Advanced Tools from Modern Cryptography

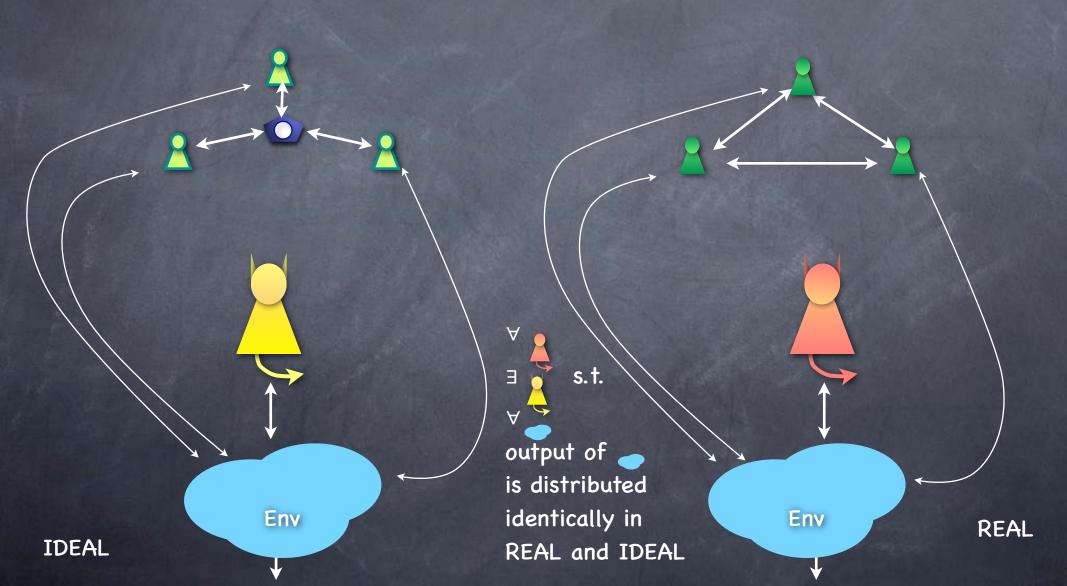
Lecture 11

MPC: UC Theorem. UC Limitations.

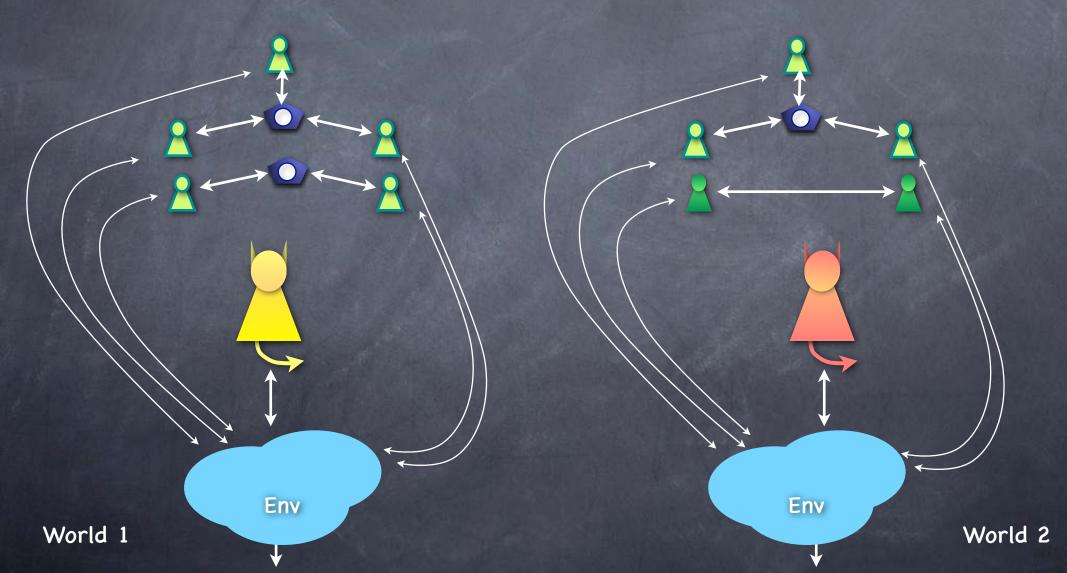
RECALL

UC Security

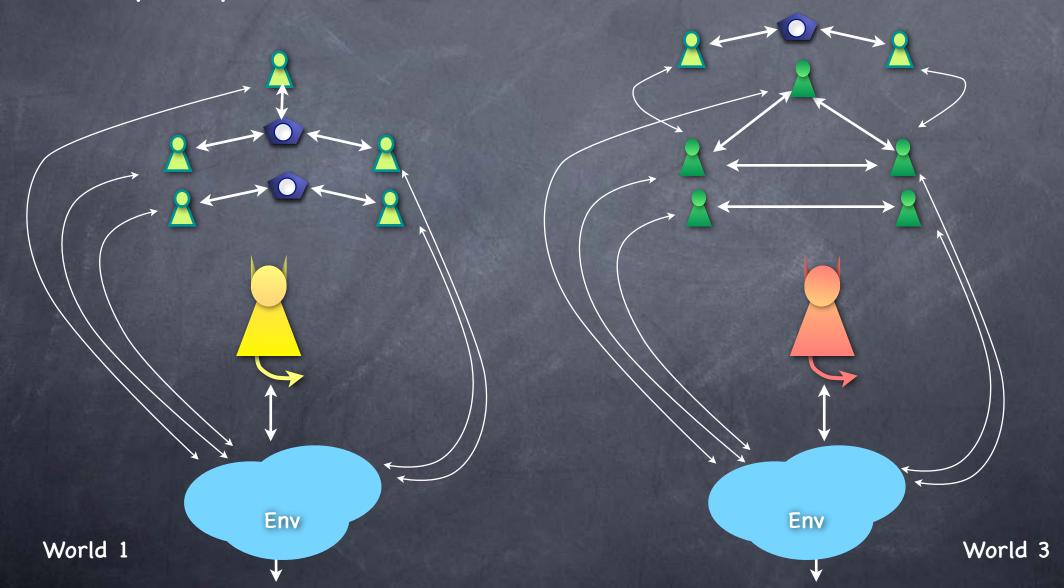
REAL is as secure as IDEAL if:



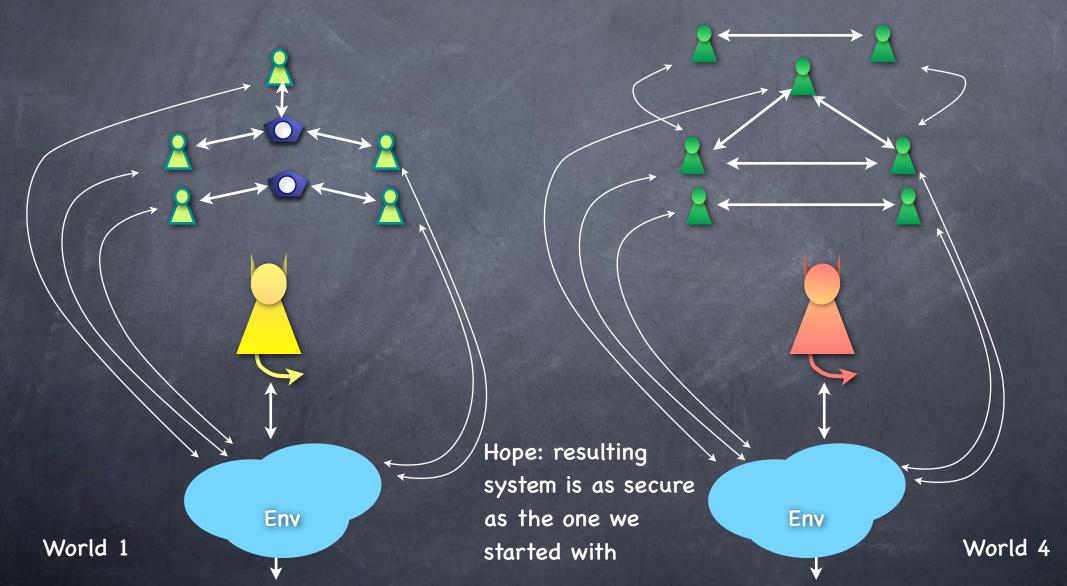
Replace protocol 🔏 📜 with 🛂 which is as secure, etc.



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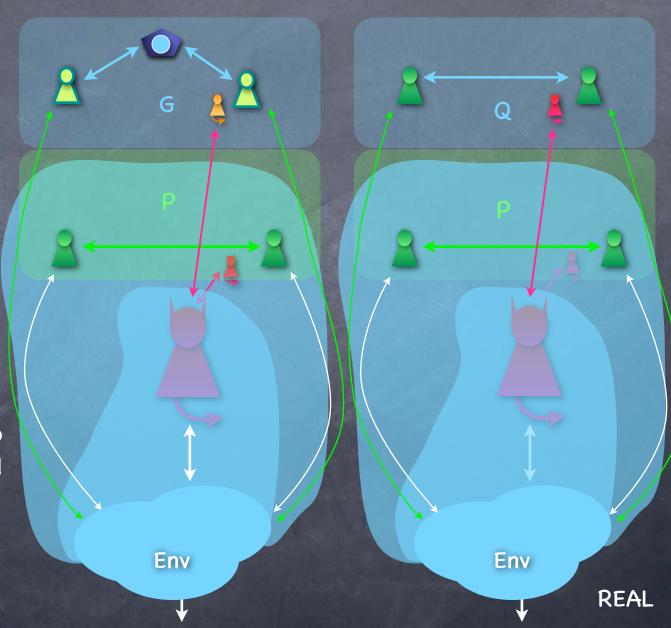


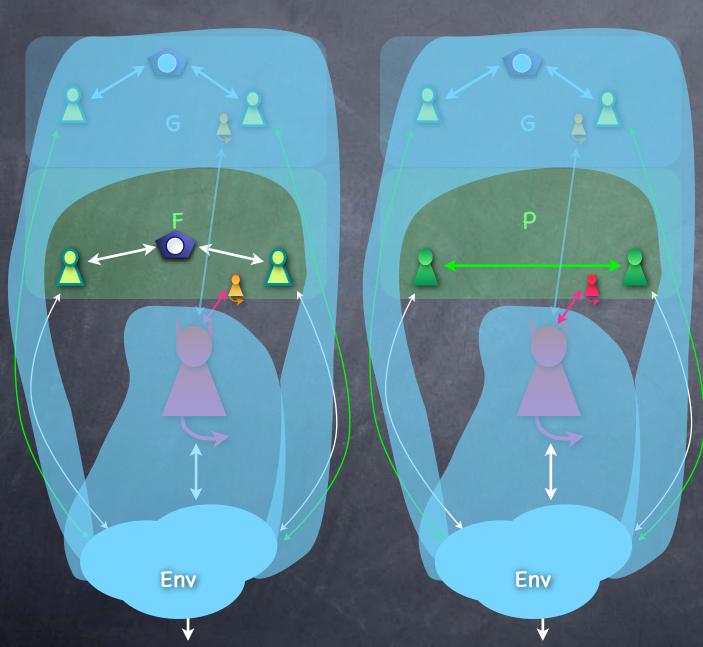
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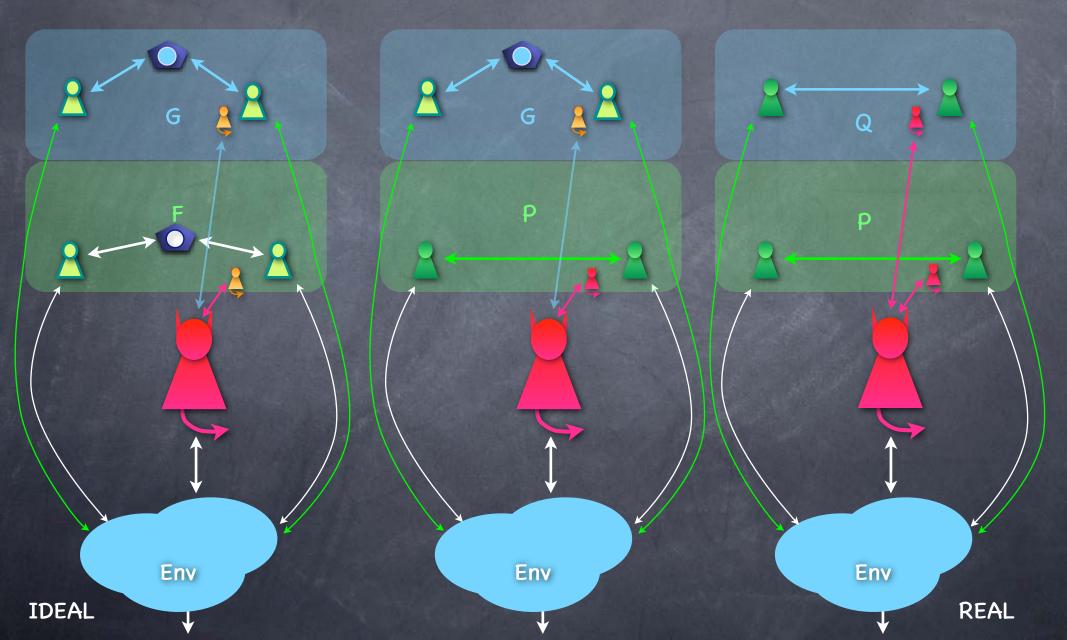
- Start from world A (think "IDEAL")
 - Repeat (for any poly number of times):
 - For some 2 "protocols" (that possibly make use of ideal functionalities) I and R such that R is as secure as I, substitute an I-session by an R-session
 - Say we obtain world B (think "REAL")
 - UC Theorem: Then world B is as secure as world A
- Gives a modular implementation of the IDEAL world

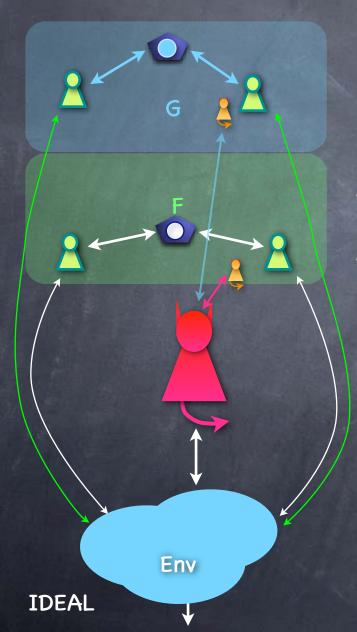
- Consider environment which runs the adversary internally, and depends on "dummy adversaries" to interface with the protocols
- Now consider new environment s.t. only Q (and its adversary) is outside it
- Use "Q is as secure as G" to get a new world with G and a new adversary





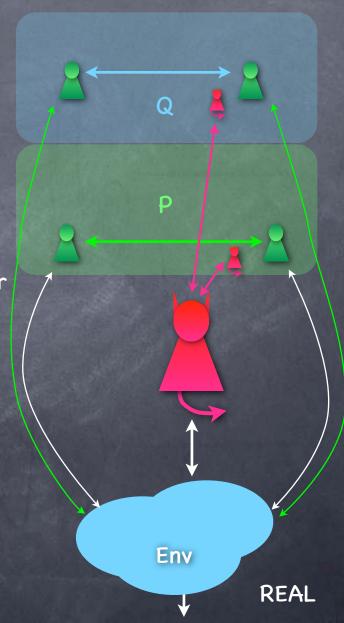
- Now consider new environment s.t. only P (and adversary) is outside it
 - Note: G and simulator for Q/G are inside the new environment
- Use "P is as secure as F" to get a new world with F and a new adversary





Hence REAL ≈ IDEAL

Main idea: Environment can model other sessions (real or ideal)



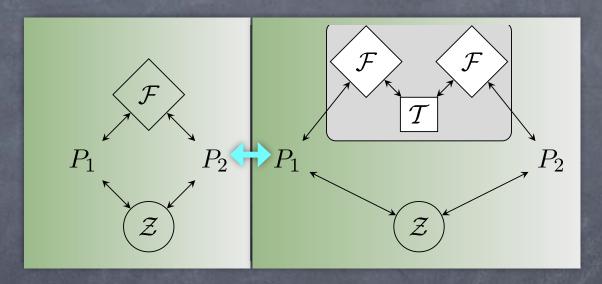
UC Secure MPC?

- UC-security is a strong security definition, and also enjoys the UC property
- But impossible to have "non-trivial" UC-secure MPC (for 2 parties)!
- Universal Composition possible when:
 - Passive corruption, or
 - Honest majority, or
 - Given trusted setups (e.g., OT), or
 - Using alternate security definitions
 (e.g., "Angel-aided simulation": still meaningful and UC)

Impossibility of UC Security $\mathcal{Z}^{\mathcal{F}_{\mathsf{split}}^T}$ functionality \mathcal{F} functionality Indist. by security \mathcal{F} functionality π_1 $\mathcal{F}_{\mathsf{split}}^{\mathcal{T}}$ functionality Identical systems \mathcal{F} No corruption \mathcal{Z} \mathcal{Z} $\mathcal{F}_{\text{split}}^T$ functionality \mathcal{F} functionality Party 1 corrupt F has a UC-secure protocol only if F is P_2 "splittable" Very few are splittable! \mathcal{Z}' \mathcal{S}_1 \mathcal{Z}'' Party 2 corrupt

Splittable Functionalities

lacktriangledown F splittable if $\exists T \ \forall Z$ the outputs of Z in the following two experiments are negligibly far from each other:



- A splittable functionality essentially involves only communication and local computation. All splittable functionalities have UC-secure protocols.
- Most interesting functionalities are unsplittable. E.g., coin-tossing, commitment, XOR, GT, ... P_2 π_1 π_2

UC Security Beyond 2 Parties Without Honest-Majority

- Any multi-party function F such that a 2-way partition of it is unsplittable is impossible to UC-securely realise
 - © Consider F with an unsplittable partition f. Protocol Π_F gives a 2-party protocol Π_f . Π_F tolerates corruption of either part $\longrightarrow \Pi_f$ tolerates corruption of either party
- So only "disseminating" and "aggregating" functionalities
- Disseminating: Only one party has input that influences the output of the others (e.g., broadcast, secret-sharing)
- Aggregating: Only one party has output that is influenced by the input of the others (e.g., group summation)
 - Not all aggregating functionalities have 2-way partitions that are splittable [Why?]

UC Security Beyond 2 Parties

- All disseminating functionalities are UC-securely realisable!
 - e.g., Broadcast protocol
 - Sender sends m to all Receivers
 - Each Receiver sends m that it received to all others
 - Each Receiver outputs m if it received the same m from all other Receivers. Else Aborts.
 - Note: Here abort allowed. UC-Secure [Why?]
- Open: which aggregating functionalities are UC-securely realisable?
 - e.g. additive-sharing based summation protocol (input parties play servers, only one output client) [Why UC-Secure?]