Advanced Tools from Modern Cryptography

Lecture 14

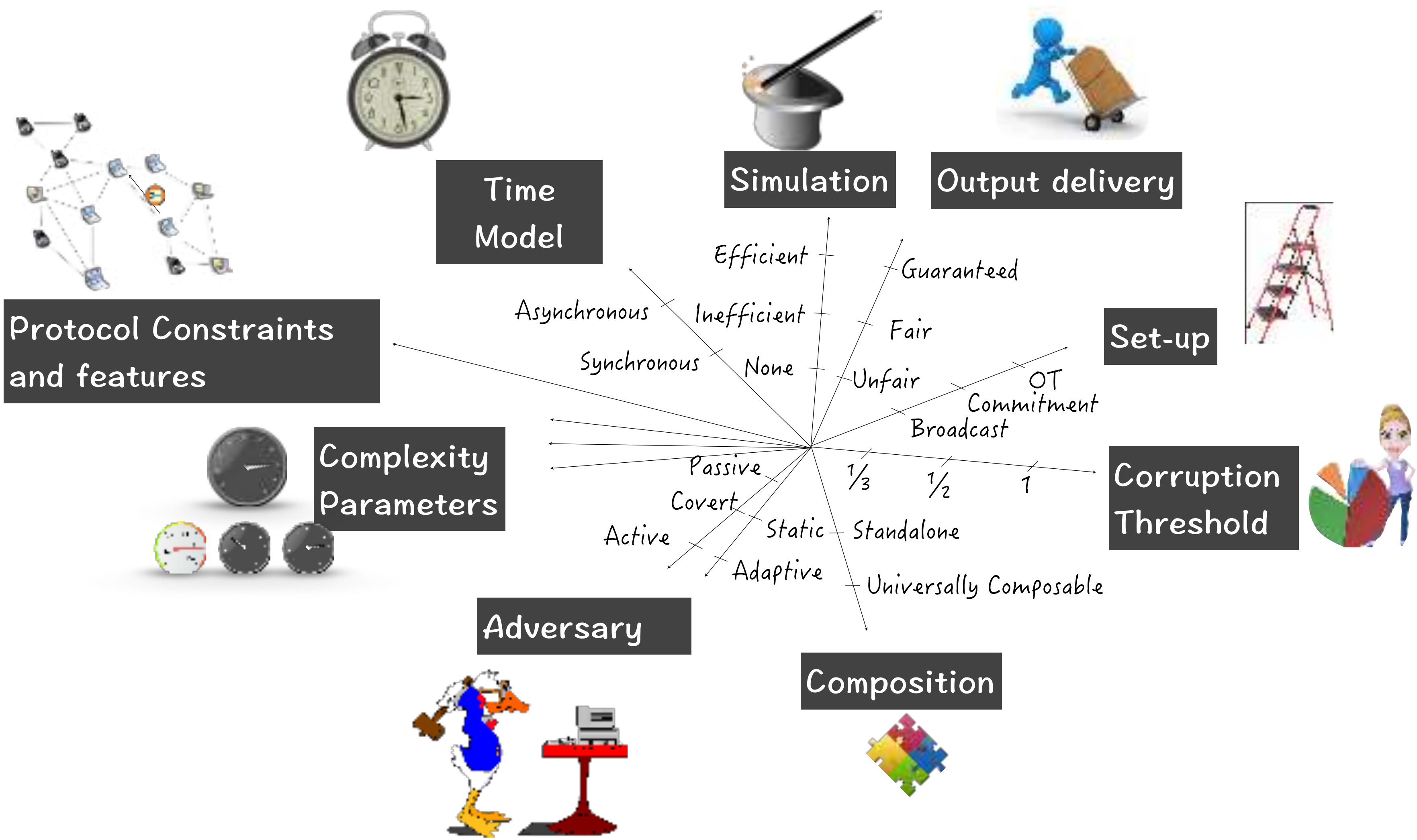
MPC: Fairness and Guaranteed Output Delivery

Some Impossibility Results

General MPC

- Information-theoretic security
  - Passive with corruption threshold $t < n/2$ — Passive BGW/CCD
  - Passive with OT setup — Passive GMW
  - Guaranteed Output UC with $t < n/3$ — BGW
  - Guaranteed Output UC with $t < n/2$ and Broadcast — Rabin-BenOr
  - Selective Abort UC, with OT — Kilian (Also IPS)
- Computational security
  - Passive — Composing Yao or Passive GMW with a passive-secure OT protocol
  - Standalone — GMW: using ZK proofs
  - Selective Abort UC, with CRS — Composing Kilian with a CRS-based UC-secure OT protocol

MPC Dimensions



Basic Dimensions

- Adversary's computational power: PPT adversary, Information-theoretic security
- Honest majority: Thresholds 1 (no honest majority), ½ and ⅓
- Security Level: Passive security, UC security with selective abort, or UC security with guaranteed output delivery
- Trusted setup: Point-to-point channels, Broadcast, Common Reference String (CRS), OT

Output Delivery

- 3 levels:
  - Unfair (a.k.a., selective abort)
    - Adversary can see its output and decide which set of honest parties receive theirs
  - Fair
    - Adversary can cause abort for all parties before seeing its output. To get its output, should let all parties get theirs.
  - Guaranteed output delivery
    - Adversary cannot prevent honest parties from producing an output. (Adversary will have well-defined inputs no matter what it does.)
- Today: Impossibility of fairness/guaranteed output delivery even for some very simple tasks, if there is no honest majority

Fair Coin-Tossing

- For 2-party functions, fair protocol $\Rightarrow$ guaranteed output delivery
  - Modify a fair protocol: if aborted, locally compute the output (with a fixed input for the other party)
- 2-party fair coin-tossing from commitment?
    - Alice commits to a random bit a, Bob sends a bit b, Alice opens and they output $a \oplus b$
    - Unfair: Alice can abort after learning the outcome
- Two parties can never obtain a fair coin, given only unfair setups, even under computational assumptions, even for standalone security, even against fail-stop adversaries
  - Unfair setup: Sends outputs to the parties one at a time. Adversary can abort at any point.

Fair Coin-Tossing

- Guaranteed output delivery: Each party has a tentative output after each message it receives, if an abort happens right after it
- Best possible unfair setup: F runs the protocol on behalf of the parties; at each round, sends each party its tentative output.
  - X_0,Y_0 if abort before start. Then F Sends X_1 (to Alice), Y_1 (to Bob), X_2, Y_2, ..., X_n, Y_n. Allows adversary to abort at any point.
- X_0,Y_0 independent; also uniform (by correctness for abort at start)
- Correctness when no abort: $\Pr[X_n=b, Y_n=b]=½$, for $b\in\{0,1\}$
- $\Pr[X_i=Y_i]$ went from ½ to 1: So $\exists i$ s.t. $\Pr[X_i=Y_i]-\Pr[X_{i-1}=Y_{i-1}] \geq 1/(2n)$. i.e., $\Pr[X_i=Y_i]-\Pr[X_i=Y_{i-1}] + \Pr[X_i=Y_{i-1}]-\Pr[X_{i-1}=Y_{i-1}] \geq 1/(2n)$
  - So, some i s.t. either $\Pr[X_i=Y_i]-\Pr[X_i=Y_{i-1}] \geq 1/(4n)$ or $\Pr[X_i=Y_{i-1}]-\Pr[X_{i-1}=Y_{i-1}] \geq 1/(4n)$
- But will show, $\forall i$ $\Pr[X_i=Y_i] \approx \Pr[X_i=Y_{i-1}]$ and $\Pr[X_i=Y_{i-1}] \approx \Pr[X_{i-1}=Y_{i-1}]$) !

Fair Coin-Tossing

- To show $\Pr[X_i=Y_i] \approx \Pr[X_i=Y_{i-1}]$ (and similarly $\Pr[X_i=Y_{i-1}] \approx \Pr[X_{i-1}=Y_{i-1}]$)
  - Firstly, $\Pr[Y_{i-1}=0] \approx 1/2$, $\Pr[Y_i=0] \approx 1/2$ (by correctness against Alice who aborts after Y_{i-1} and one who aborts after Y_i)
  - Consider two more attackers for corrupt Alice:
    A_0: If $X_i=0$, abort immediately, else abort after Y_i delivered
    A_1: If $X_i=1$, abort immediately, else abort after Y_i delivered
  - Under attack by A_0,
    $\Pr[\text{Bob outputs } 0] = \Pr[X_i=0,Y_{i-1}=0] + \Pr[X_i=1,Y_i=0]$
    $= \Pr[X_i=0,Y_{i-1}=0] - \Pr[X_i=0,Y_i=0] + \Pr[Y_i=0]$
    $\Rightarrow \Pr[X_i=0,Y_{i-1}=0] \approx \Pr[X_i=0,Y_i=0]$
  - Similarly, from A_1, $\Pr[X_i=1,Y_{i-1}=1] \approx \Pr[X_i=1,Y_i=1]$
  - So, $\Pr[X_i=Y_{i-1}] \approx \Pr[X_i=Y_i]$

Recall

Broadcast

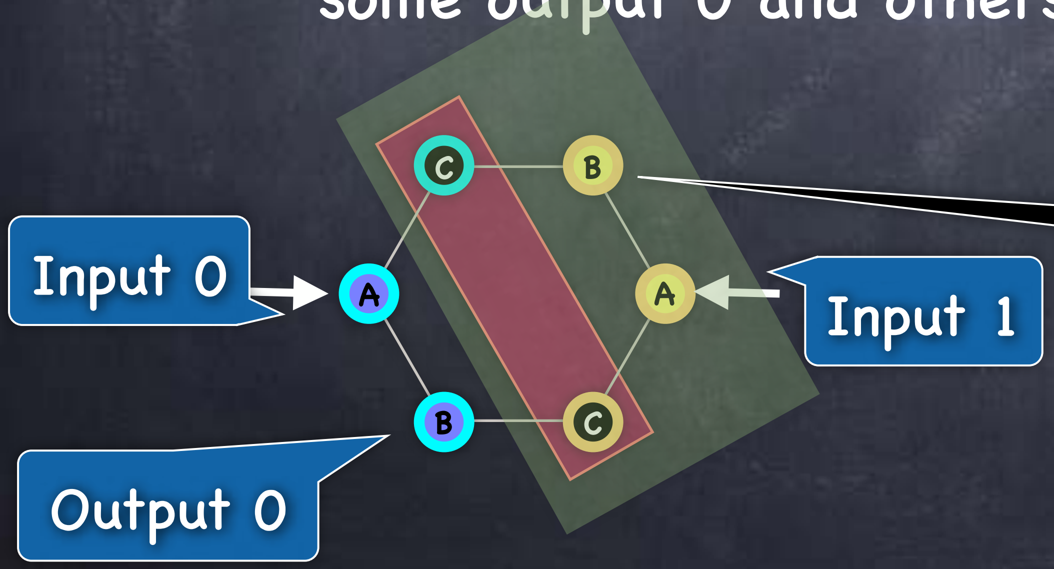
- BGW protocol relied on broadcast to ensure all honest parties have the same view of disputes, resolution etc.
- Concern addressed by broadcast: a corrupt sender can send different values to different honest parties
- Broadcast with selective abort can be implemented easily, even without honest majority
    - Sender sends message to everyone. Every party cross-checks with everyone else, and aborts if there is any inconsistency.
- If corruption threshold t < n/3, then it turns out that broadcast with guaranteed output delivery can be implemented

Otherwise not!

- If broadcast given as a setup, can do MPC with guaranteed output delivery for up to t < n/2

No Broadcast with Guaranteed Output if 1/3 Corrupt

- Broadcast requirements (message being a single bit):
  - If sender honest, all honest parties should output the bit it sends (can't abort)
  - All honest parties should agree on the outcome (can't have some output 0 and others 1)



Consider 6 parties running the code for A, B, C (A is the sender)

Adversary corrupting C

Note: can't do this if A, B allowed to have a priori shared secrets (say message authentication keys)

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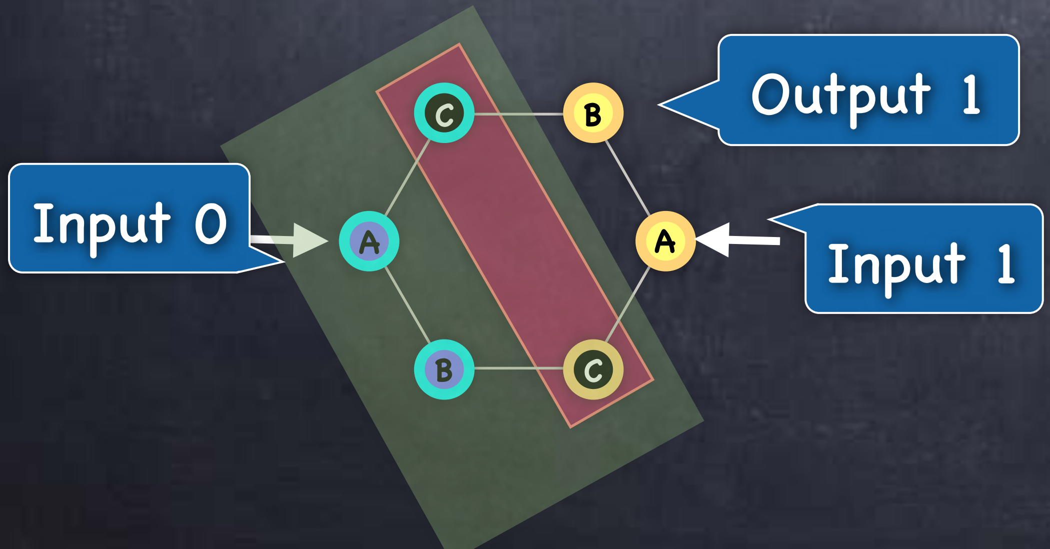

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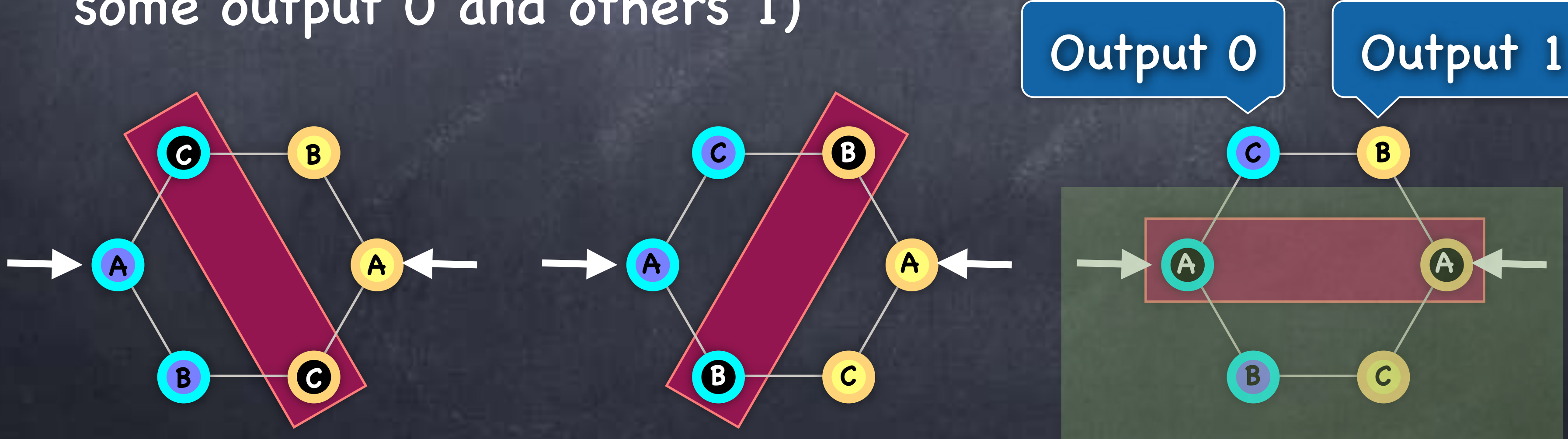

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- Broadcast requirements (message being a single bit):
  - If sender honest, all honest parties should output the bit it sends (can't abort)
  - All honest parties should agree on the outcome (can't have some output 0 and others 1)
- Impossible to satisfy both constraints simultaneously, if 1/3 can be corrupt
  - Irrespective of what computational assumptions are used!
  - But a priori shared keys can give broadcast with guaranteed output delivery against unrestricted corruption (in the synchronous model)