Advanced Tools from Modern Cryptography

Lecture 10

MPC: GMW Paradigm. Composition.

MPC: Story So Far

- Security against passive corruption
  - "Basic GMW" using OT, Yao's Garbled Circuits using OT, "Passive-BGW" with honest majority
- Security against active corruption (2-party, no honest majority)
  - Commitment, ZK proofs
- Today
  - GMW paradigm for upgrading from passive security to active security
  - Composition

GMW Paradigm

- Run a passive-secure protocol Π, but let each party "verify" that the others are following the protocol correctly
  - Correctly: pick arbitrary inputs and arbitrary randomness first, but then follow the specified program
- Need to prove that each message was correctly computed, right when it is sent
  - If proofs checked only at the end, too late!
- Proving ∃ input, rand, s.t. next-message$_\Pi$(input,rand,messages) equals the message being sent
  - Should use the same input and randomness through out!
  - ZK proofs not enough

Commit & Prove

- To prove ∃ input, rand, s.t. next-message$_\Pi$(input,rand,messages) equals the message being sent
- Commit-and-Prove functionality: F_{CaP}
  - Alice sends v to F_{CaP}, which sends "committed" to Bob
  - Subsequently, for i=1,2,... Alice sends a function f_i (represented as a circuit) to F_{CaP}, which sends $(f_i, f_i(v))$ to Bob
    - More generally, Alice sends (f_i, w_i) and F_{CaP} sends $(f_i, f_i(v, w_i))$ to Bob (i.e., without revealing w_i)
  - Note: same v used in all rounds

Exercise

- Could "securely implement" F_{CaP} using a "plain" commitment of v (i.e., not using F_{com}), and proving statements about it using F_{ZK}

GMW Paradigm

- Run a passive-secure protocol Π, but let each party "verify" that the others are following the protocol correctly
  - Correctly: pick arbitrary inputs and arbitrary randomness first, but then follow the specified program
- Each party proves to everyone, using F_{CaP}, that each message was correctly computed, for the same committed inputs and randomness
  - f_i defined so that $f_i(v) = 1$ iff Π produces message m_i on input/randomness v for the proving party, given the transcript so far (Π, m_i and the transcript are hard-coded into f_i)
    - Since verifiers need to refer to all the messages received by the prover, all communication in Π assumed to be over public channels (say, using public-key cryptography)

Composition

- We built an active-secure protocol using access to ideal F_{CaP} functionality
  - Is it OK to "replace" it by a secure protocol for F_{CaP}?
  - More generally, can we replace an ideal functionality running in an arbitrary environment with a secure protocol?
  - Depends on the exact definition of security!
    - Looking ahead: OK for both UC security and passive security
    - **Not OK for standalone security**
      - OK if it can be forced that no sub-protocol sessions run concurrently at any point (sequential composition)

Composition Issues
An example

- An auction, with Alice and Bob bidding:
  - A bid is an integer in the range [0,100]
  - Alice can bid only even integers and Bob odd integers
  - Person with the higher bid wins
- Goal: find out the winning bid (winner & amount) without revealing anything more about the losing bid (beyond what is revealed by the winning bid)
  - F_{max} : Output the higher bid to both parties (Domains are disjoint)

Composition Issues
An example

- Secure protocol:
  - Count down from 100
  - At each even round Alice announces whether her bid equals the current count; at each odd round Bob does the same
  - Stop if a party says yes
- Dutch flower auction

Perfect Standalone Security
But doesn't compose!

Attack on Dutch Flower Auction

- Alice and Bob are taking part in two auctions
- Alice's goal: ensure that Bob wins at least one auction with some bid z, and the winning bid in the other auction $\in \{z, z-1\}$
- Easy in the protocol: run the two protocols lockstep. Wait till Bob says yes in one. Done if Bob says yes in the other simultaneously. Else Alice will say yes in the next round.
- Why is this an attack?
  - Impossible for Alice to ensure this in IDEAL!

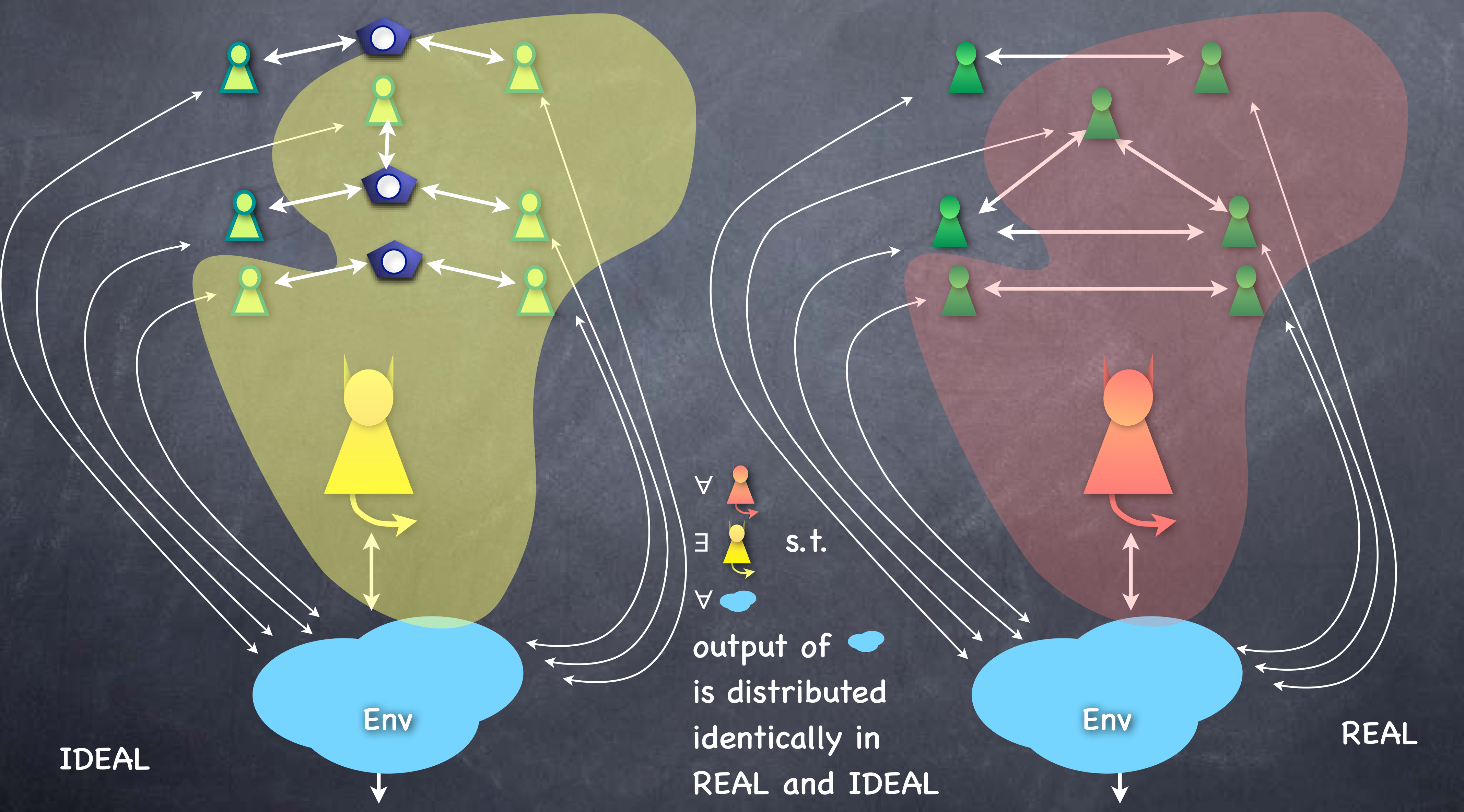
Attack on Dutch Flower Auction

- Alice's goal: ensure that Bob wins at least one auction with some bid z, and the winning bid in the other auction $\in \{z, z-1\}$
- Impossible to ensure this in IDEAL!
- Alice can get a result in one session, before running the other. But what should she submit as her input x in the first one?
  - Trouble if $x \neq 0$, because she could win (i.e., $z-1=x$) and Bob's input in the other session may be $\neq x+1$
  - Trouble if $x=0$, because Bob could win with input 1 (i.e., $z=1$) and in the other session his input > 1

Composition Issues

- Standalone security definition does not ensure security when composed
- Different modes of composition
  - Sequential composition: protocols executed one after the other. Adversary communicates with the environment between executions.
  - Concurrent composition: multiple sessions (typically of the same protocol) are active at the same time, and the adversary can coordinate its actions across the sessions

Concurrent Executions



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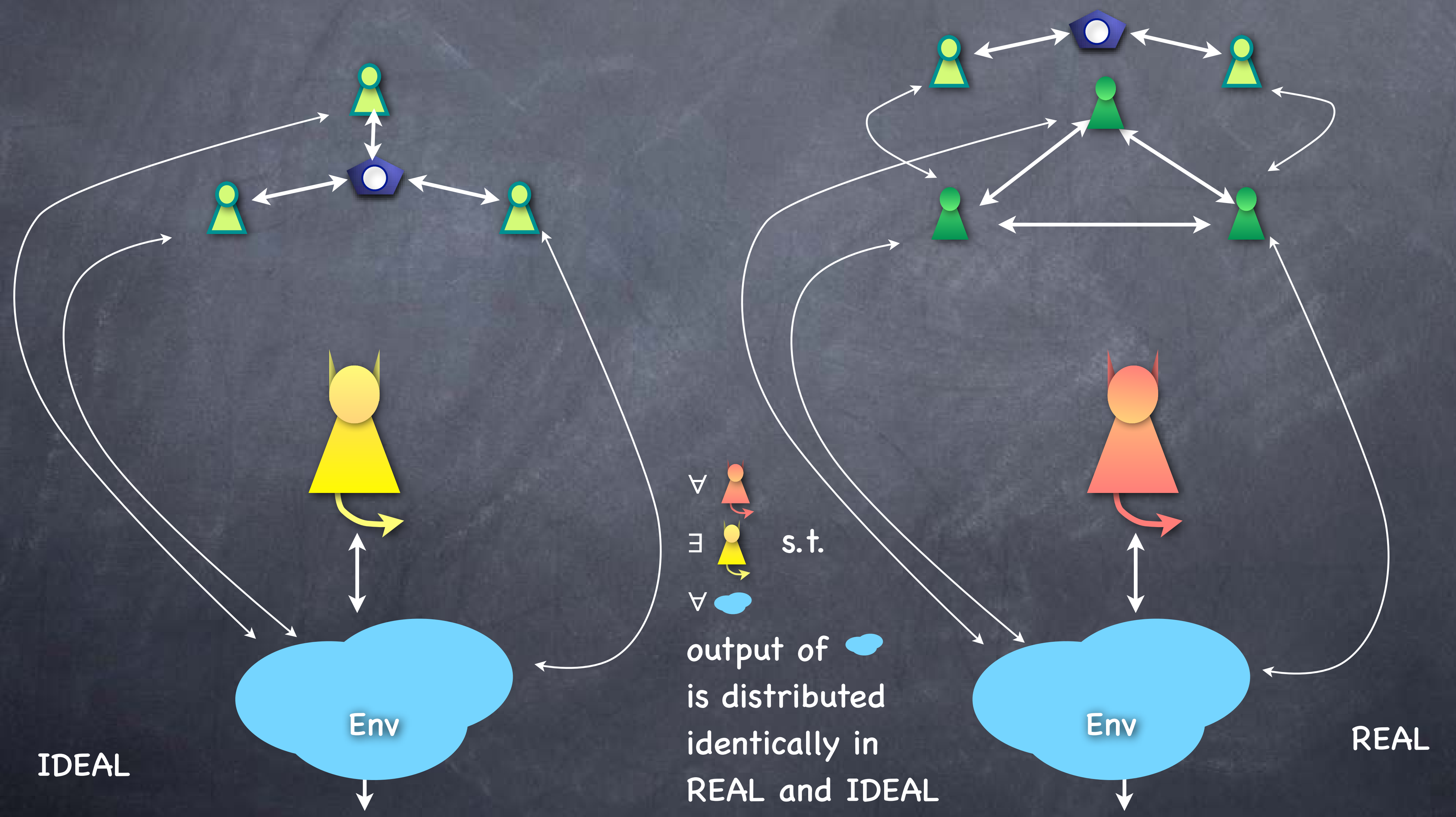
    - Also, subroutine calls

Subroutines

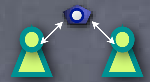
A "REAL" protocol in which parties access (another) IDEAL protocol

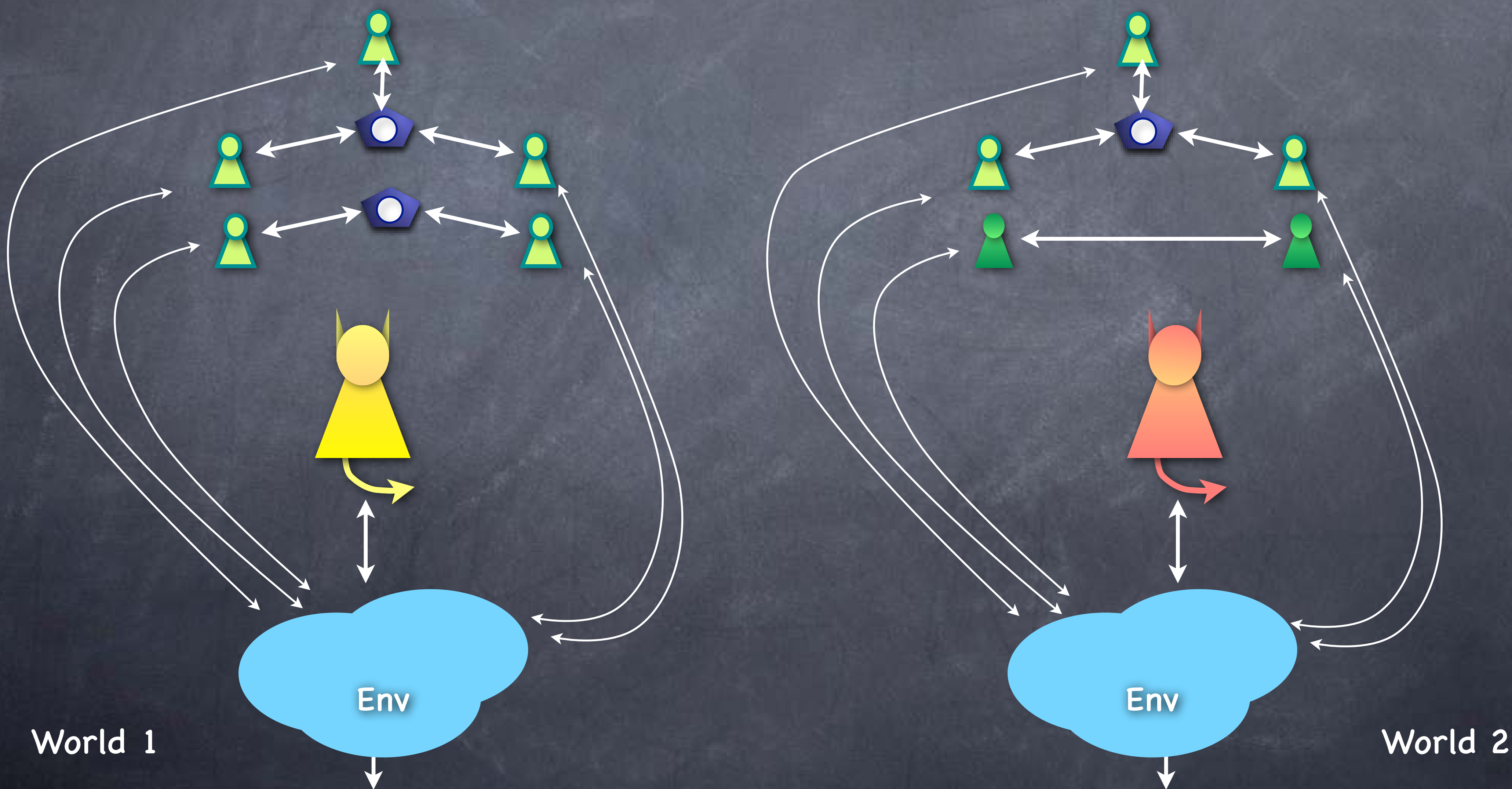


Composition Issues

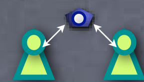
- Standalone security definition doesn't ensure security when composed
- Different modes of composition
  - Sequential composition: protocols executed one after the other. Adversary communicates with the environment between executions. (OK by standalone security definition.)
  - Concurrent composition: multiple sessions (typically of the same protocol) are active at the same time, and the adversary can coordinate its actions across the sessions
    - Also, subroutine calls
  - Universal composition: Executed in an arbitrary environment which may include other protocol sessions (possibly calling this session as a subroutine). Live communication between environment and adversary.

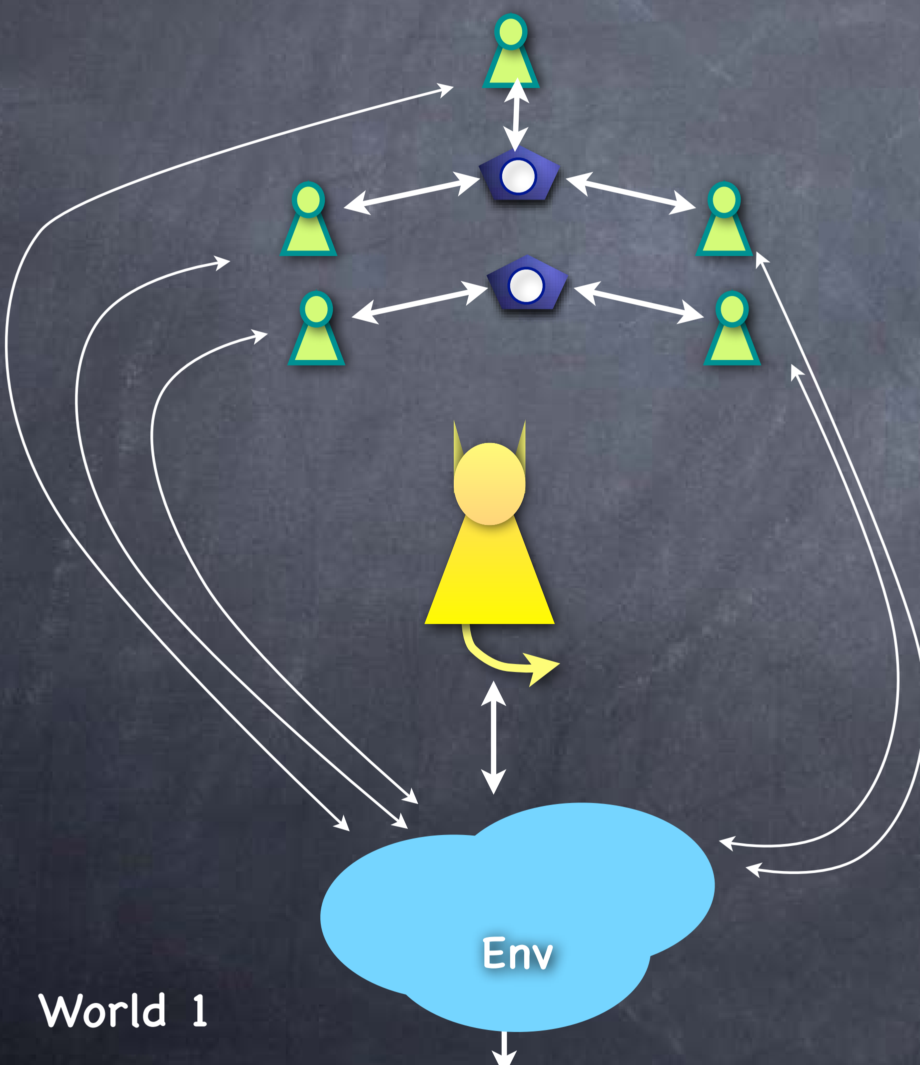
Universal Composition

Replace protocol  with  which is as secure, etc.

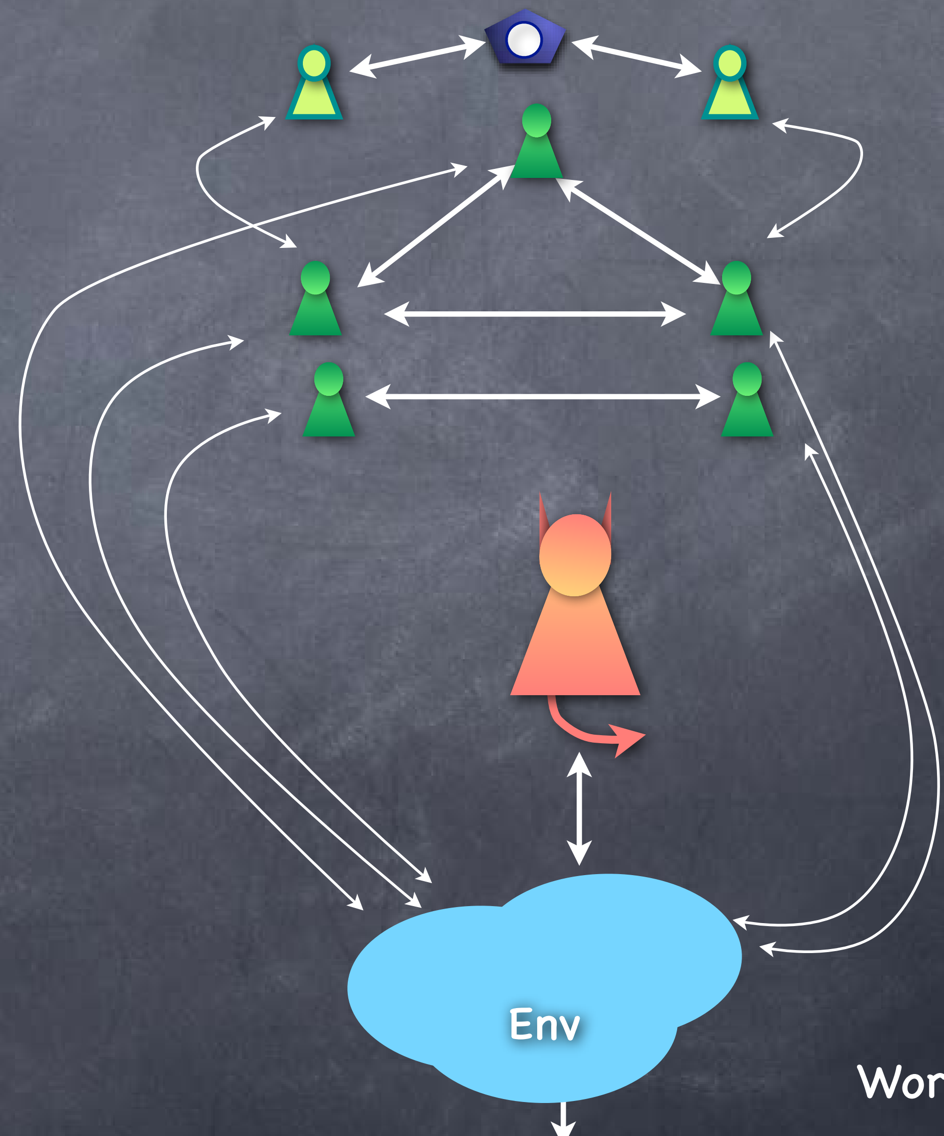


Universal Composition

Replace protocol  with  which is as secure, etc.

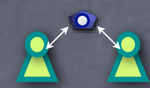


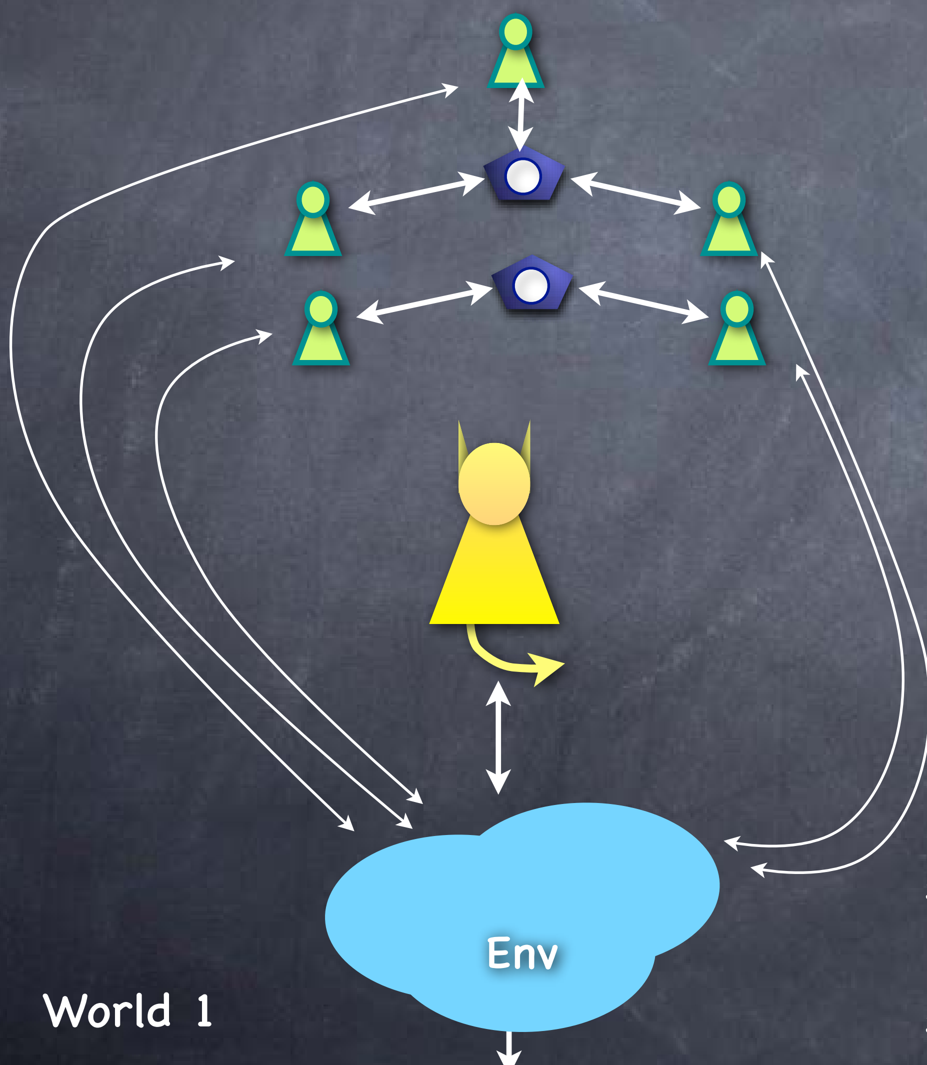
World 1



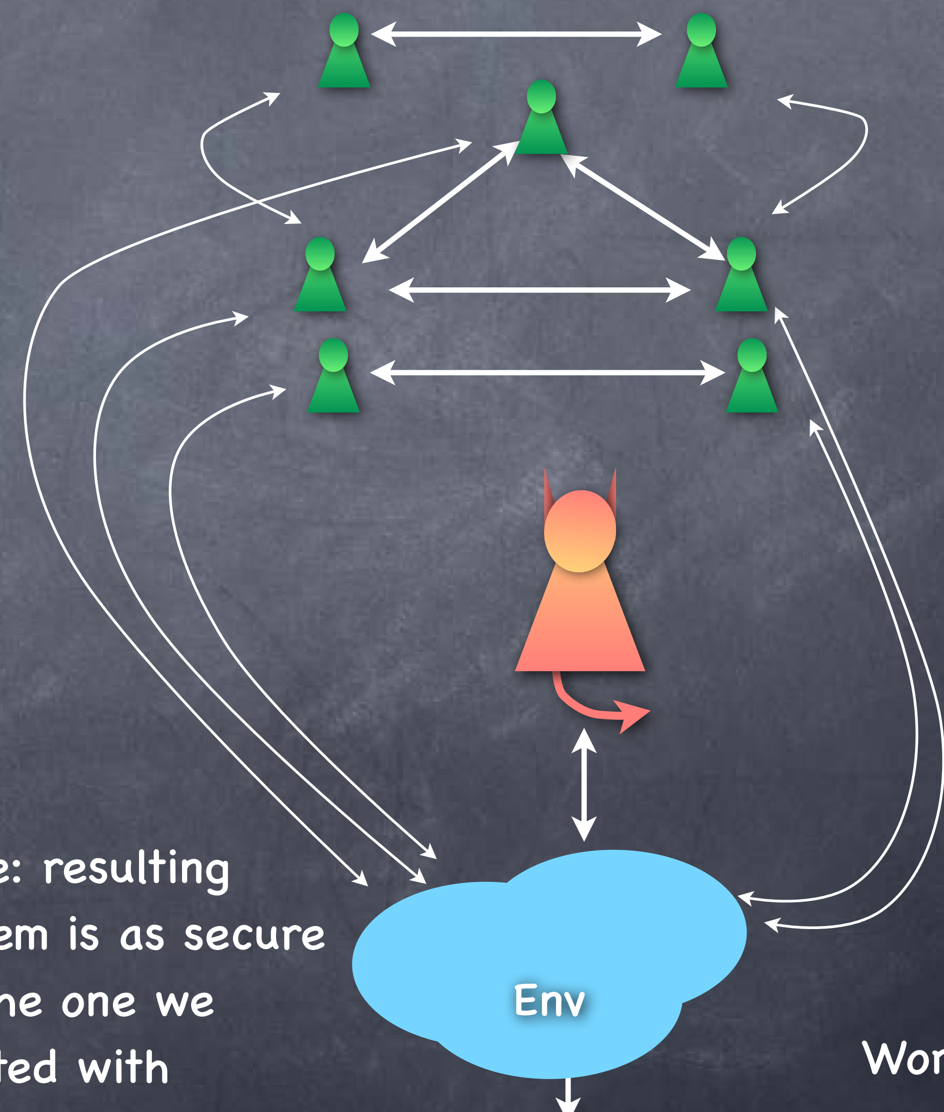
World 3

Universal Composition

Replace protocol  with  which is as secure, etc.



Hope: resulting system is as secure as the one we started with



Universal Composition

- Start from world A (think "IDEAL")
  - Repeat (for any poly number of times):
    - For some 2 "protocols" (that possibly make use of ideal functionalities) I and R such that R is as secure as I, substitute an I-session by an R-session
  - Say we obtain world B (think "REAL")
  - UC Theorem: Then world B is as secure as world A
- Gives a modular implementation of the IDEAL world