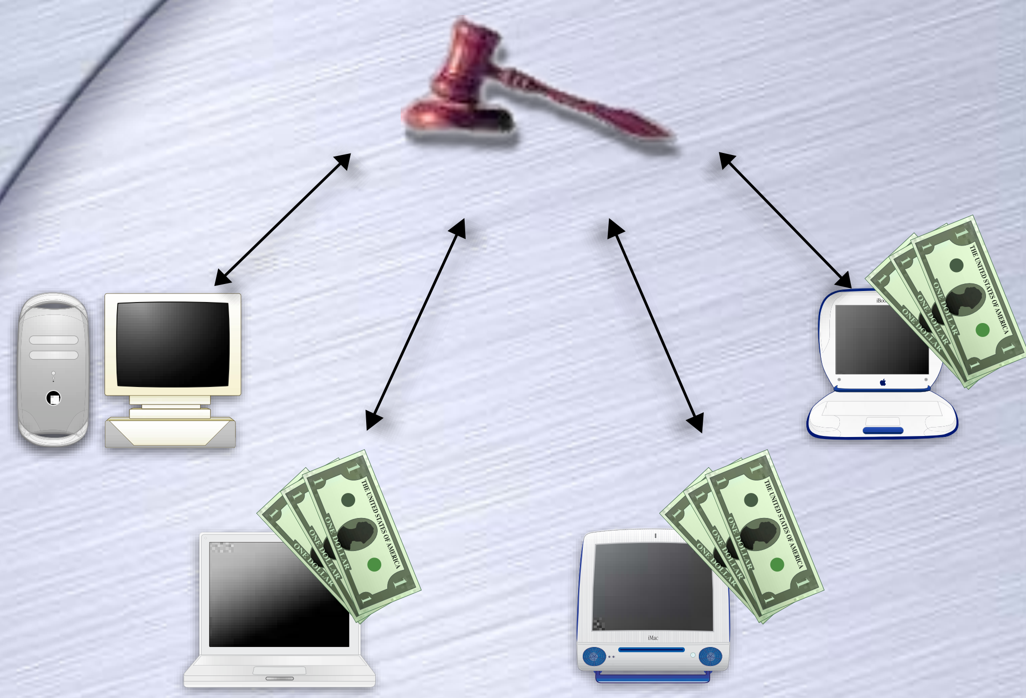
Secure Multi-Party Computation:

Passive Corruption,

Linear Functions

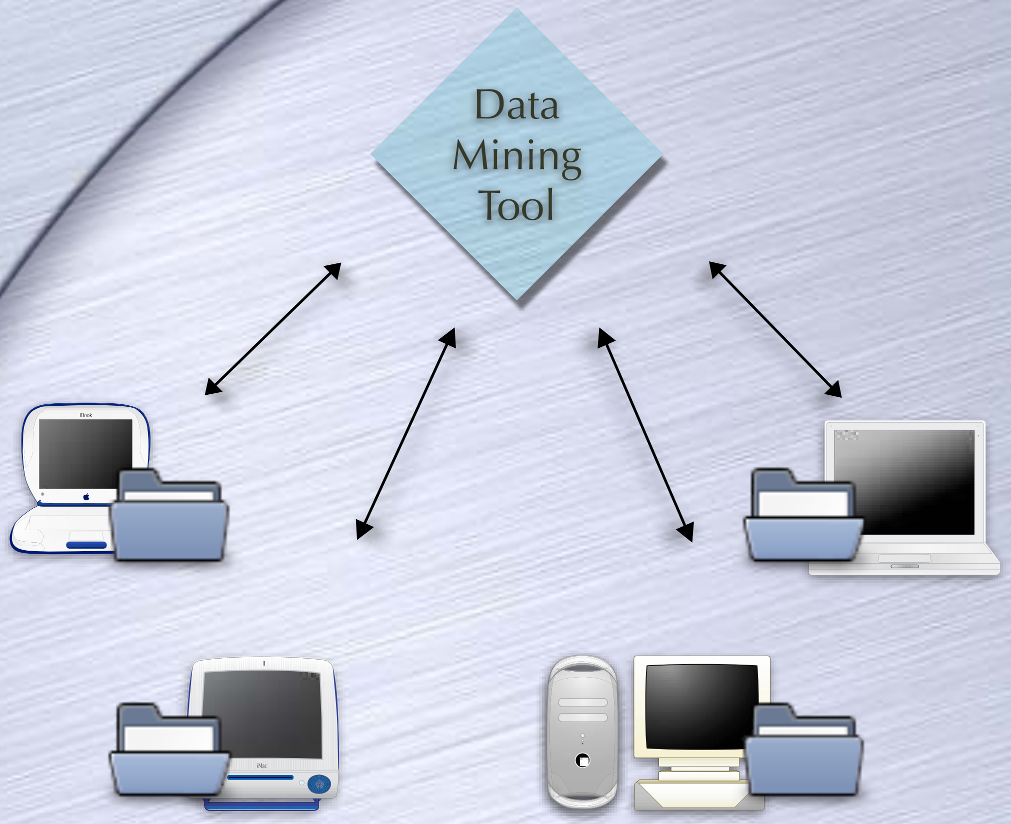
Must We Trust eBay ?

- Can we have an auction without an auctioneer?!
  - Declared winning bid should be correct
  - Only the winner and winning bid should be revealed



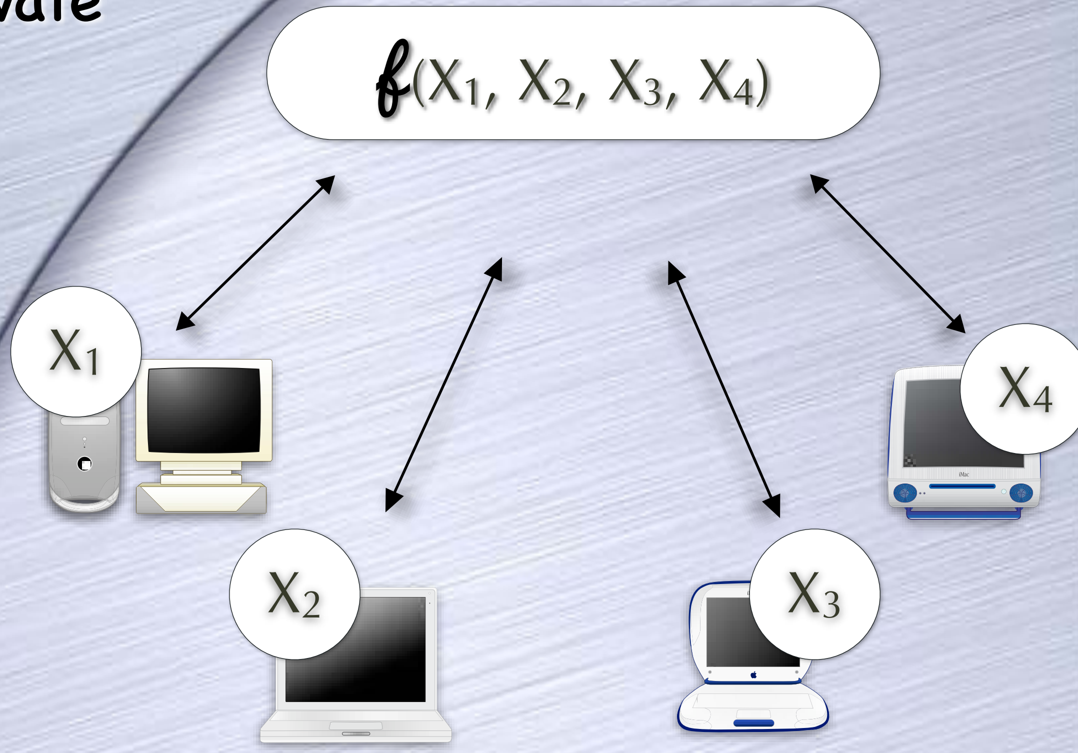
Using data without sharing?

- Hospitals which can't share their patient records with anyone
    - But want to data-mine on combined data



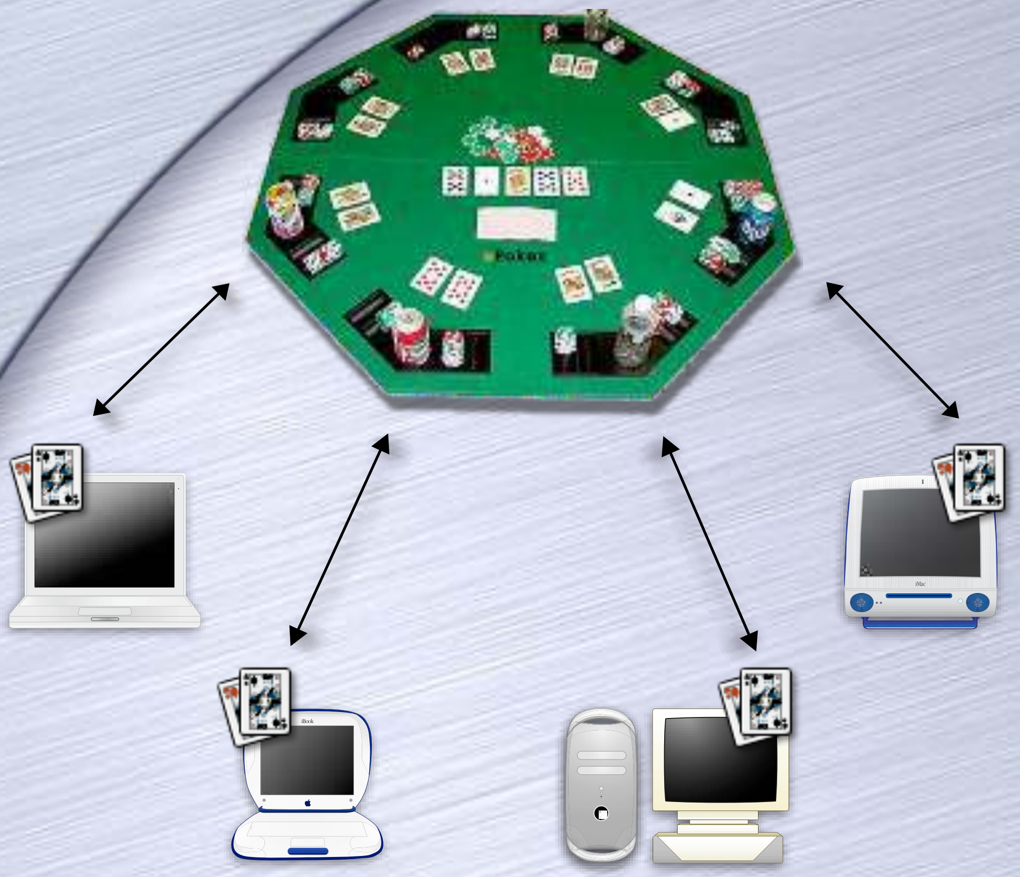
Secure Function Evaluation

- A general problem
- To compute a function of private inputs without revealing information about the inputs
  - Beyond what is revealed by the function



Poker With No Dealer?

- Need to ensure
  - Cards are shuffled and dealt correctly
  - Complete secrecy
  - No "cheating" by players, even if they collude
- No universally trusted dealer



The Ambitious Goal

- Without any trusted party, securely do
  - Distributed Data mining
  - E-commerc
  - Network G
  - E-voting
  - Secure fun
  - ....

Secure
Multi-Party Computation
(MPC)

Any task that uses a trusted party!

Mental Poker

Adi Shamir, Ronald L. Rivest and Leonard M. Adleman

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ABSTRACT

Can two potentially dishonest players play a fair game of poker without using any cards—for example, over the phone? This paper provides the following answers:

1. No. (Rigorous mathematical proof supplied.)
2. Yes. (Correct and complete protocol given.)

Emulating Trusted Computation

- Encryption/Authentication allow us to emulate a trusted channel
- Secure MPC: to emulate a source of trusted computation
  - Trusted means it will not “leak” a party’s information to others
  - And it will not cheat in the computation
- A tool for mutually distrusting parties to collaborate

Is it for Real?

- Getting there!
- Many implementations/platforms
  - Fairplay, VIFF, Sharemind, SCAPI, Obliv-C, JustGarble, SPDZ/MASCOT, ObliVM, ...
- See multipartycomputation.com

Is it for Real?

- And many practical systems using some form of MPC
  - Danish company Partisia with real-life deployments (since 2008)
    - sugar beet auction, electricity auction, spectrum auction, key management
  - A prototype for credit rating, supported by Danish banks
  - A proposal to the Estonian Tax & Customs Board
  - A proposal for Satellite Collision Analysis
  - Legislation in the US to use MPC for applications like a "higher education data system"
  - MPC Alliance
  - ...

MPC

- Several dimensions
  - Passive (Semi-Honest) vs. Active corruption
    - Passive: corrupt parties still follow the protocol
  - Honest-Majority vs. Unrestricted corruption
  - Information-theoretic vs. Computational security
  - ...

Security Definition

- Simplest case: Passive corruption, Information-theoretic security
  - In general, need honest-majority (or similar restriction)
- In passive corruption, the adversary can see the internals of all the corrupt parties, but cannot control their actions
  - Main concern will be secrecy (correctness is automatic, provided the protocol is correct in the absence of corruption)
  - Will ask for Perfect Secrecy
    - Similar to secret-sharing

Security Definition

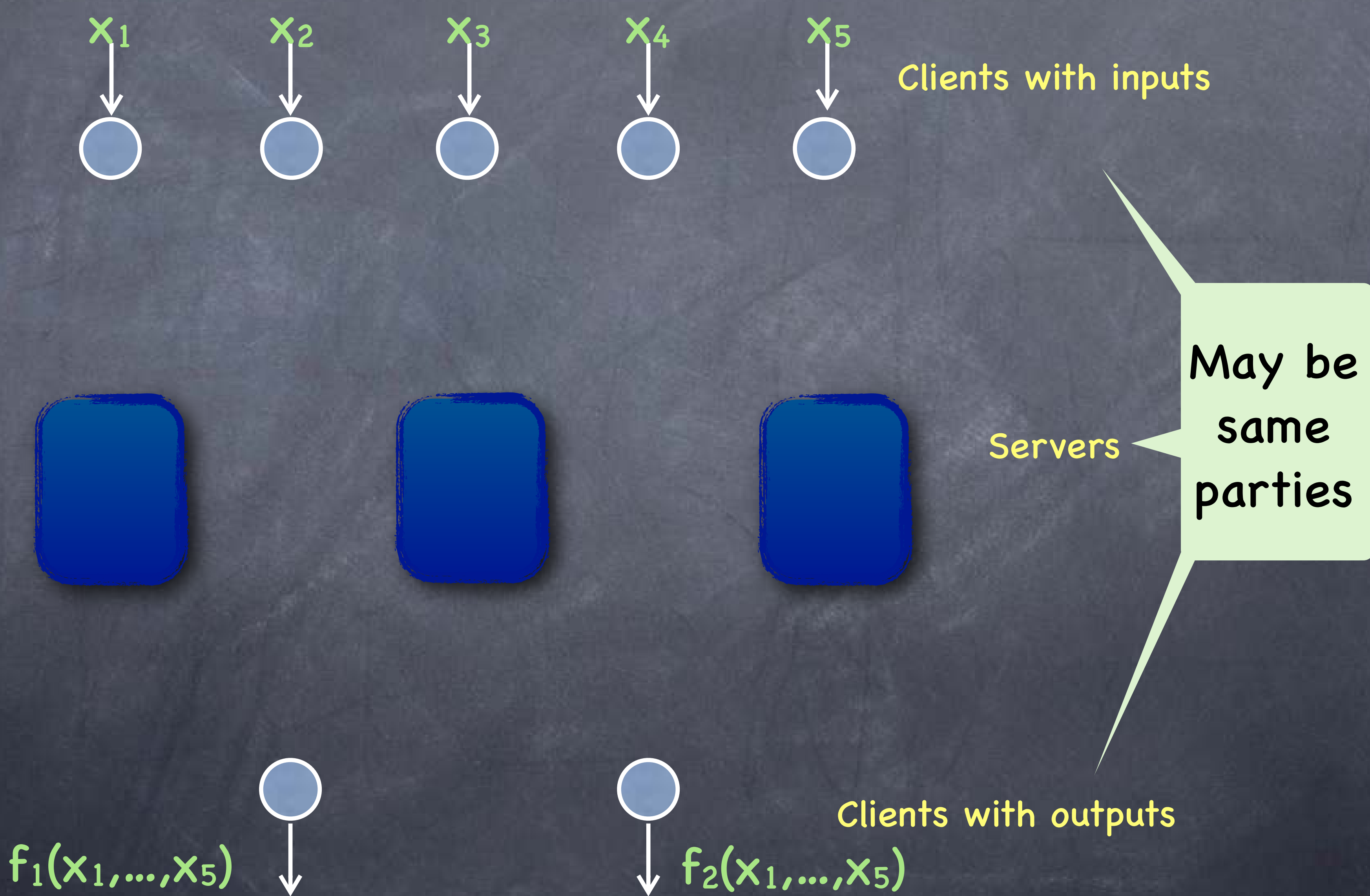
- Multiple parties in a protocol could be corrupt
  - Collusion
  - Modelled using a single adversary who corrupts the parties
    - Its view contains all the corrupt parties' views
- Security guarantee given against an "adversary structure"
  - Sets of parties that could be corrupt together

Security Definition

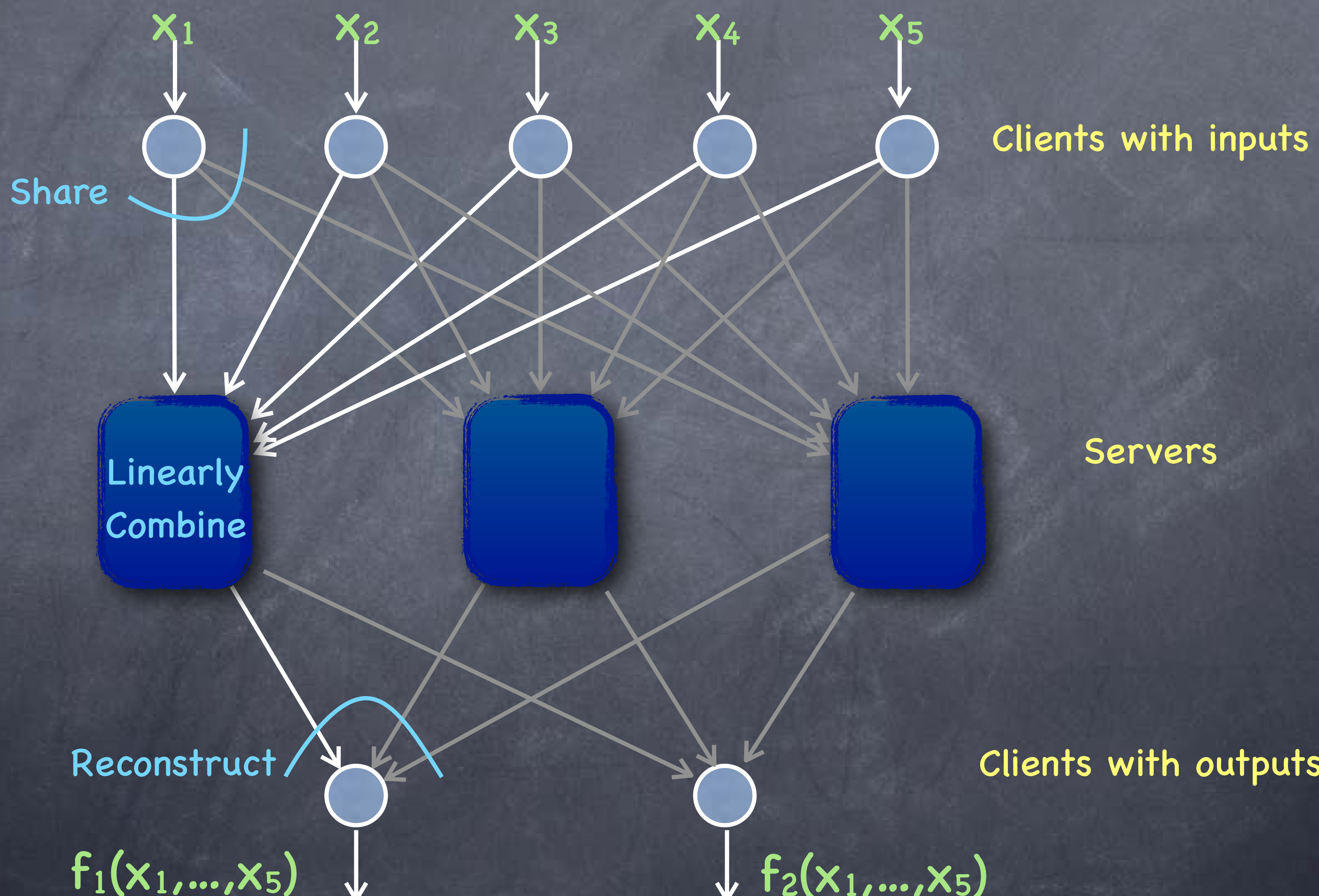
- For secret sharing we needed to formalise "x is secret"
- Now want to say: x is **secret except for f(x)** which is revealed
- $\forall$ x, x′ s.t. f(x)=f(x′), { view | input=x} $\equiv$ { view | input=x′ }
  - Here f(x) consists of the coordinates of input as well as the coordinates of outputs that correspond to corrupted parties
    - i.e., what the collusion is allowed to learn about x
  - Later: More complicated when considering active corruption and/or computational security

MPC for Linear Functions

- Client-server setting



MPC for Linear Functions: Using Linear Secret-Sharing



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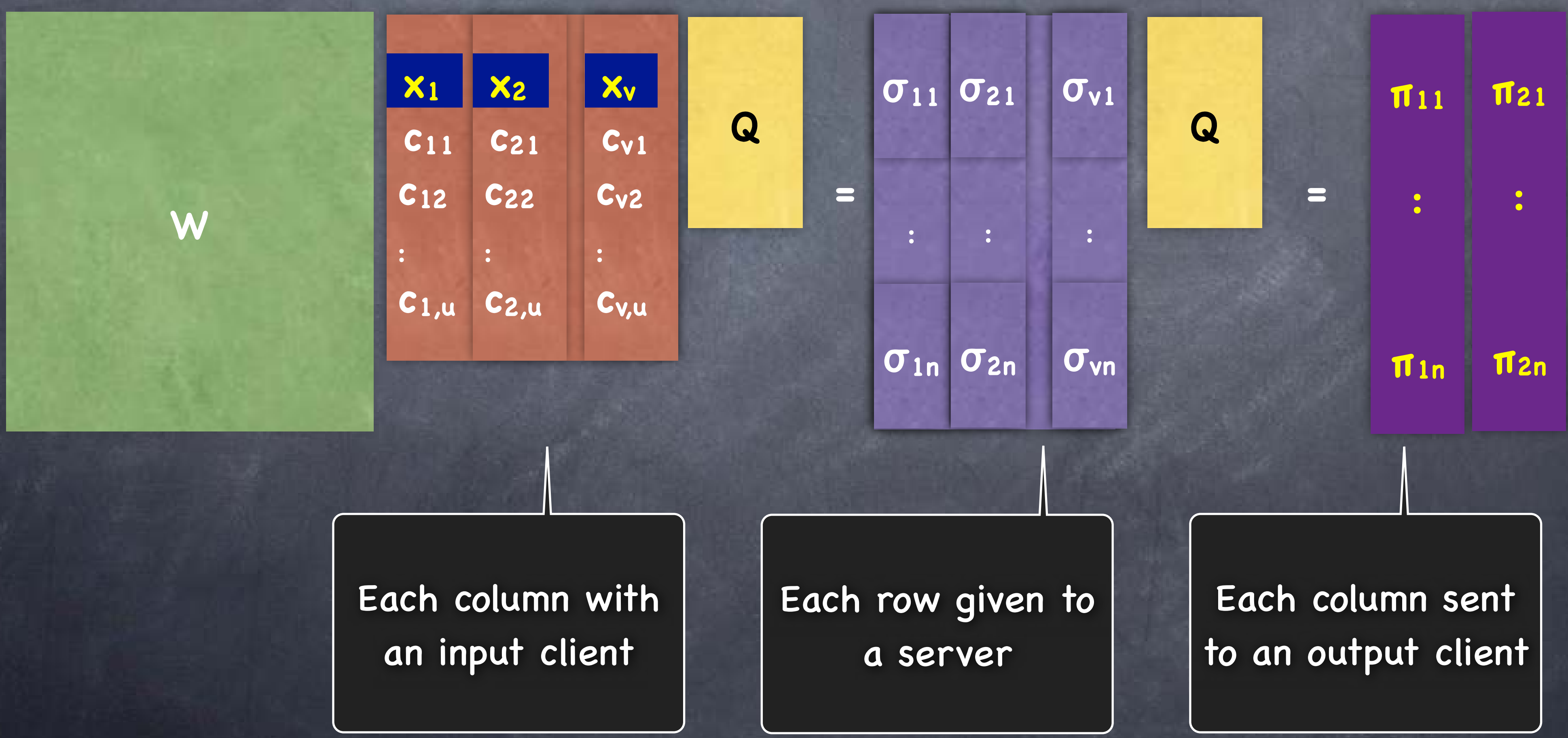
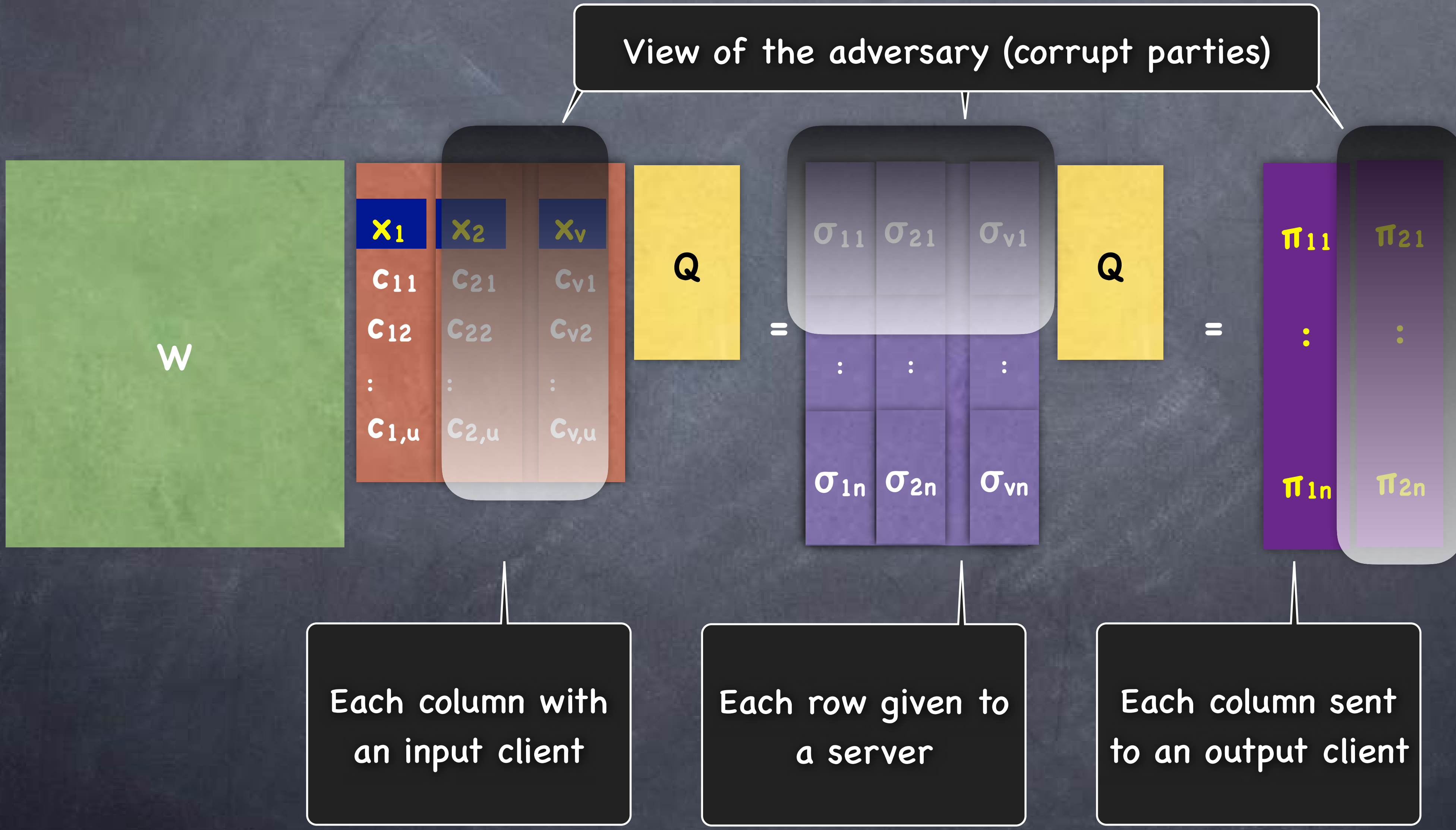


MPC for Linear Functions: Using Linear Secret-Sharing



Security

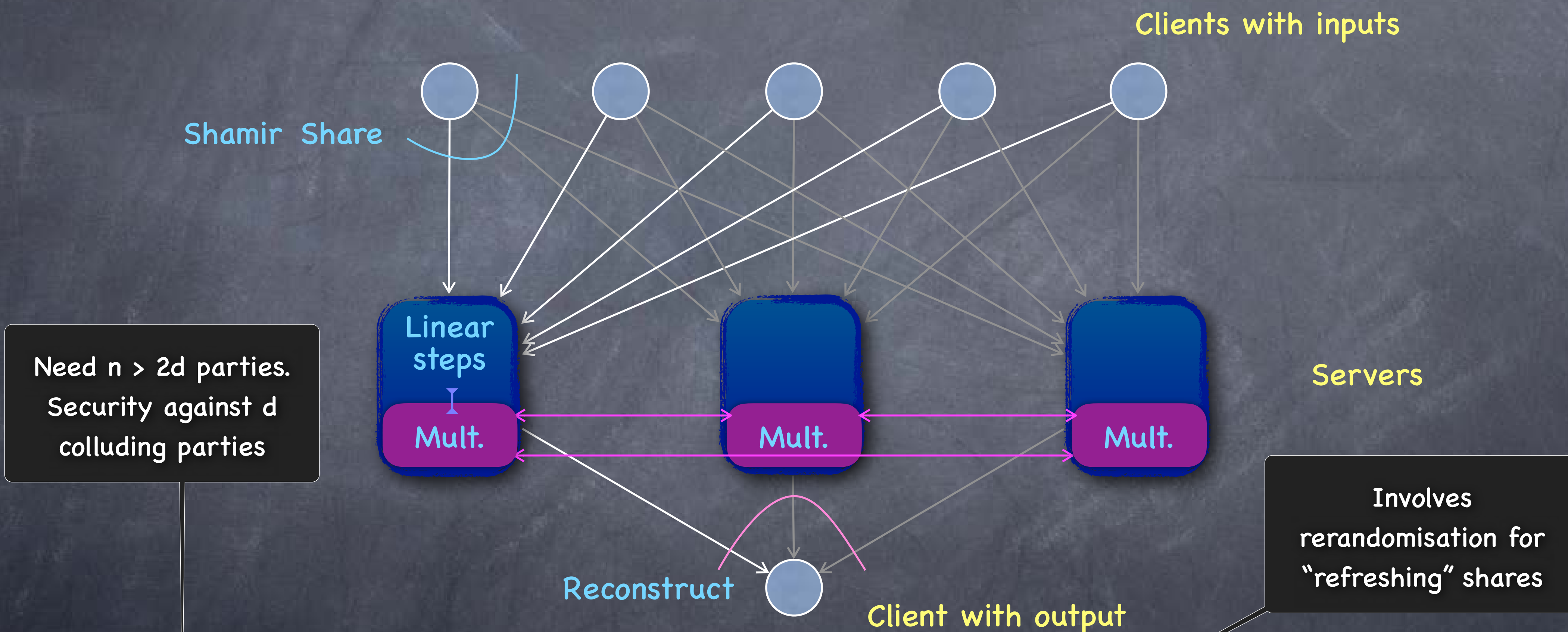
- Adversary allowed to corrupt any set of input and output clients and any subset T of servers s.t. T is not a privileged set (i.e., not in the access structure) for the secret-sharing scheme
- View of adversary should reveal nothing beyond the inputs and outputs of the corrupted clients
  - Claim: Consider any input y of corrupt clients. If x, x′ of uncorrupted clients such that for each corrupt output client i $f_i(x,y)=f_i(x',y)$, then the view of the adversary in the two cases are identically distributed
    - Because for any given view of the adversary, in each of the two cases (x and x′), the solution space of randomness is non-empty and then it has the same dimension
    - Exercise

MPC for General Functions?

- So far: a 2-round protocol for any <u>linear</u> function
  - Could use additive secret-sharing
- How about other functions?
- Any function over a finite field can be computed using addition and multiplication
  - Interested in functions which are efficiently computable
  - Arithmetic circuit: representation of the computation using addition and multiplication
- Goal: MPC Protocol for f, which is efficient if we are given an efficient arithmetic circuit for f

MPC from Shamir Secret-Sharing: Overview

- A function f given as a program with linear steps and multiplications: arithmetic circuit (over a finite field)



- Locally multiplying degree d shares of M_1 and M_2 gives a degree 2d share of $M_1 \cdot M_2$. Then securely switch back to a degree d share (involves communicating degree d shares of degree 2d shares)