# Advanced Tools from Modern Cryptography

Lecture 6

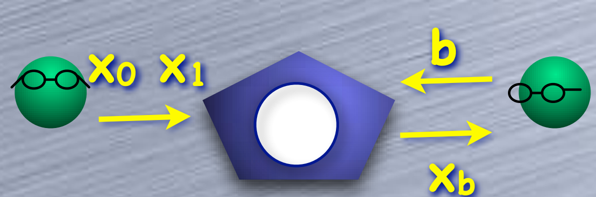
Secure Multi-Party Computation without Honest Majority: "GMW" Protocol

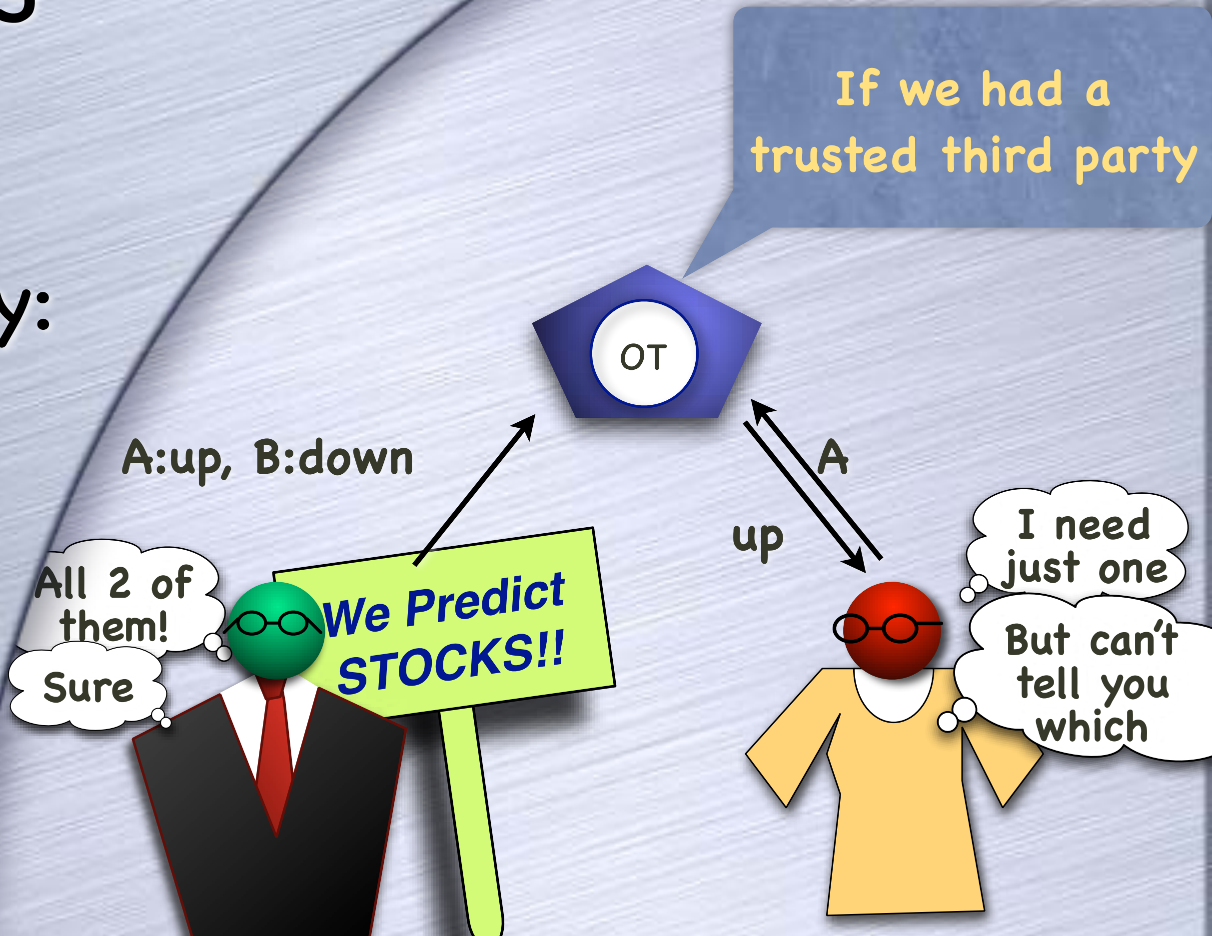
# MPC without Honest-Majority

- Plan (Still sticking with passive corruption):
- Two protocols, that are secure computationally
  - The "passive-GMW" protocol for any number of parties
  - A 2-party protocol using Yao's Garbled Circuits
  - Both rely on a computational primitive called Oblivious Transfer
- Today: OT and Passive-GMW

# Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"



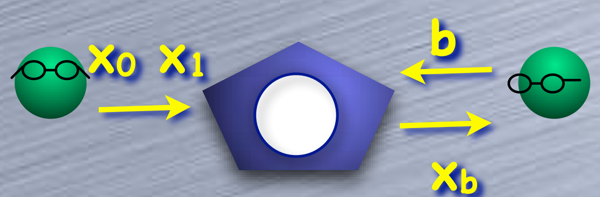



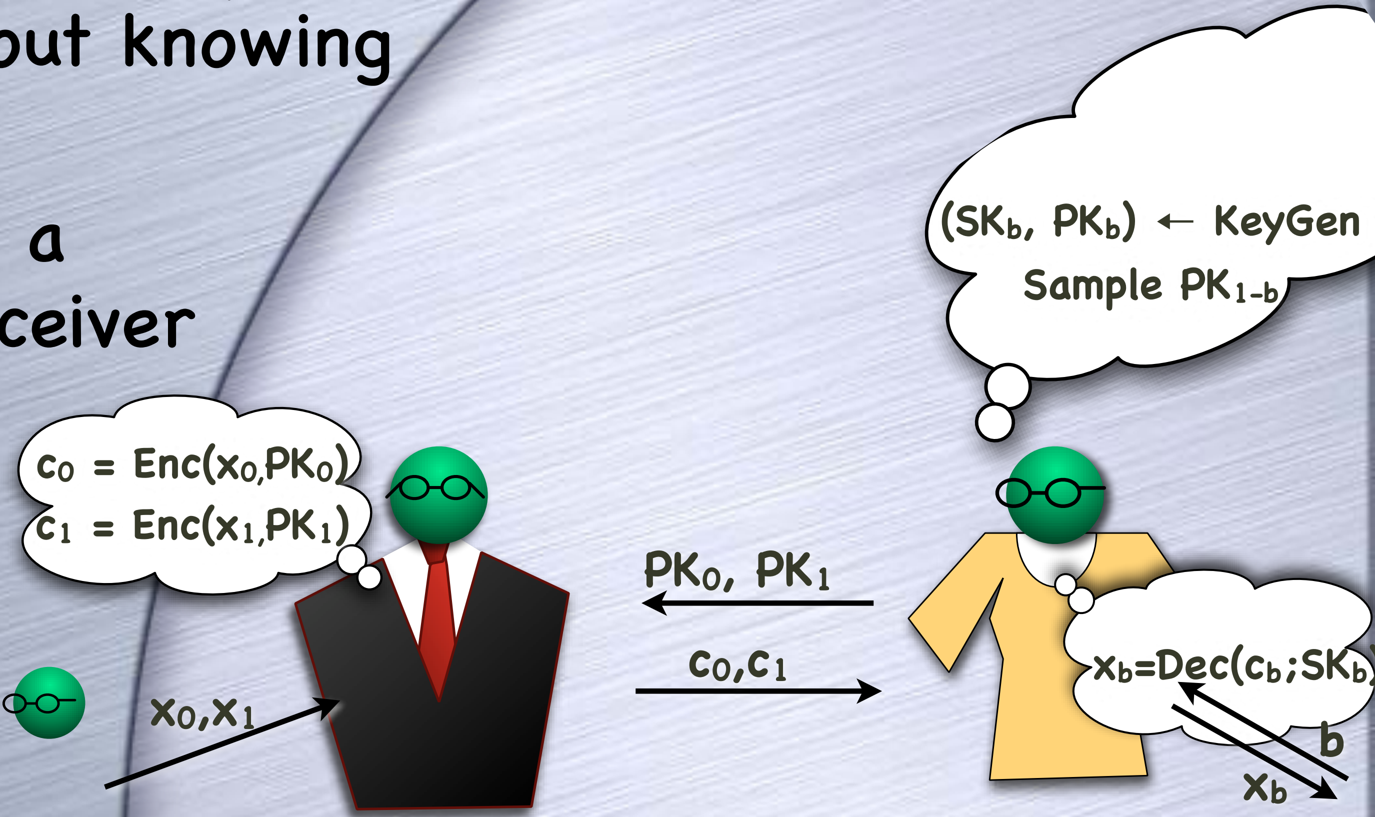
# Is OT Possible?

- No information theoretically secure 2-party protocol for OT
  - Because OT can be used to carry out information-theoretically secure 2-party AND (coming up)
- Computationally secure OT protocols exist under various computational hardness assumptions
  - Will define computational security of MPC later, comparing the protocol to the ideal functionality

# An OT Protocol (against passive corruption)

- Using (a special) public-key encryption
  - In which one can sample a public-key without knowing secret-key
- $c_{1-b}$ inscrutable to a passive corrupt receiver
- Sender learns nothing about b



# Why is OT Useful?
# Naïve 2PC from OT

- Say Alice's input x, Bob's input y, and only Bob should learn f(x,y)
- Alice (who knows x, but not y) prepares a table for f(x,·) with $D = 2^{|y|}$ entries (one for each y)
- Bob uses y to decide which entry in the table to pick up using 1-out-of-D OT (without learning the other entries)
- Bob learns only f(x,y) (in addition to y). Alice learns nothing beyond x.
- OT captures the essence of MPC:
  Secure computation of any function f can be **reduced** to OT

  Secure protocol for f using access to ideal OT
- Problem: D is exponentially large in |y|
  - Plan: somehow exploit efficient computation (e.g., circuit) of f

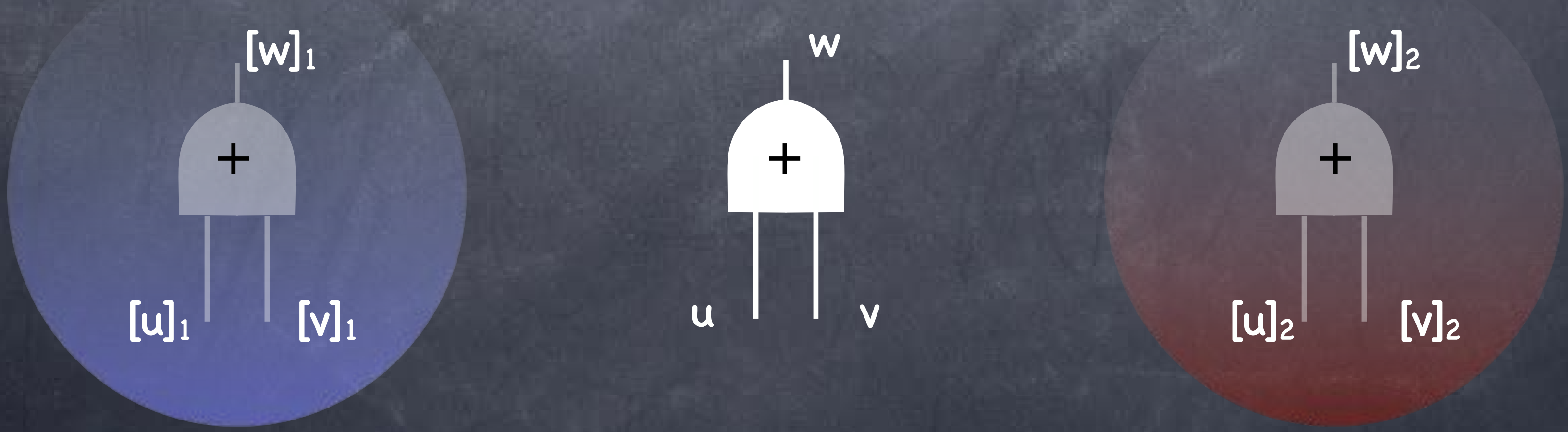
Goldreich-Micali-Wigderson (1987). As simplified in later work.

# Passive GMW

- Passive secure MPC based on OT, without any other computational assumptions
  - Will assume that a trusted party is available to carry out OT between any pair of parties (replaced by a cryptographic protocol, later)
  - Tolerates any number of corrupt parties
- Idea: Computing on **additively secret-shared values**
  - For a variable (wire value) s, will write $[s]_i$ to denote its share held by the $i^{th}$ party

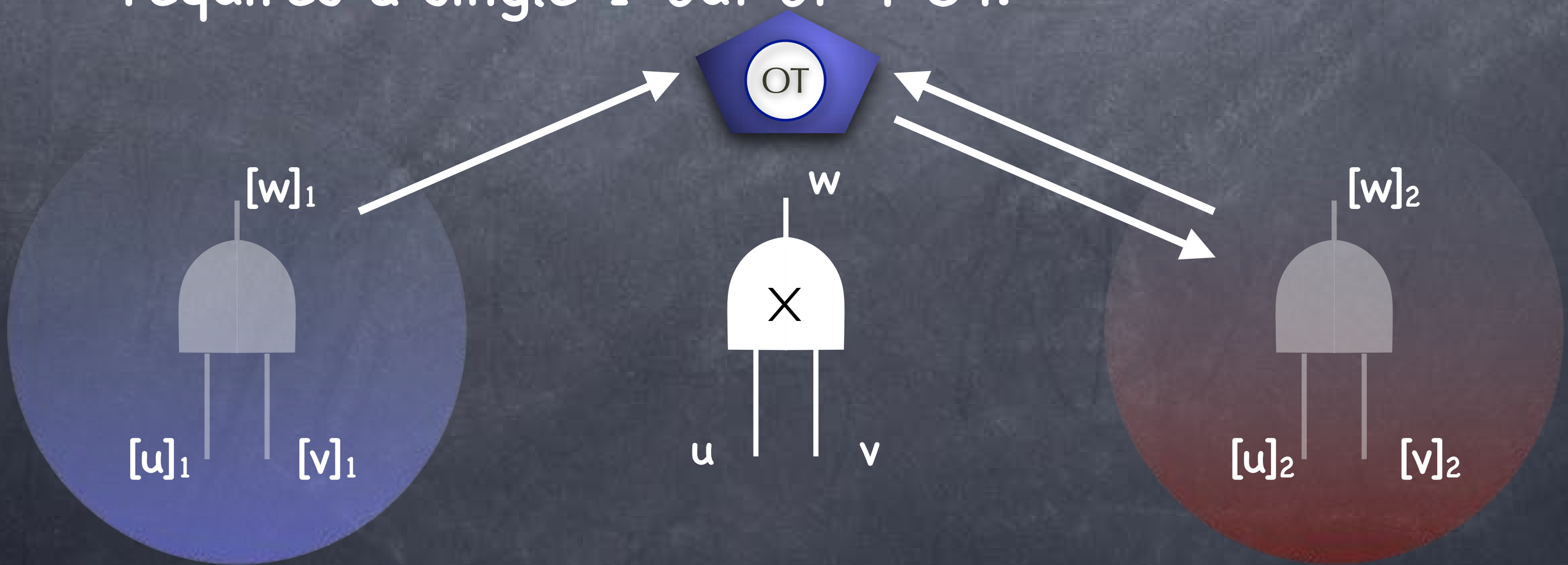
# Computing on Shares: 2 Parties

- Let gates be + & × (XOR & AND for Boolean circuits)
- Plan: Similar to BGW: shares of each wire value will be computed, with Alice holding one share and Bob the other. At the end, Alice sends her share of output wire to Bob.
- w = u + v : Each one locally computes $[w]_i = [u]_i + [v]_i$



# Computing on Shares: 2 Parties

- What about $w = u \times v$ ?
  - $[w]_1 + [w]_2 = ( [u]_1 + [u]_2 ) \times ( [v]_1 + [v]_2 )$
  - Alice picks $[w]_1$ and lets Bob compute $[w]_2$ using the naive (proof-of-concept) protocol
    - Note: Bob's input is $([u]_2,[v]_2)$. Over the binary field, this requires a single 1-out-of-4 OT.



# Passive GMW

- Secure?
- View of Alice:
  - Input x and random values it picks through out the protocol ✓
- View of Bob:
  - Input y and random values it picks through out the protocol
  - A random value (picked via OT) for each wire out of a × gate
  - f(x,y) - own share, for the output wire
- This distribution is the same for x, x′ if f(x,y)=f(x′,y) ✓
- Exercise: What goes wrong in the above claim if Alice reuses $[w]_1$ for two × gates?

# Computing on Shares: m Parties

- m-way sharing: $s = [s]_1 + ... + [s]_m$
- Addition, local as before
- Multiplication: For $w = u \times v$
  $[w]_1 + .. + [w]_m = ( [u]_1 + .. + [u]_m ) \times ( [v]_1 + .. + [v]_m )$
  - Party i computes $[u]_i[v]_i$
  - For every pair (i,j), i≠j, Party i picks random $a_{ij}$ and lets Party j securely compute $b_{ij}$ s.t. $a_{ij} + b_{ij} = [u]_i[v]_j$ using the naive protocol (a single 1-out-of-2 OT)
  - Party i sets $[w]_i = [u]_i[v]_i + \Sigma_j ( a_{ij} + b_{ji} )$

# Computing on Shares: m Parties Arithmetic Version

- m-way sharing: $s = [s]_1 + \ldots + [s]_m$


- Addition, local as before
- Multiplication: For $w = u \times v$

  $[w]_1 + .. + [w]_m = ( [u]_1 + .. + [u]_m ) \times ( [v]_1 + .. + [v]_m )$

  - Party i computes $[u]_i[v]_i$
  - For every pair (i,j), i≠j, Party i picks random $a_{ij}$ and lets Party j securely compute $b_{ij}$ s.t. $a_{ij} + b_{ij} = [u]_i[v]_j$ using Oblivious Linear-function Evaluation (OLE)
  - Party i sets $[w]_i = [u]_i[v]_i + \Sigma_j ( a_{ij} + b_{ji} )$

# MPC for Passive Corruption

- Story so far:
  - For honest-majority: Information-theoretically secure protocol, using Shamir secret-sharing [BGW]
  - Without honest-majority: Using Oblivious Transfer (OT), using additive secret-sharing [GMW]

Oblivious Linear-function Evaluation (OLE) for arithmetic over larger fields

- Up next
  - A 2-party protocol (so no honest-majority) using Oblivious Transfer and Yao's Garbled Circuits
    - Uses additional computational primitives and is limited to arithmetic circuits over small fields (e.g., boolean circuits)
    - Needs just one round of interaction