Advanced Tools from Modern Cryptography

Lecture 7

Secure 2-Party Computation:

Yao's Garbled Circuit

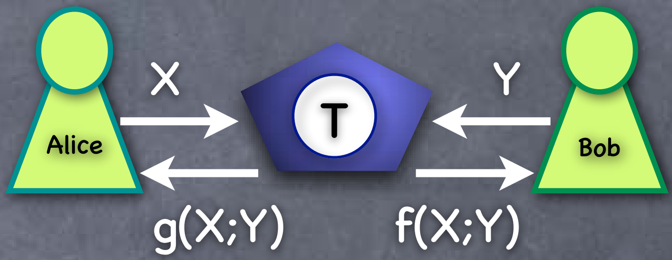
Recall

MPC without Honest-Majority

- Plan (Still sticking with passive corruption):
- Two protocols, that are secure computationally
  - The "passive-GMW" protocol for any number of parties
  - A 2-party protocol using Yao's Garbled Circuits
  - Both rely on a computational primitive called Oblivious Transfer
- Last time: OT and Passive-GMW
  - (Not exactly the version from the GMW'87 paper.)
- Today: 2-Party protocol using Yao's Garbled Circuits

2-Party SFE

- Secure Function Evaluation (SFE) IDEAL:
  - Trusted party takes (X;Y). Outputs g(X;Y) to Alice, f(X;Y) to Bob



    - Randomized Functions: g(X;Y;r) and f(X;Y;r) s.t. neither party knows r (beyond what is revealed by output)
- OT is an instance of a (deterministic) 2-party SFE
  - $g(x_0,x_1;b)$ = none; $f(x_0,x_1;b) = x_b$
  - Single-Output SFE: only one party gets any output

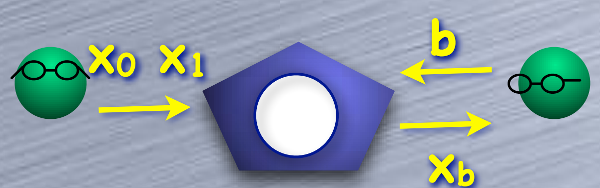
2-Party SFE

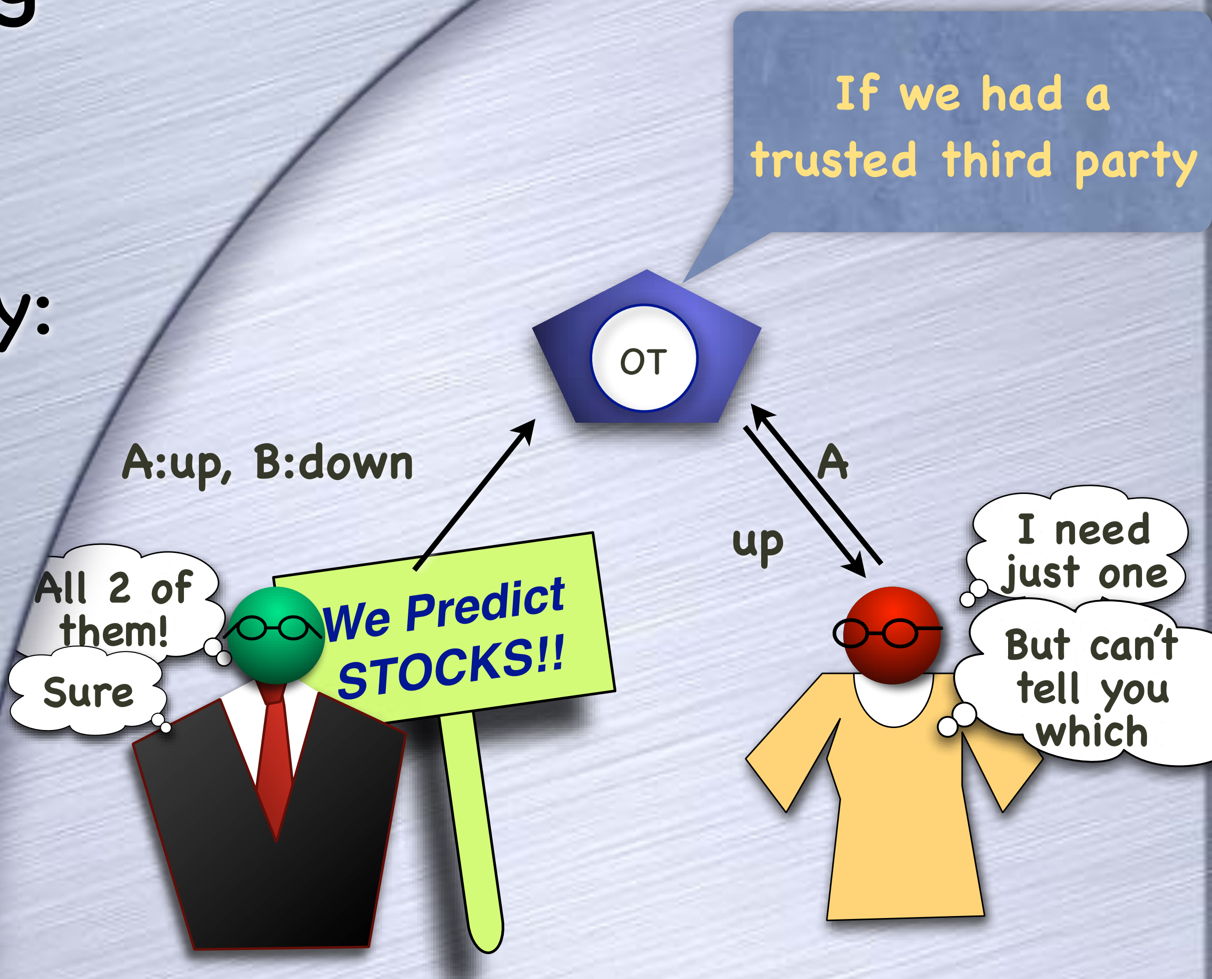
- Can reduce general SFE (even randomized) to a single-output deterministic SFE
  - $f'(X, M, r_1; Y, r_2) = (\ g(X; Y; r_1 \oplus r_2) \oplus M,\ f(X; Y; r_1 \oplus r_2)\ )$. Compute $f'(X, M, r_1; Y, r_2)$ with random M, r_1, r_2
  - Bob sends $g(X, Y; r_1 \oplus r_2) \oplus M$ to Alice
  - Passive secure
  - For active security too: f' authenticates (one-time MAC) as well as encrypts $g(X; Y; r_1 \oplus r_2)$ using keys input by Alice
  - Generalizes to more than 2 parties too [Exercise]
- Yao: Reduces single-output deterministic 2-party SFE to OT
  - Single round of interaction, but with only computational security (cf. GMW: information-theoretic, but many rounds)

Recall

Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"





Recall

Why is OT Useful?
Naïve 2PC from OT

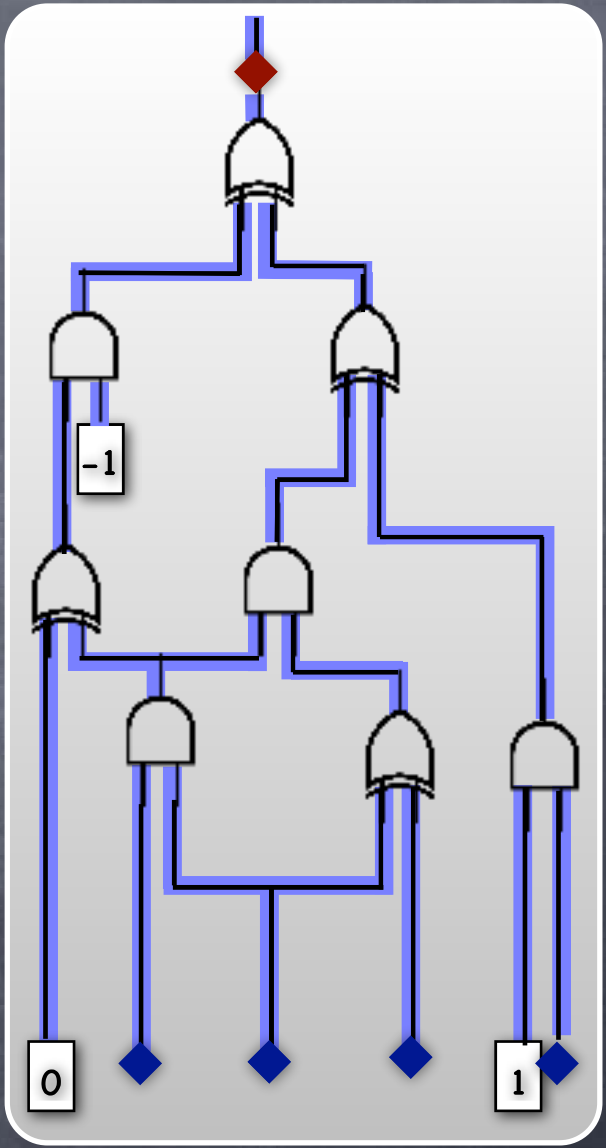
- Say Alice's input x, Bob's input y, and only Bob should learn f(x,y)
- Alice (who knows x, but not y) prepares a table for f(x,·) with $D = 2^{|y|}$ entries (one for each y)
- Bob uses y to decide which entry in the table to pick up using 1-out-of-D OT (without learning the other entries)
- Bob learns only f(x,y) (in addition to y). Alice learns nothing beyond x.
- OT captures the essence of MPC:
  Secure computation of any function f can be **reduced** to OT

  Secure protocol for f using access to ideal OT
- Problem: D is exponentially large in |y|
  - Plan: somehow exploit efficient computation (e.g., circuit) of f

Recall

Functions as Circuits

- Directed acyclic graph
  - Nodes: multiplication and addition gates, constant gates, inputs, output(s)
  - Edges: wires carrying values from F
  - Each wire comes out of a unique gate, but a wire might fan-out
  - Can evaluate wires according to a topologically sorted order of gates they come out of



2-Party MPC for General Circuits

	0	1
0	0	1
1	1	1

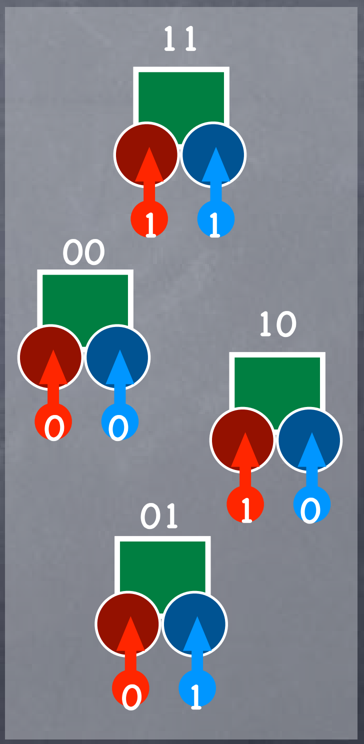
- "General": evaluate any arbitrary (boolean) circuit
  - One-sided output: both parties give inputs, only one party gets outputs
  - Either party maybe corrupted passively
- Consider evaluating OR (single gate circuit)
  - Alice holds x=a, Bob has y=b; Bob should get OR(x,y)

A Physical Protocol

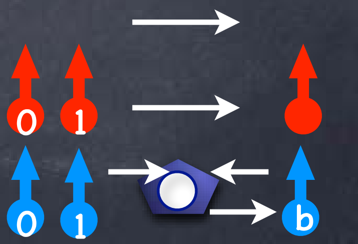
- Alice prepares 4 boxes B_{xy} corresponding to 4 possible input scenarios, and 4 padlocks/keys $K_{x=0}$, $K_{x=1}$, $K_{y=0}$ and $K_{y=1}$

	0	1
0	0	1
1	1	1

- Inside $B_{xy=ab}$ she places the bit OR(a,b) and locks it with two padlocks $K_{x=a}$ and $K_{y=b}$ (need to open both to open the box)

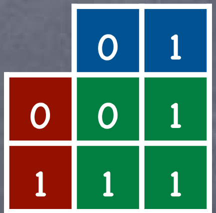


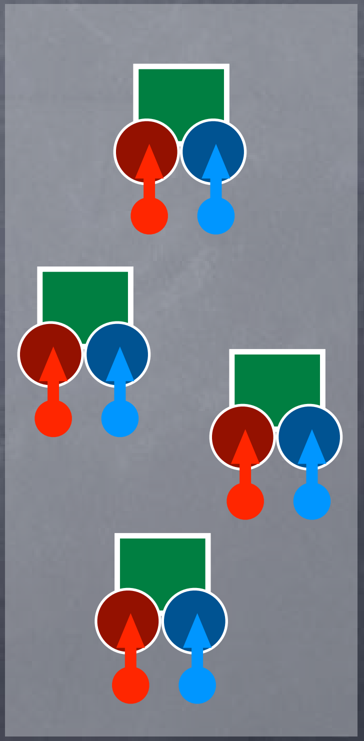
- She un-labels the four boxes and sends them in random order to Bob. Also sends the key $K_{x=a}$ (labeled only as K_x).
    - So far Bob gets no information
- Bob "obliviously picks up" $K_{y=b}$, and tries the two keys K_x,K_y on the four boxes. For one box both locks open and he gets the output.

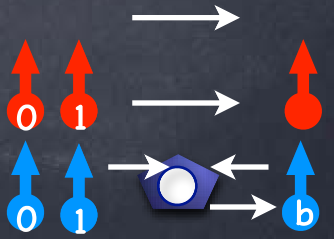


A Physical Protocol

- Secure?
- For curious Alice: only influence from Bob is when he picks up his key $K_{y=b}$
  - But this is done "obliviously", so she learns nothing
- For curious Bob: What he sees is predictable (i.e., can be simulated), given the final outcome
  - What Bob sees: His key opens K_y in two boxes, Alice's opens K_x in two boxes; only one random box fully opens. It has the outcome.
  - Note when y=1, cases x=0 and x=1 appear same

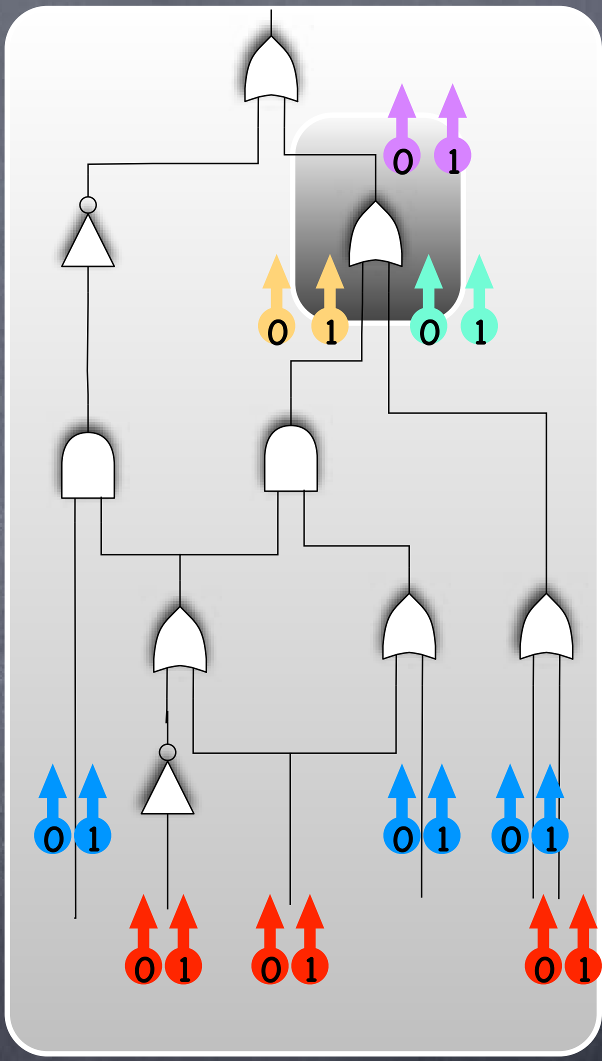




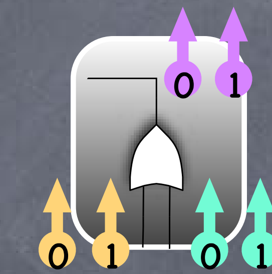


Larger Circuits

- Idea: For each gate in the circuit Alice will prepare locked boxes, but will use it to keep keys for the next gate
  - For each wire w in the circuit (i.e., input wires, or output of a gate) pick 2 keys $K_{w=0}$ and $K_{w=1}$



Larger Circuits



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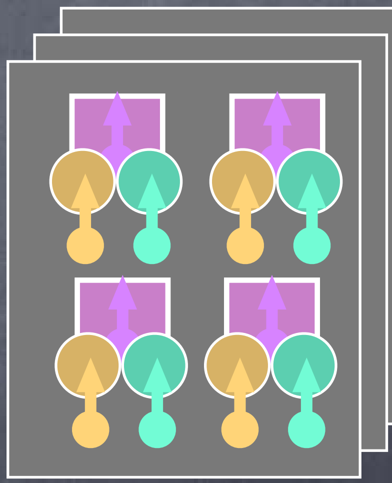
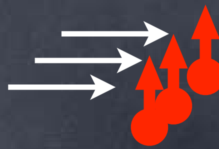
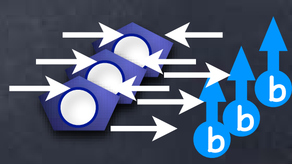


- For each gate G with input wires (u,v) and output wire w, prepare 4 boxes B_{uv} and place $K_{w=G(a,b)}$ inside box $B_{uv=ab}$. Lock $B_{uv=ab}$ with keys $K_{u=a}$ and $K_{v=b}$

- Give to Bob: Boxes for each gate, one key for each of Alice's input wires



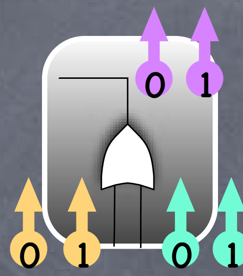
  - Obliviously: one key for each of Bob's input wires

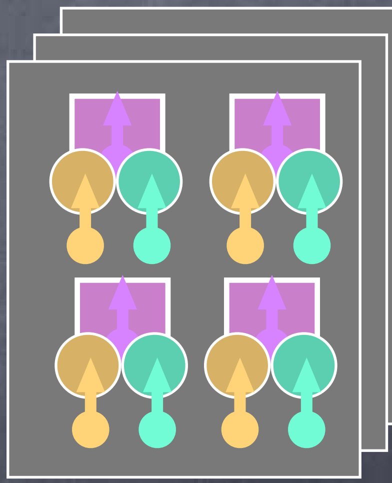


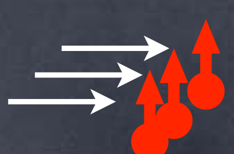
- Boxes for output gates have values instead of keys

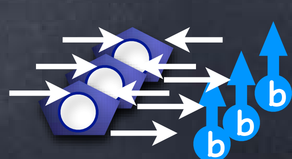
Larger Circuits

- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds
  - Gets output from a box for the output gate
- Security similar to before
  - Curious Alice sees nothing
  - Bob can simulate his view given final output: Bob could prepare boxes and keys (stuffing unopenable boxes arbitrarily); for an output gate, place the output bit in the box that opens









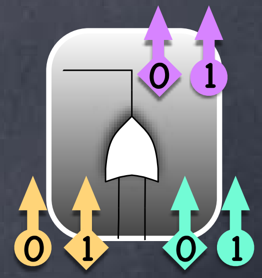
Garbled Circuit

Coming up

- That was too physical!
- Yao's Garbled circuit: boxes/keys replaced by Symmetric Key Encryption (specifically, using a Pseudorandom Function or PRF)
  - $\mathrm{Enc}_K(m) = \mathrm{PRF}_K(\mathrm{index}) \oplus m$, where index is a wire index (distinct for different wires fanning-out of the same gate)
  - Double lock: $\mathrm{Enc}_{K_x}(\mathrm{Enc}_{K_y}(m))$
  - PRF in practice: a block-cipher, like AES
- Uses Oblivious Transfer for strings: For passive security, can just repeat bit-OT several times to transfer longer keys
- Security? Need to first define security when computational primitives are used! (Next time!)

Garbled Circuit

- One issue when using encryption instead of locks
  - Given four doubly locked boxes (in random order) and two keys, we simply tried opening all locks until one box fully opened
  - With encryption, cannot quite tell if a box opened or not! Outcome of decryption looks random in either case.
  - Simple solution: encode the keys so that wrong decryption does not result in outputs that look like valid encoding of keys
  - Better solution: For each wire 0 & 1 keys have distinct "shape" labels, assigned at random. Each locked box marked with the shape of the two keys needed to unlock it.



Pseudorandomness

- Basic notions in (symmetric-key) cryptography
  - A Pseudorandomness Generator (PRG) and a Pseudorandom Function (PRF)
- PRG takes a short seed and (deterministically) outputs a longer string: $G_k: \{0,1\}^k \rightarrow \{0,1\}^{n(k)}$ where $n(k) > k$
- A PRF is essentially a PRG with a "long" output, with an extra input (index) which specifies a block to be selected from this output
- Security definitions based on computational indistinguishability

Recall

Indistinguishability

$A_k \approx B_k$

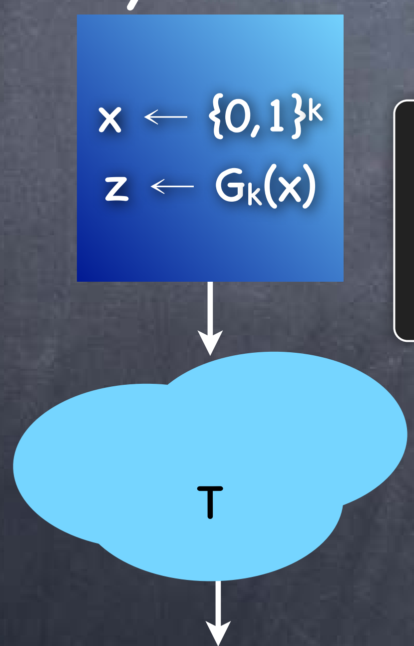
- Distribution ensembles $\{A_k\}$, $\{B_k\}$ **computationally indistinguishable** if $\forall$ Probabilistic Polynomial Time tests T, $\exists$ negligible $\nu(k)$ s.t.

$$| \Pr_{x \leftarrow A_k}[T(x)=1] - \Pr_{x \leftarrow B_k}[T(x)=1] | \leq \nu(k)$$

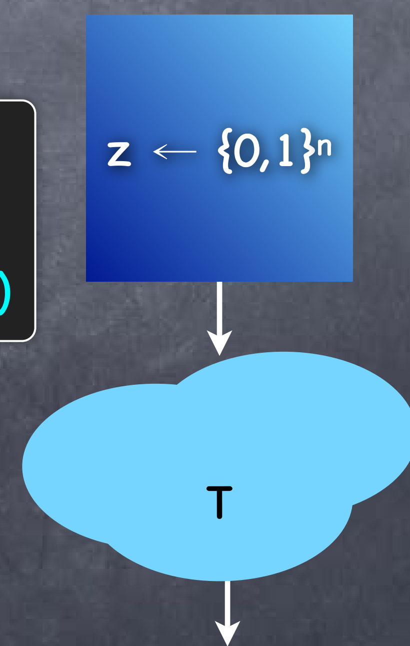
Pseudorandomness Generator (PRG)

- Takes a short seed and (deterministically) outputs a long string
  - $G_k: \{0,1\}^k \longrightarrow \{0,1\}^{n(k)}$ where $n(k) > k$
- Security definition: $\boxed{\{G_k(x)\}_{x \leftarrow \{0,1\}^k} \approx U_{n(k)}}$

$\{G_k(x)\}_{x \leftarrow \{0,1\}^k}$ cannot be statistically indistinguishable from $U_{n(k)}$ unless $n(k) \leq k$ (Why?)



REAL



IDEAL

$\forall$ PPT

REAL $\approx$ IDEAL

Pseudorandom Function (PRF)

- A PRF is essentially a PRG with a "long" output
  - A function F(s;i) outputs the i^{th} block of the pseudorandom string corresponding to seed s
  - When the number of blocks is small (polynomial in the security parameter), this is the same as a PRG with a longer output
    - This suffices for Garbled Circuits
- More generally a PRF supports exponentially many blocks (i.e., large domain for i)
  - Needs a new security definition as the adversary can't be given the entire string

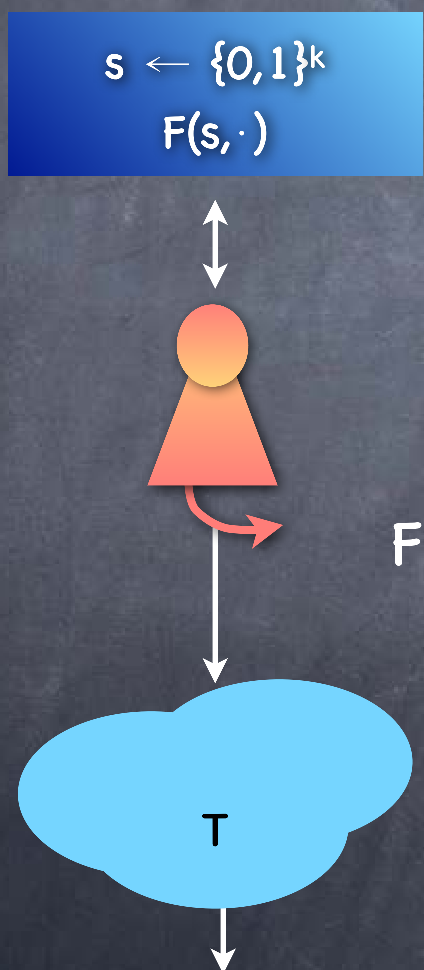
Pseudorandom Function (PRF)

- A compact representation of an exponentially long (pseudorandom) string
  - Allows "random-access" (instead of just sequential access)
    - A function F(s;i) outputs the i^{th} block of the pseudorandom string corresponding to seed s
    - Exponentially many blocks (i.e., large domain for i)

If the domain of i is polynomial sized (as is sufficient for Garbled Circuits), can implement PRF using a PRG

- Security definition
  - Need to define pseudorandomness for a function (not a string)
  - Idea: the view of an adversary arbitrarily interacting with the function is indistinguishable from its view when interacting with a random function

Pseudorandom Function (PRF)



REAL

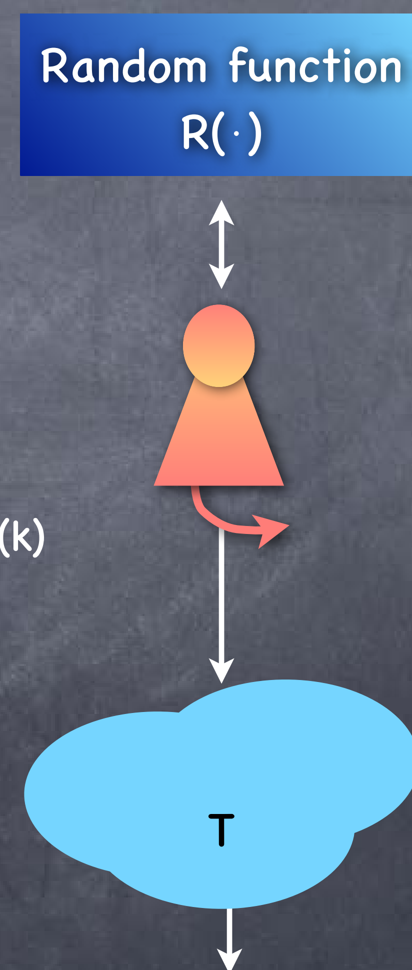
$F: \{0,1\}^k \times \{0,1\}^{m(k)} \rightarrow \{0,1\}^{n(k)}$

is a PRF if

$\forall$ PPT

$\forall$ PPT

REAL ≈ IDEAL



IDEAL