

Advanced Tools from Modern Cryptography

Lecture 4

Secure Multi-Party Computation:
Passive Corruption + Honest-Majority

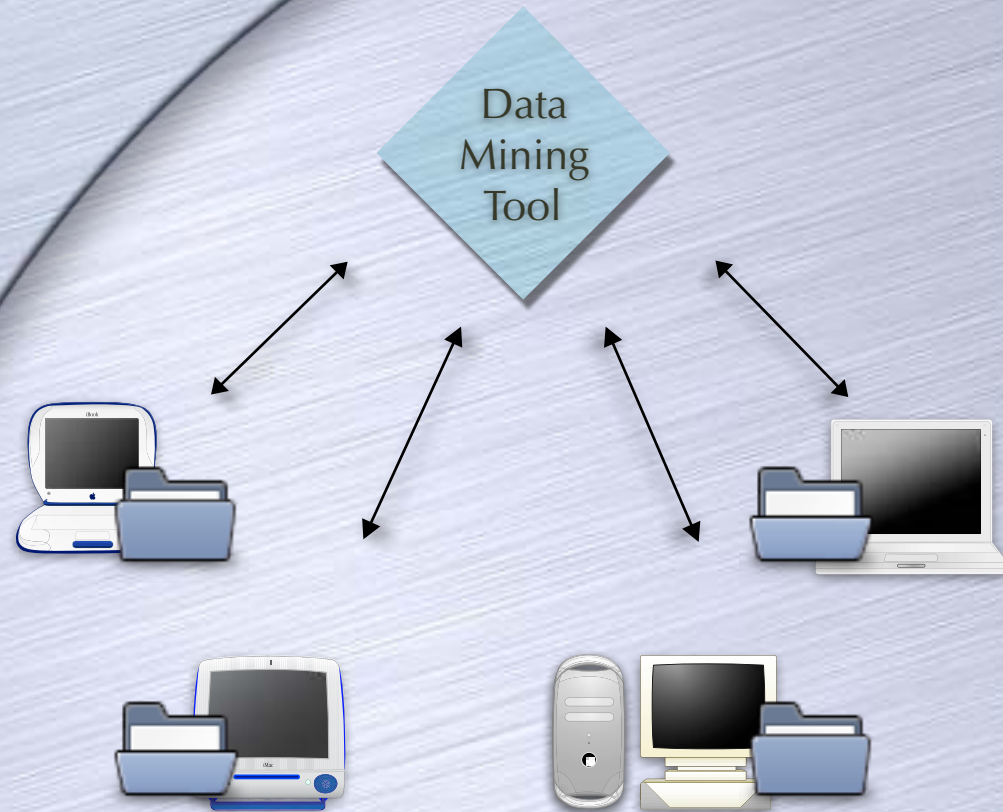
Must We Trust ?

- Can we have an auction without an auctioneer?!
- Declared winning bid should be correct
- Only the winner and winning bid should be revealed



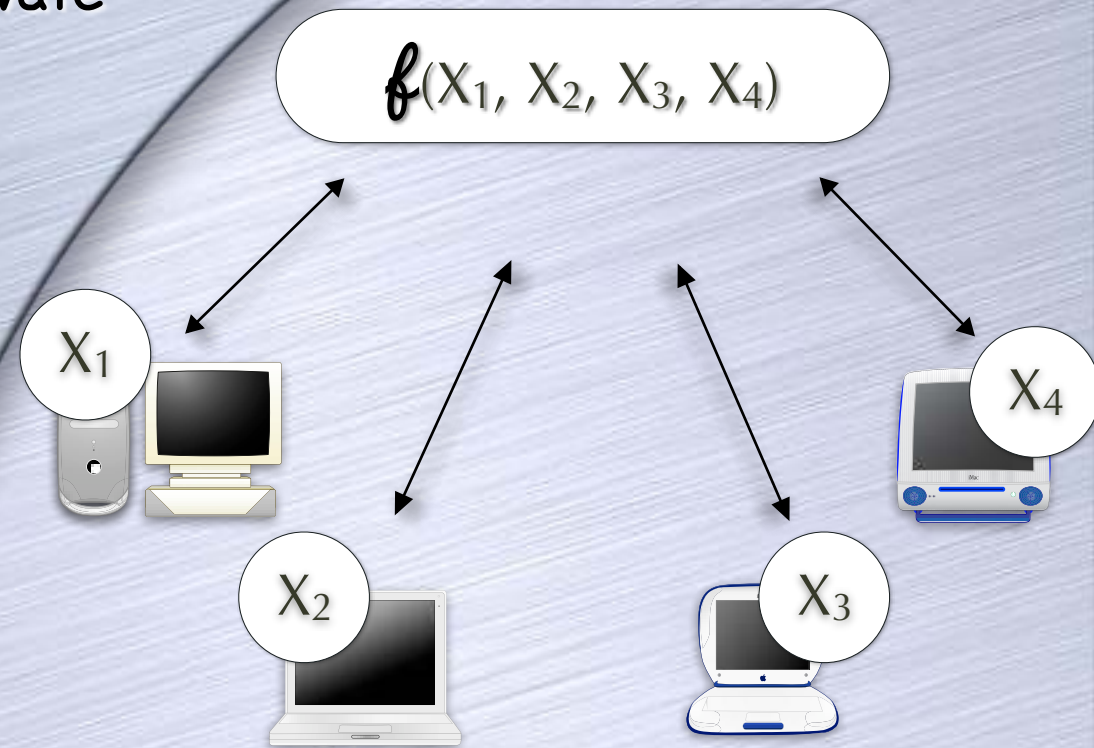
Using data without sharing?

- Hospitals which can't share their patient records with anyone
- But want to data-mine on combined data



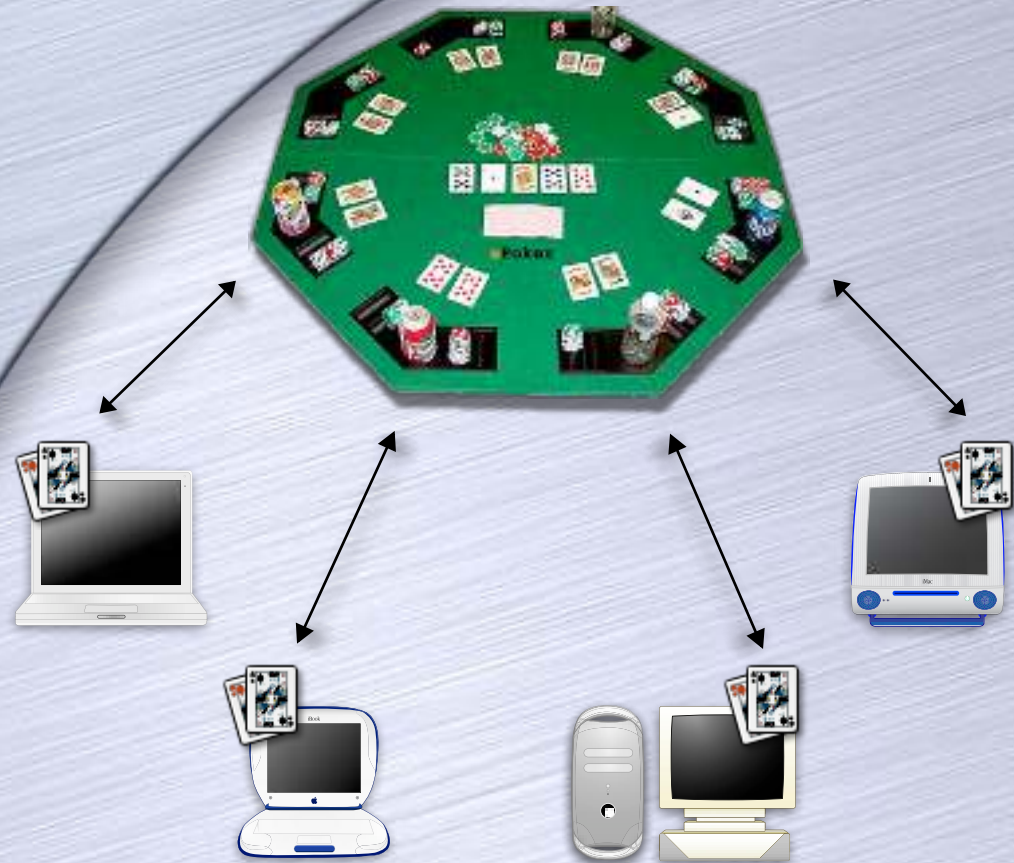
Secure Function Evaluation

- A general problem
- To compute a function of private inputs without revealing information about the inputs
- Beyond what is revealed by the function



Poker With No Dealer?

- Need to ensure
 - Cards are shuffled and dealt correctly
 - Complete secrecy
 - No "cheating" by players, even if they collude
- No universally trusted dealer



The Ambitious Goal

- Without any trusted party, securely do
 - Distributed Data mining
 - E-commerce
 - Network G
 - E-voting
 - Secure fun
 -

Secure
Multi-Party Computation
(MPC)

Any task that
uses a trusted
party!



Mental Poker



**Adi Shamir, Ronald L. Rivest
and Leonard M. Adleman**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

ABSTRACT

Can two potential, dishonest players play a fair game of poker without using any cards—for example, over the phone? This paper provides the following answers:

1. No. (Rigorous mathematical proof supplied.)
2. Yes. (Correct and complete protocol given.)

Emulating Trusted Computation

- Encryption/Authentication allow us to emulate a trusted channel
- Secure MPC: to emulate a source of trusted computation
 - Trusted means it will not “leak” a party’s information to others
 - And it will not cheat in the computation
- A tool for mutually distrusting parties to collaborate

Is it for Real?

- Getting there! Many implementations/platforms
 - Fairplay, VIFF
 - Sharemind
 - SCAPI
 - Obliv-C
 - JustGarble
 - SPDZ/MASCOT
 - OblivM
 - ...
 - multipartycomputation.com/mpc-software

Is it for Real?

- And many practical systems using some form of MPC
 - Danish company Partisia with real-life deployments (since 2008)
 - sugar beet auction, electricity auction, spectrum auction, key management
 - A prototype for credit rating, supported by Danish banks
 - A proposal to the Estonian Tax & Customs Board
 - A proposal for Satellite Collision Analysis
 - Legislation in the US to use MPC for applications like a "higher education data system"
 - ...

MPC

- Several dimensions
 - Passive (Semi-Honest) vs. Active corruption
 - Passive: corrupt parties still follow the protocol
 - Honest-Majority vs. Unrestricted corruption
 - Information-theoretic vs. Computational security
 - ...

Security Definition

- Simplest case: Passive corruption, Information-theoretic security
 - Need honest-majority (or similar restriction)
- In passive corruption, the adversary can see the internals of all the corrupt parties, but cannot control their actions
 - Main concern will be secrecy (correctness is automatic, provided the protocol is correct in the absence of corruption)
 - Will ask for Perfect Secrecy
 - Similar to secret-sharing

Security Definition

- Multiple parties in a protocol could be corrupt
 - Collusion
 - Modelled using a single adversary who corrupts the parties
 - Its view contains all the corrupt parties' views
- Security guarantee given against an "adversary structure"
 - Sets of parties that could be corrupt together

Security Definition

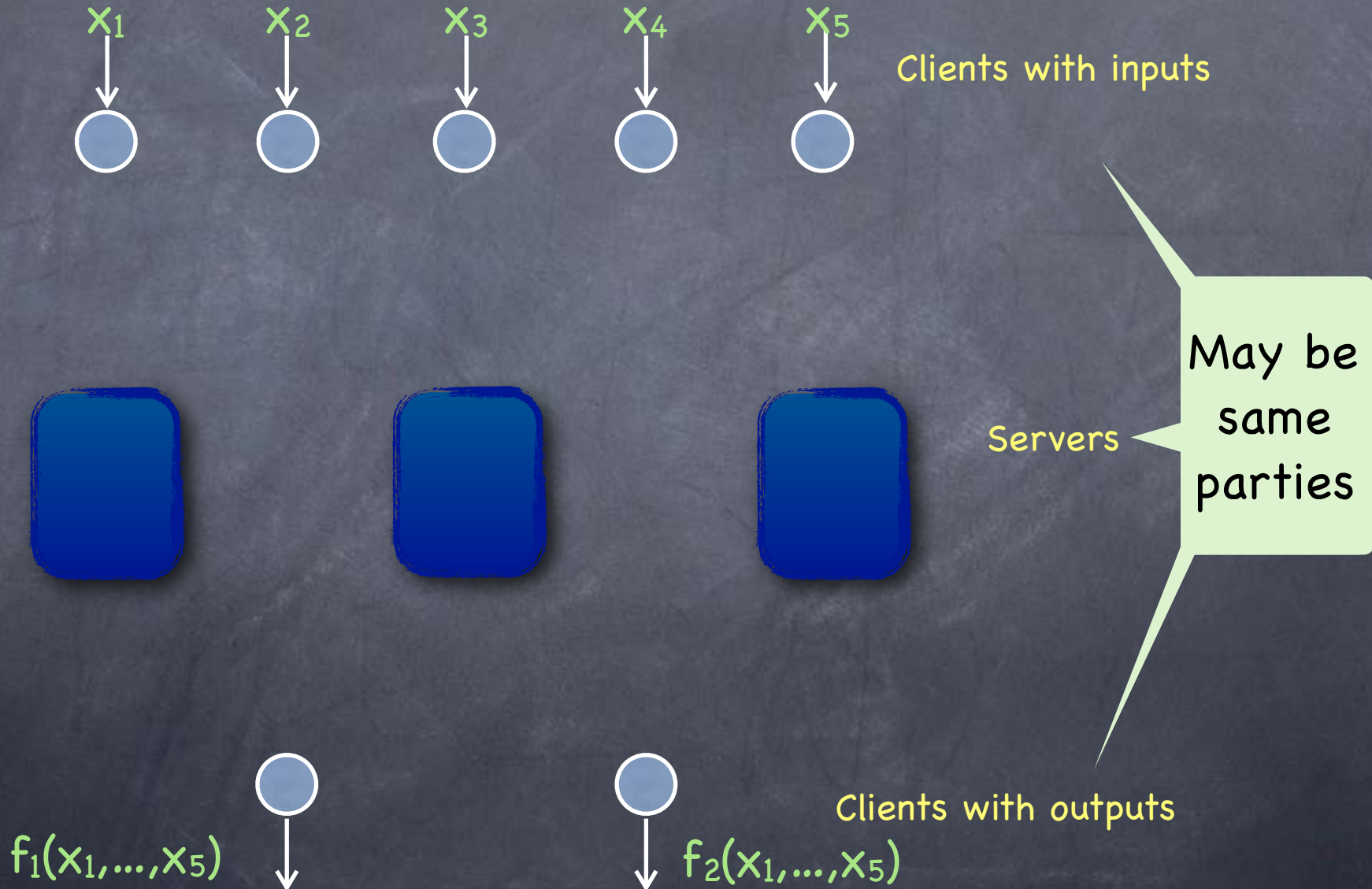
- For secret sharing we needed to formalise "x is secret"
- Now want to say: x is **secret except for f(x)** which is revealed
- $\forall x, x' \text{ s.t. } \underline{f(x)=f(x')}, \{ \text{view} \mid \text{input}=x \} \equiv \{ \text{view} \mid \text{input}=x' \}$

Information-Theoretic Passive-Secure MPC

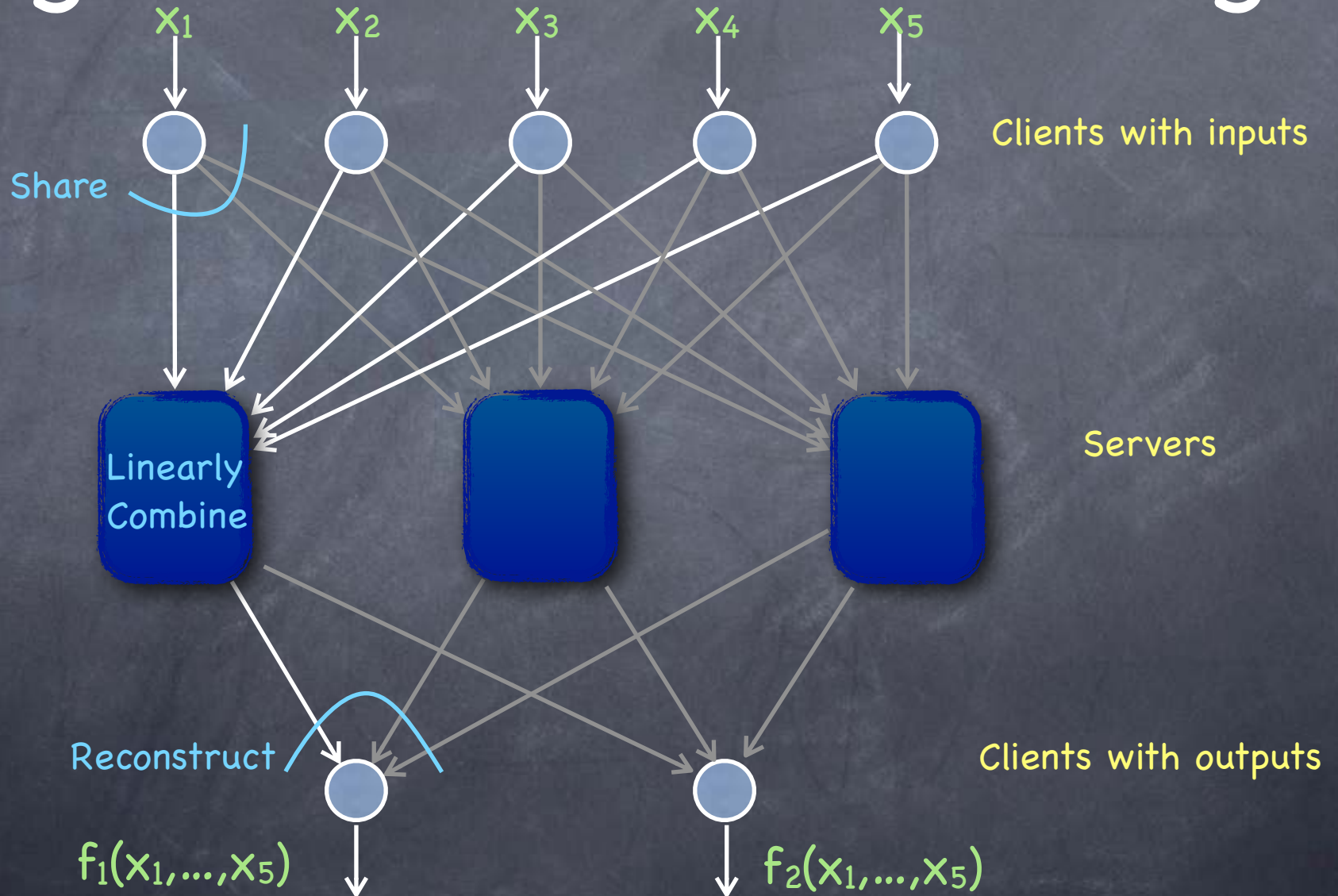
- **Perfectly secure MPC against passive corruption**
- Today: For linear functions
- Next time: For general functions

MPC for Linear Functions

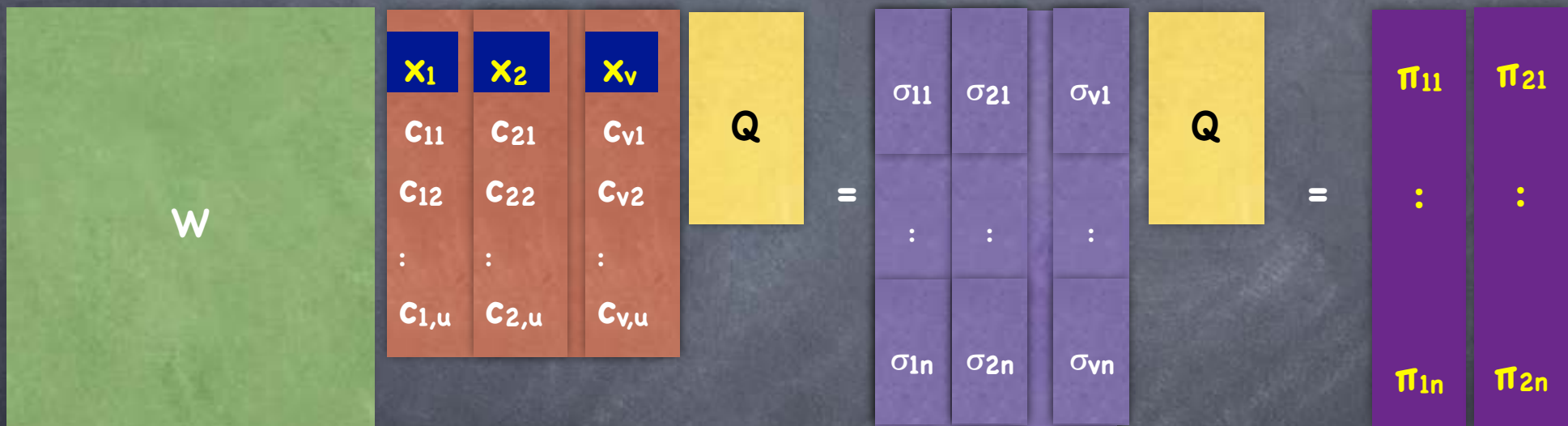
- Client-server setting



MPC for Linear Functions: Using Linear Secret-Sharing



MPC for Linear Functions: Using Linear Secret-Sharing

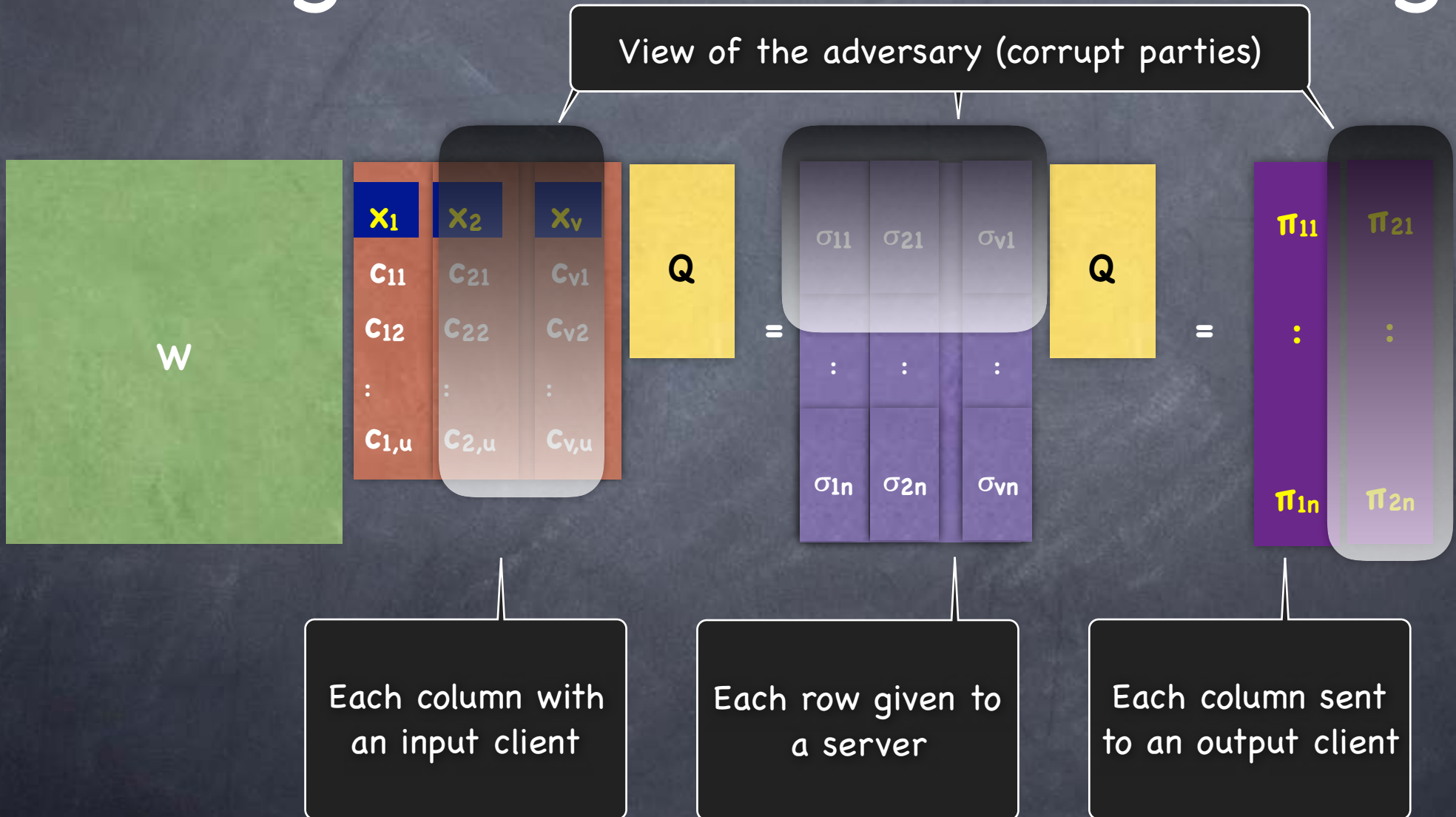


Each column with
an input client

Each row given to
a server

Each column sent
to an output client

MPC for Linear Functions: Using Linear Secret-Sharing



Security

- Adversary allowed to corrupt any set of input and output clients and any subset T of servers s.t. T is not a privileged set (i.e., not in the access structure) for the secret-sharing scheme
- View of adversary should reveal nothing beyond the inputs and outputs of the corrupted clients
 - Claim: Consider any input y of corrupt clients. If x, x' of uncorrupted clients such that for each corrupt output client i $f_i(x,y)=f_i(x',y)$, then the view of the adversary in the two cases are identically distributed
 - Because for any given view of the adversary, the solution space of randomness has the same dimension in the two cases
 - Exercise