

# Active Adversary

Lecture 7  
CCA Security  
MAC

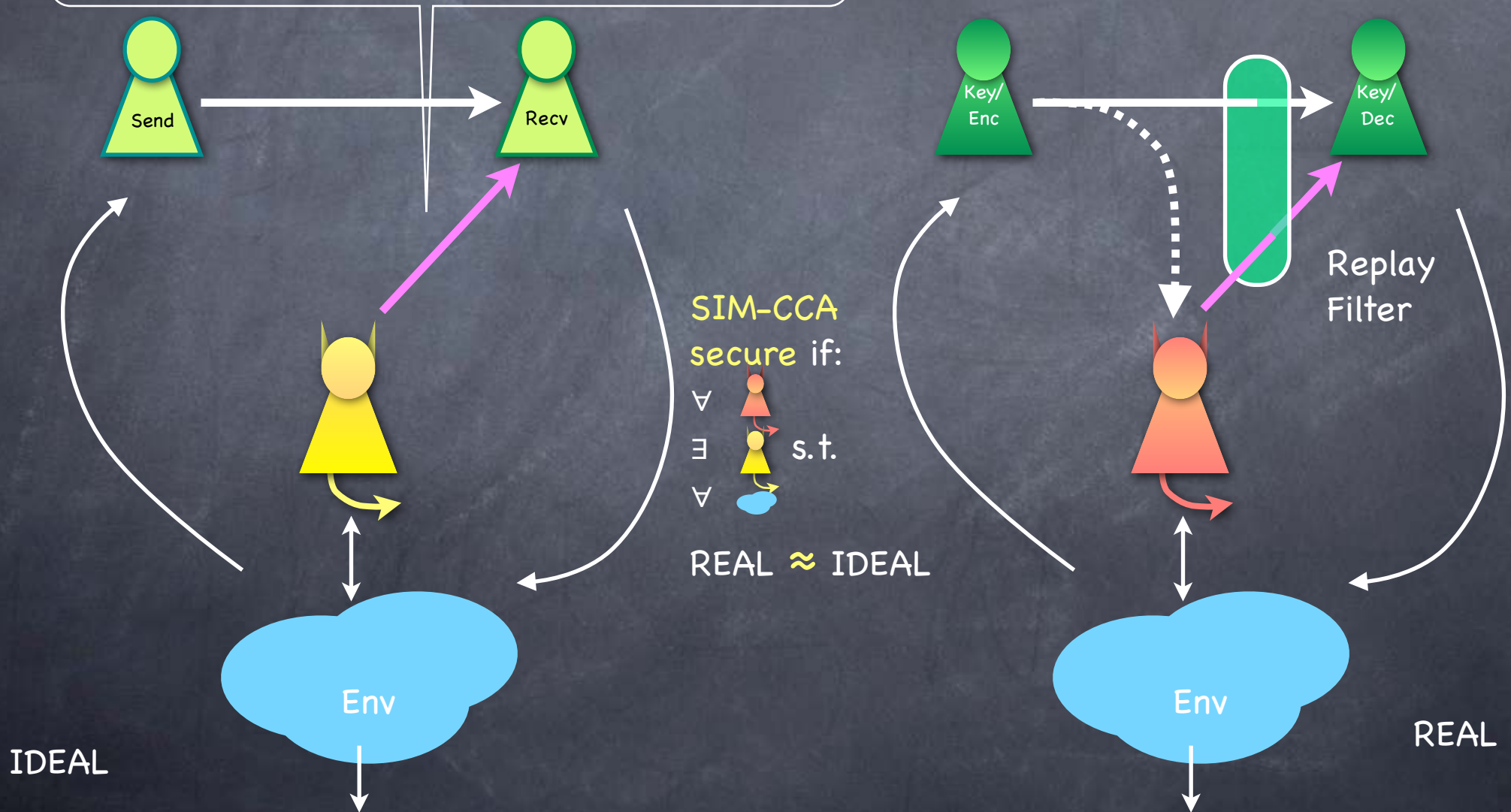
# Active Adversary

- An active adversary can inject messages into the channel
  - Eve can send ciphertexts to Bob and get them decrypted
    - Chosen Ciphertext Attack (CCA)
- If Bob decrypts all ciphertexts for Eve, no security possible
  - What can Bob do?

# Symmetric-Key Encryption

## SIM-CCA Security

Authentication not required.  
Adversary allowed to send own messages





# Symmetric-Key Encryption

## IND-CCA Security

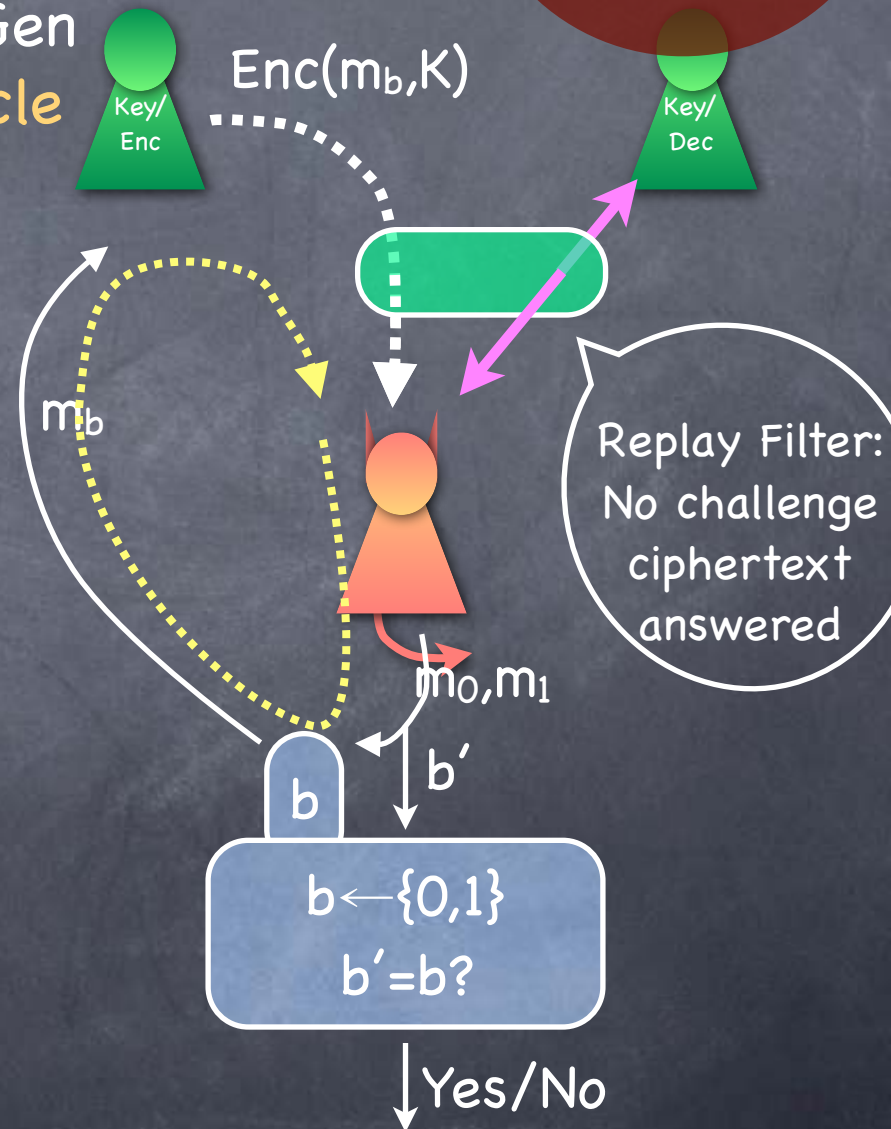
IND-CCA +  
~correctness  
equivalent to  
SIM-CCA

- Experiment picks  $b \leftarrow \{0,1\}$  and  $K \leftarrow \text{KeyGen}$   
Adv gets (guarded) access to  $\text{Dec}_K$  oracle

- For as long as Adversary wants

- Adv sends two messages  $m_0, m_1$  to the experiment
- Expt returns  $\text{Enc}(m_b, K)$  to the adversary

- Adversary returns a guess  $b'$
- Experiment outputs 1 iff  $b' = b$
- IND-CCA secure if for all feasible adversaries  $\Pr[b' = b] \approx 1/2$



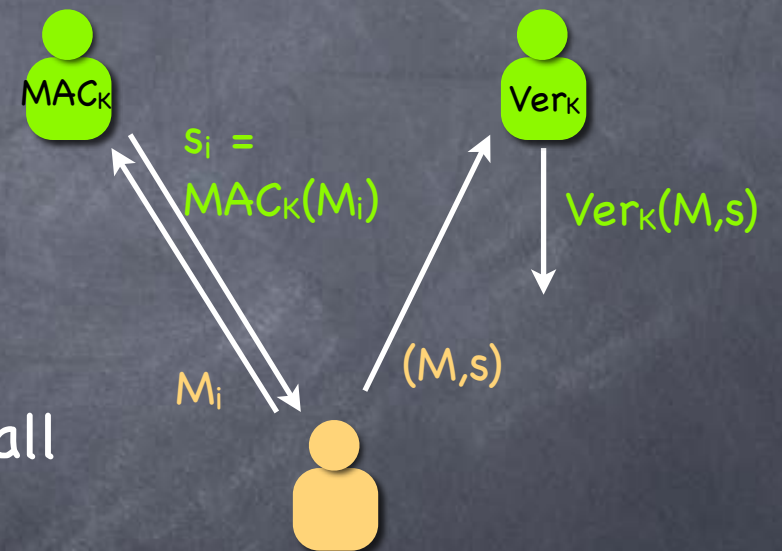
# CCA Security

- How to obtain CCA security?
- Use a CPA-secure encryption scheme, but make sure Bob “accepts” and decrypts only ciphertexts produced by Alice
  - i.e., Eve can’t create new ciphertexts that will be accepted by Bob
  - Achieves the stronger guarantee: in IDEAL, Eve can’t send its own messages to Bob
- CCA secure SKE reduces to the problem of CPA secure SKE and (shared key) message authentication
  - Symmetric-key solution for message authentication: Message Authentication Code (MAC)

# Message Authentication Codes

- A single short key shared by Alice and Bob
  - Can sign any (polynomial) number of messages

- A triple (KeyGen, MAC, Verify)
- Correctness: For all  $K$  from KeyGen, and all messages  $M$ ,  $\text{Verify}_K(M, \text{MAC}_K(M))=1$
- Security: probability that an adversary can produce  $(M,s)$  s.t.  $\text{Verify}_K(M,s)=1$  is negligible unless Alice produced an output  $s=\text{MAC}_K(M)$



Advantage

$$= \Pr[ \text{Ver}_K(M,s)=1 \text{ and } (M,s) \notin \{(M_i,s_i)\} ]$$



# CCA Secure SKE

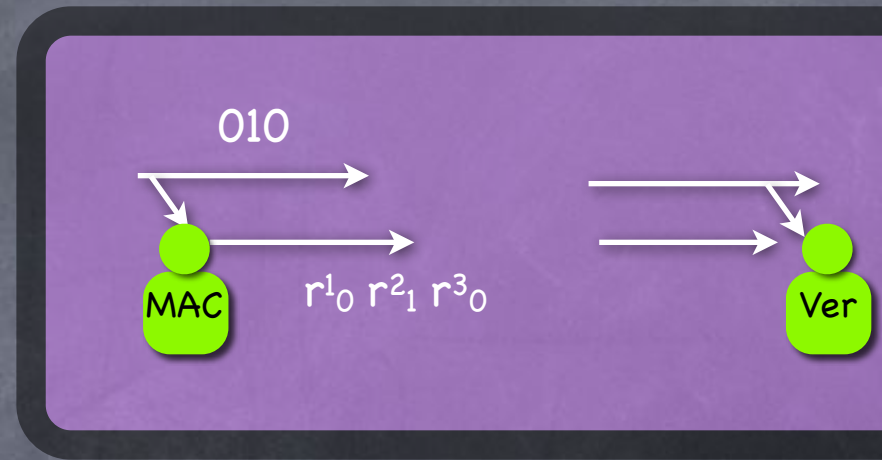
- $\text{CCA-Enc}_{K_1, K_2}(m) = ( c := \text{CPA-Enc}_{K_1}(m), t := \text{MAC}_{K_2}(c) )$ 
  - CPA secure encryption: Block-cipher/CTR mode construction
  - MAC: from a PRF or Block-Cipher (coming up)
- **SKE can be entirely based on Block-Ciphers**
  - A tool that can make things faster: Hash functions (later)
  - Or, in principle, from any One-Way Function

# Making a MAC



# One-time MAC

- To sign a single  $n$  bit message
- A simple (but inefficient) scheme
  - Shared secret key:  $2n$  random strings (each  $k$ -bit long)  $(r^i_0, r^i_1)_{i=1..n}$
  - Signature for  $m_1...m_n$  be  $(r^i_{m_i})_{i=1..n}$
  - Negligible probability that Eve can produce a signature on  $m' \neq m$
- Doesn't require any computational restrictions on adversary!
  - Has a statistical security parameter  $k$  (unlike one-time pad which has perfect security)
- More efficient one-time MACs exist (later)



$r^1_0$	$r^2_0$	$r^3_0$
$r^1_1$	$r^2_1$	$r^3_1$

# (Multi-msg) MAC from PRF

When Each Message is a Single Block

- PRF is a MAC!

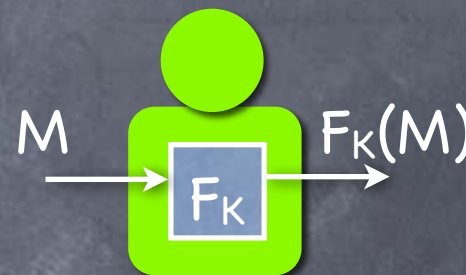
- $\text{MAC}_K(M) := F_K(M)$  where  $F$  is a PRF

- $\text{Ver}_K(M, S) := 1$  iff  $S = F_K(M)$

- Output length of  $F_K$  should be big enough

- If an adversary forges MAC with probability  $\epsilon_{\text{MAC}}$ , then can break PRF with advantage  $O(\epsilon_{\text{MAC}} - 2^{-m(k)})$  ( $m(k)$  being the output length of the PRF) [How?]

- If random function  $R$  used as MAC, then probability of forgery,  $\epsilon_{\text{MAC}}^* = 2^{-m(k)}$



Recall: Advantage in breaking a PRF  $F$  = diff in prob test has of outputting 1, when given  $F$  vs. truly random  $R$

# MAC for Multiple-Block Messages

- What if message is longer than one block?
- MAC'ing each block separately is not secure (unlike in the case of CPA secure encryption)
  - Eve can rearrange the blocks/drop some blocks
- Coming up: two solutions
  1. A simple but inefficient scheme from MAC for single-block messages
  2. From a PRF (block cipher), build a PRF that takes longer inputs



# MAC for Multiple-Block Messages

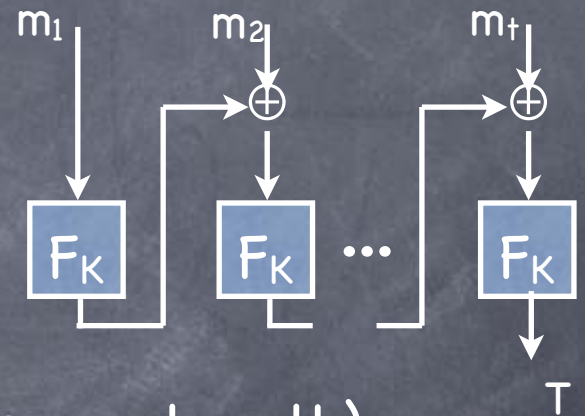
- A simple solution: “tie the blocks together”
  - Add to each block a random string  $r$  (same  $r$  for all blocks), total number of blocks, and a sequence number
    - $B_i = (r, t, i, M_i)$
    - $MAC(M) = (r, (MAC(B_i))_{i=1..t})$
    - $r$  prevents mixing blocks from two messages,  $t$  prevents dropping blocks and  $i$  prevents rearranging
- Inefficient! Tag length increases with message length

# CBC-MAC

- PRF domain extension: Chaining the blocks

- cf. CBC mode for encryption (which is not a MAC!)

- t-block messages, a single block tag



- Can be shown to be secure

- If restricted to t-block messages (i.e., same length)

- Else attacks possible (by extending a previously signed message)

- Security crucially relies on not revealing intermediate output blocks

# Patching CBC-MAC

- Patching CBC MAC to handle message of any (polynomial) length but still producing a single block tag (secure if block-cipher is):
  - Derive  $K$  as  $F_{K'}(t)$ , where  $t$  is the number of blocks
  - Use first block to specify number of blocks
    - Important that first block is used: if last block, message extension attacks still possible
  - EMAC: Output not the last tag  $T$ , but  $F_{K'}(T)$ , where  $K'$  is an independent key (after padding the message to an integral number of blocks). No need to know message length a priori.
  - CMAC: XOR last message block with a key (derived from the original key using the block-cipher). Also avoids padding when message is integral number of blocks. NIST Recommendation. 2005
- **Later**: Hash-based HMAC used in TLS and IPSec IETF Standard. 1997