Defining Encryption

Lecture 2

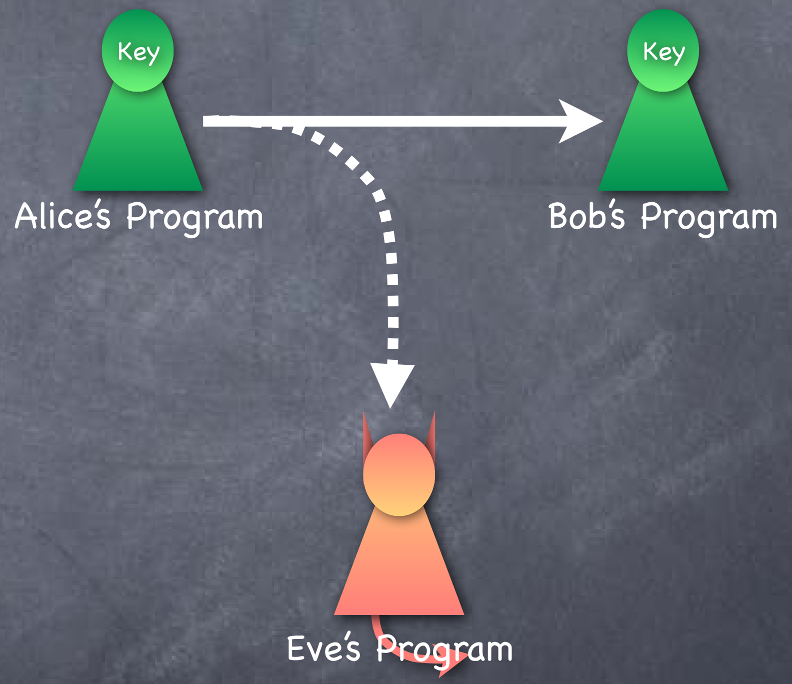
Simulation & Indistinguishability

Roadmap

- First, Symmetric Key Encryption
- Defining the problem
  - We'll do it elaborately, so that it will be easy to see different levels of security
- Solving the problem
  - In theory and in practice
- Today: one-time symmetric-key encryption

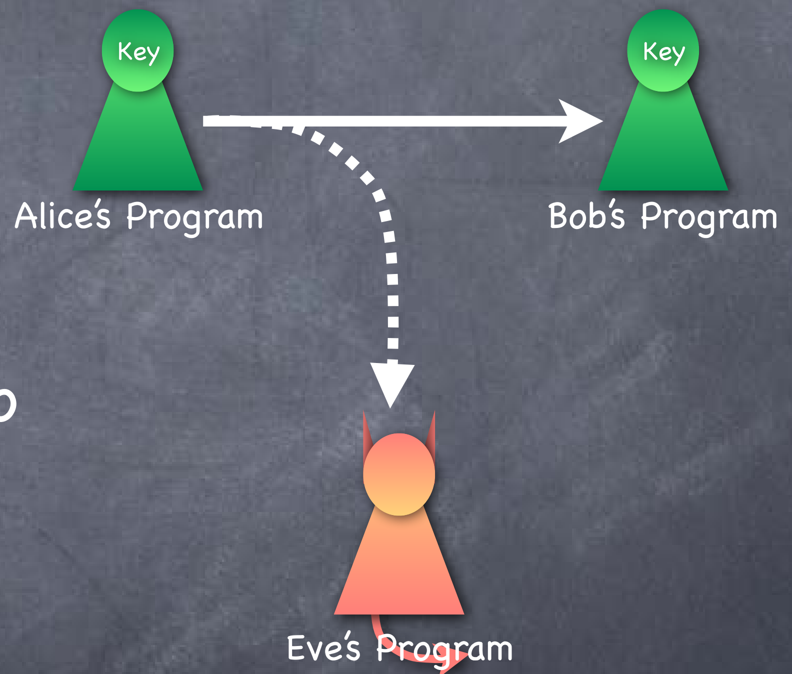
Building the Model

- Alice, Bob and Eve. Alice and Bob share a key (a bit string)
- Alice wants Bob to learn a message, "without Eve learning it"
- Alice can send out a bit string on the channel. Bob and Eve both get it



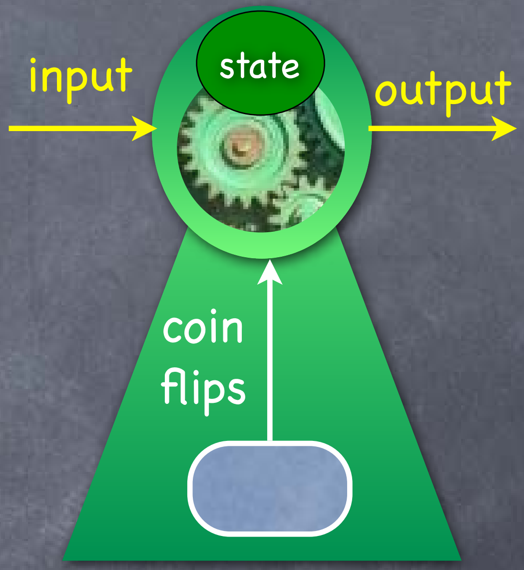
Encryption: Syntax

- Three algorithms
  - Key Generation: What Alice and Bob do a priori, for creating the shared secret key
  - Encryption: What Alice does with the message and the key to obtain a "ciphertext"
  - Decryption: What Bob does with the ciphertext and the key to get the message out of it
- All of these are (probabilistic) computations



Modeling Computation

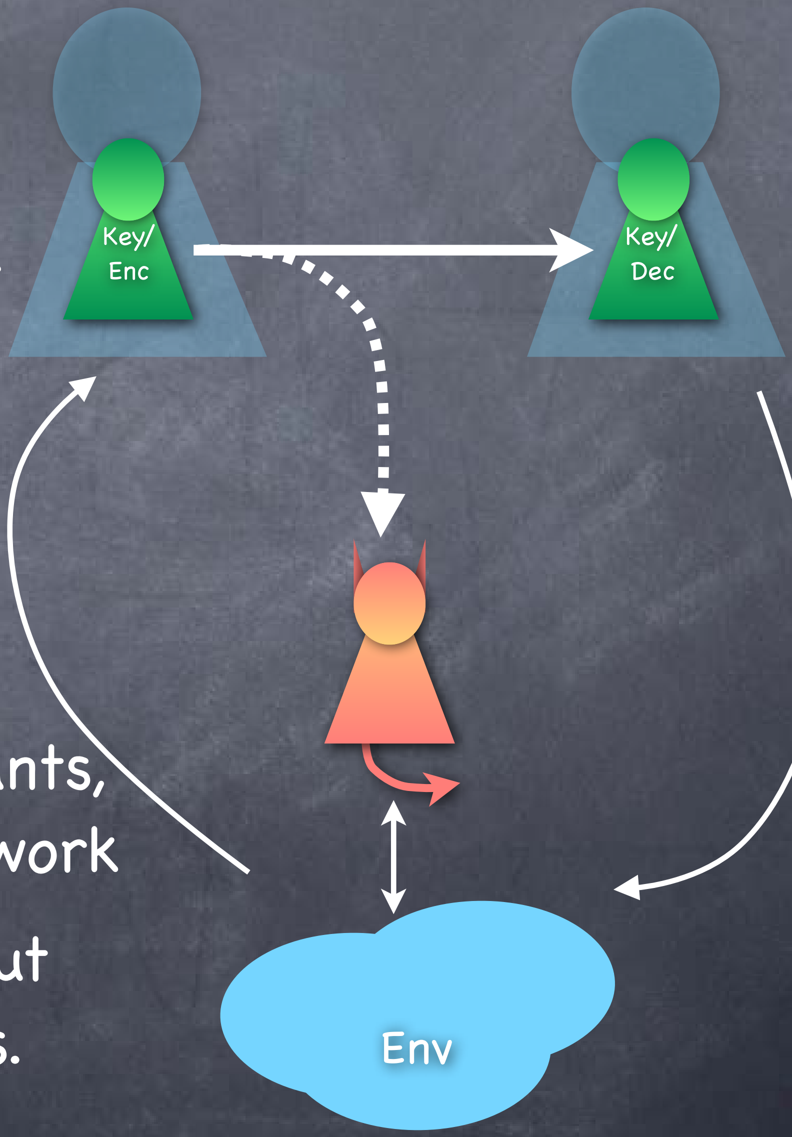
- In our model (standard model) parties are programs (computations, say Turing Machines)
- Effect of computation limited to be in a blackbox manner (only through input/output functionality)
  - No side-information (timing, electric signals, ...) unless explicitly modeled
  - Can be probabilistic
  - Sometimes stateful



Ideal coin flips: If n coins flipped, each outcome has probability 2^{-n}

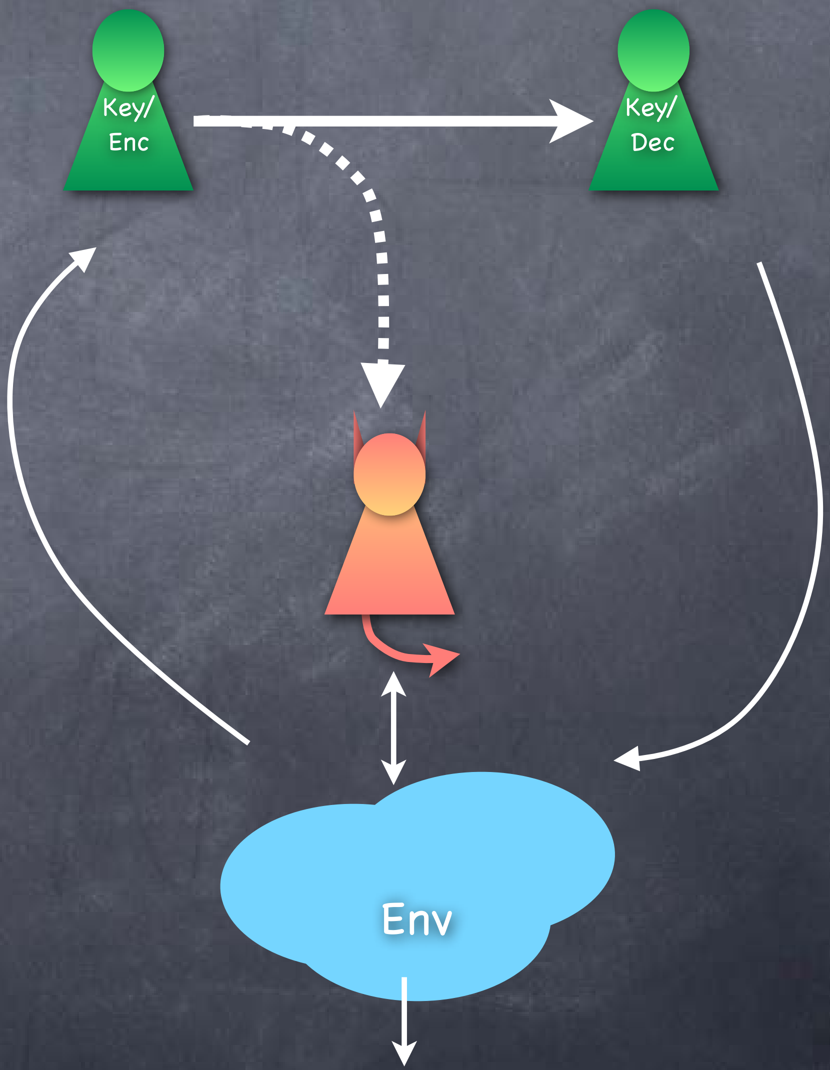
The Environment

- Where does the message come from?
  - Eve might already have partial information about the message, or might receive such information later
  - In fact, Eve might influence the choice of the message
- The environment
  - Includes the operating systems and other programs run by the participants, as well as other parties, if in a network
  - Abstract entity from which the input comes and to which the output goes. Arbitrarily influenced by Eve



Defining Security

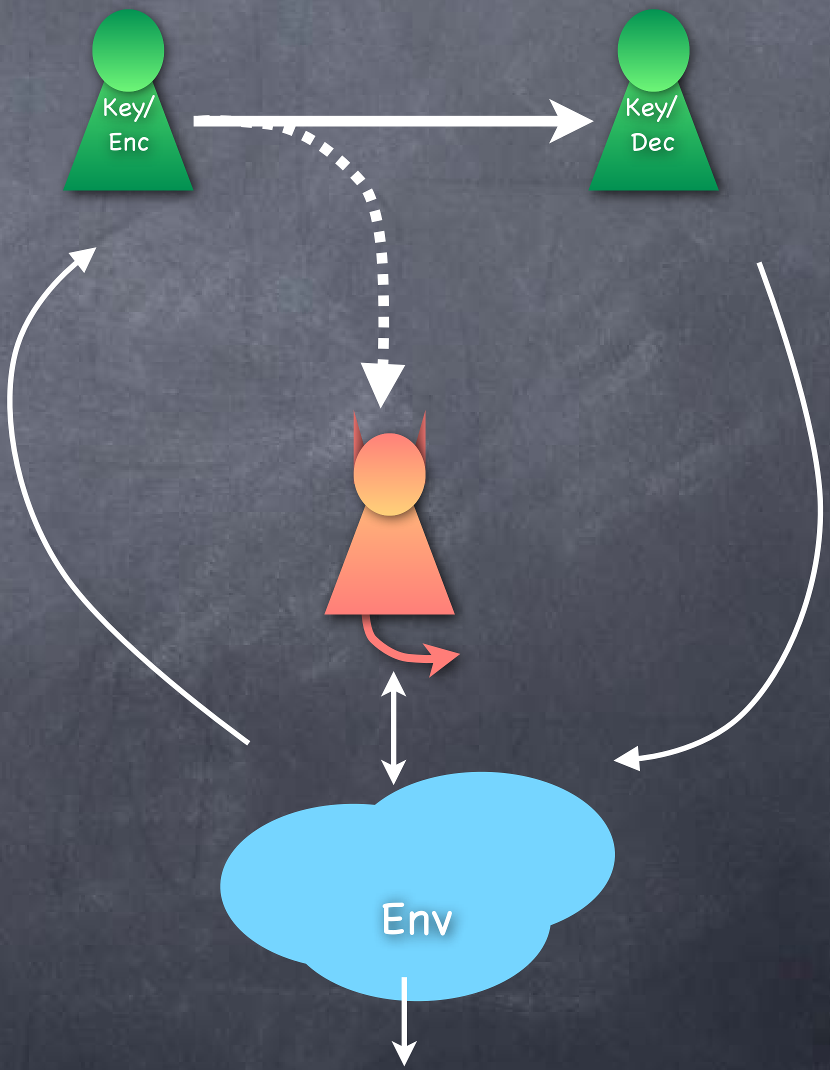
- Eve shouldn't be able to produce any "bad effects" in any environment
  - Or increase the probability of "bad effects"
- Effects in the environment: modeled as a bit in the environment (called the output bit)
- What is bad?
  - Anything that Eve couldn't have caused if an "ideal channel" was used



Defining Security

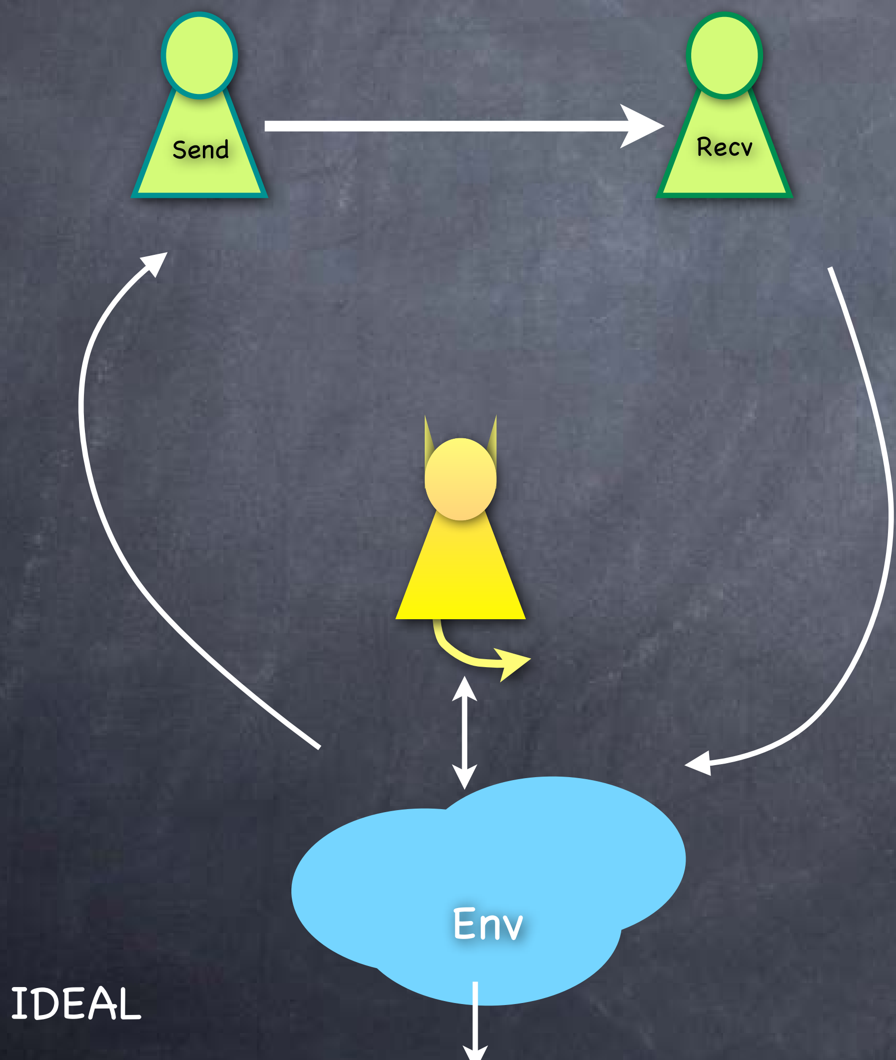
The REAL/IDEAL Paradigm

- Eve shouldn't produce any more effects than she could have in the ideal world
  - IDEAL world: Message sent over a (physically) secure channel. No encryption in this world.
  - REAL world: Using encryption
  - Encryption is secure if whatever Eve can do in the REAL world (using some strategy), she can do in the IDEAL world too (using an appropriate strategy)

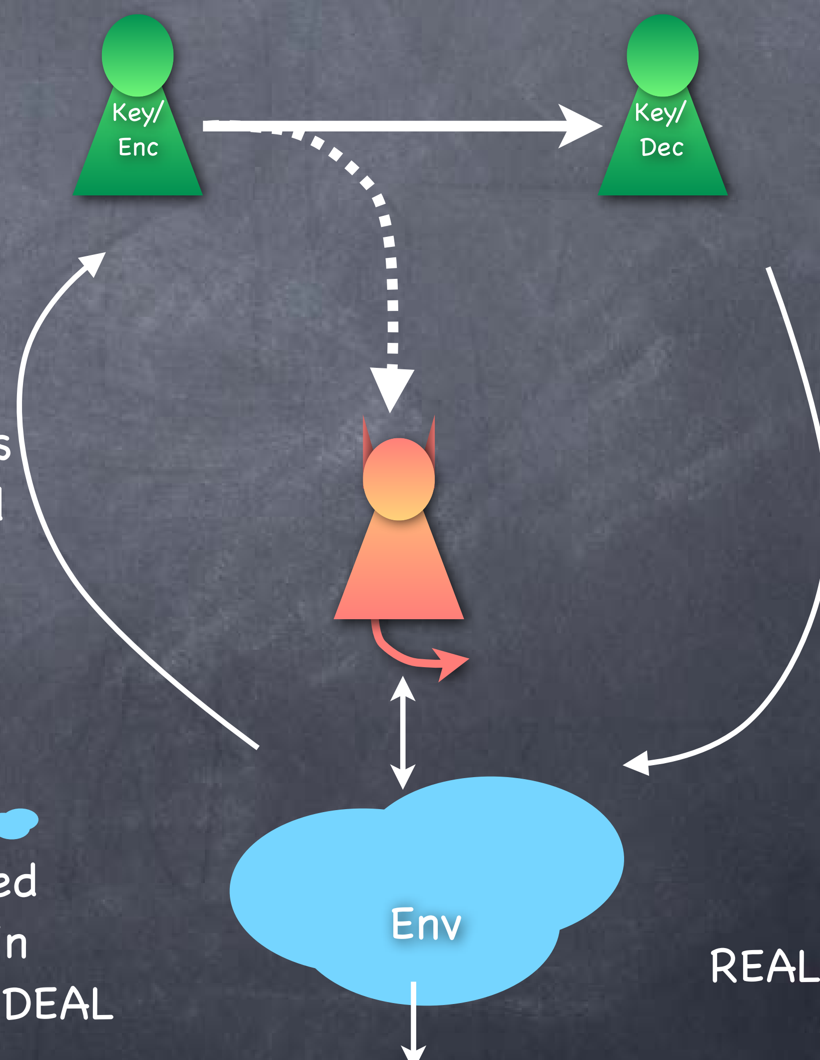


Defining Security

The REAL/IDEAL Paradigm



A scheme is secure (and correct) if:
∀
∃ s.t.
∀
output of
is distributed identically in REAL and IDEAL



Ready to go...

- REAL/IDEAL (a.k.a simulation-based) security forms the basic template for a large variety of security definitions
- We will see three definitions of symmetric-key encryption
  - Security of "one-time encryption" — today
  - Security of (muti-message) encryption
  - Security against "active attacks"
- Will also see alternate (but essentially equivalent) security definitions

Onetime Encryption

The Syntax

- Shared-key (Private-key) Encryption
  - Key Generation: Randomized
    - $K \leftarrow \mathcal{K}$, uniformly randomly drawn from the key-space (or according to a key-distribution)
  - Encryption: Deterministic
    - Enc: $\mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
  - Decryption: Deterministic
    - Dec: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$

Will change later (for more-than-once encryption)

Onetime Encryption

Perfect Secrecy

- **Perfect secrecy:** $\forall$ m, m′ $\in \mathcal{M}$
  - $\{Enc(m,K)\}_{K \leftarrow KeyGen} = \{Enc(m',K)\}_{K \leftarrow KeyGen}$

Distribution of the ciphertext

- Distribution of the ciphertext is defined by the randomness in the key
- In addition, require **correctness**
  - $\forall$ m, K, Dec(Enc(m,K), K) = m
- E.g. One-time pad: $\mathcal{M} = \mathcal{K} = \mathcal{C} = \{0,1\}^n$ and Enc(m,K) = m⊕K, Dec(c,K) = c⊕K
  - More generally $\mathcal{M} = \mathcal{K} = \mathcal{C} = \mathcal{G}$ (a finite group) and Enc(m,K) = m+K, Dec(c,K) = c−K

$\mathcal{M}$ \ $\mathcal{K}$	0	1	2	3
a	x	y	y	z
b	y	x	z	y

Assuming K uniformly drawn from $\mathcal{K}$

Pr[Enc(a,K)=x] = ¼,
Pr[Enc(a,K)=y] = ½,
Pr[Enc(a,K)=z] = ¼

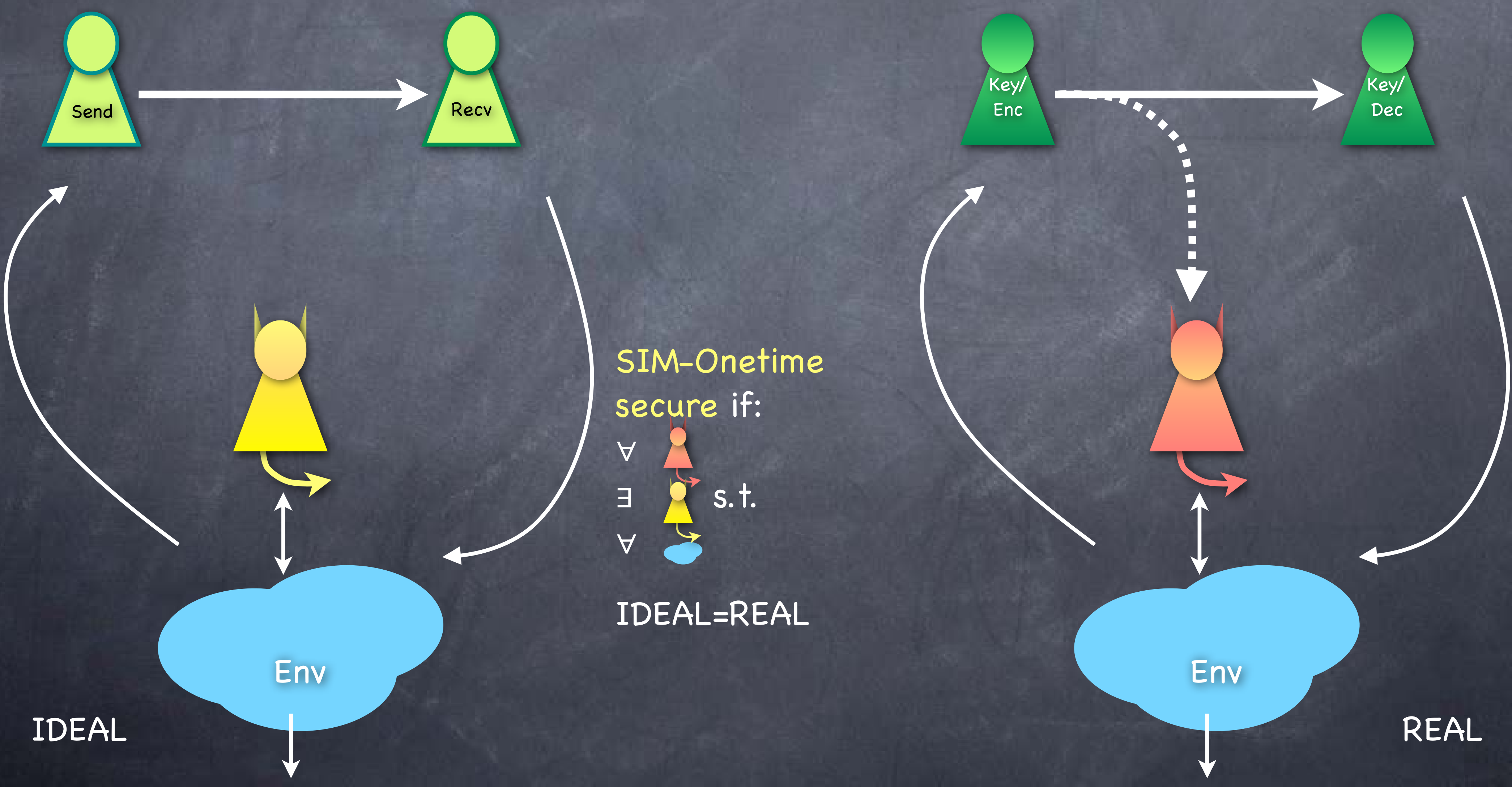
Same for Enc(b,K).

Onetime Encryption

SIM-Onetime Security

Equivalent to perfect secrecy + correctness

- Class of environments which send only one message



SIM-Onetime secure if:

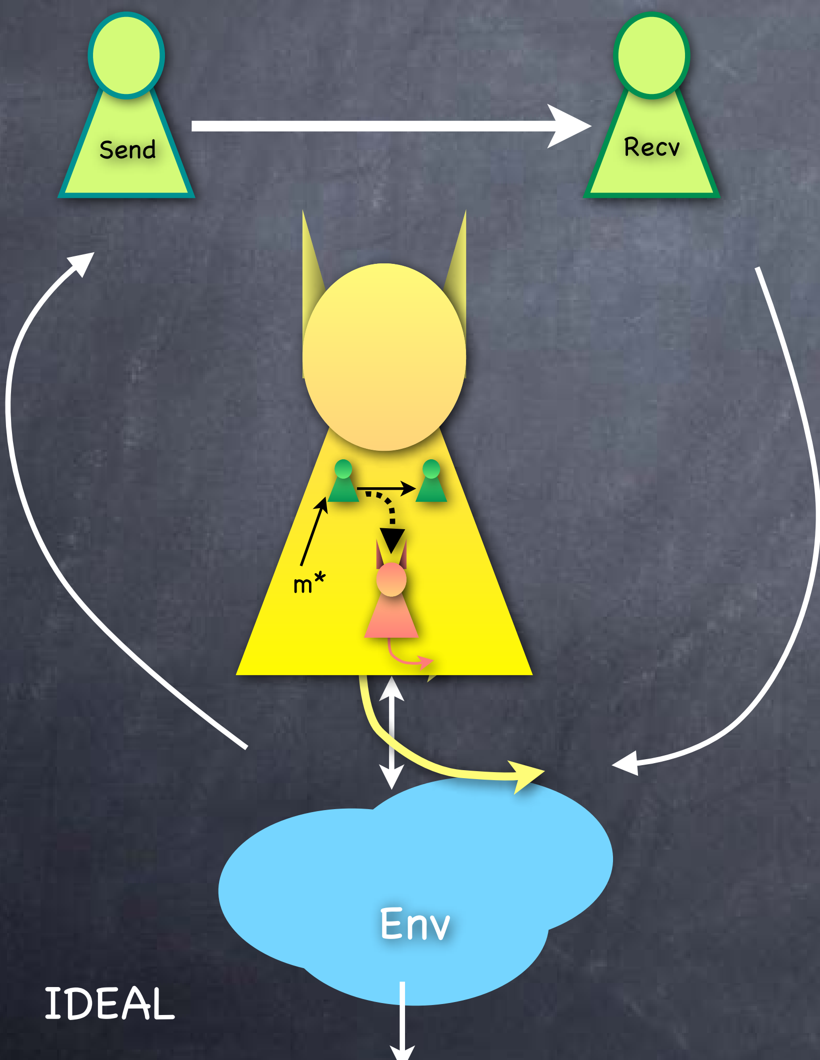
∀ [adversary]

∃ [simulator] s.t.

∀ [environment]

IDEAL=REAL

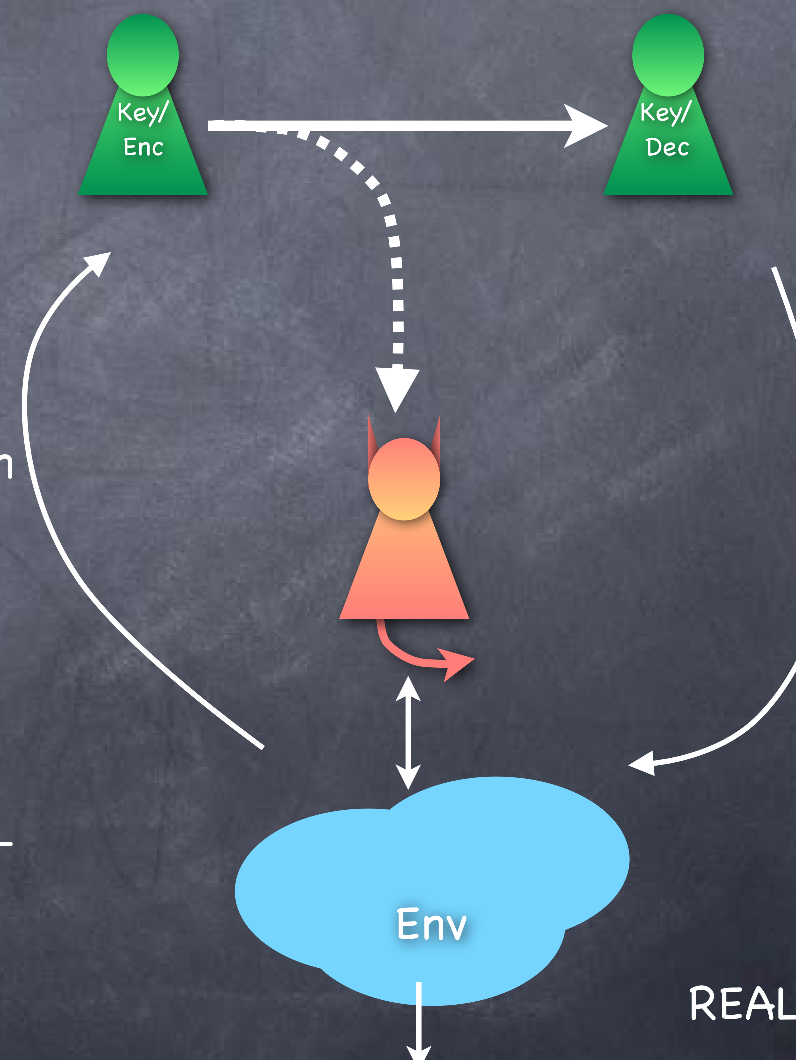
Perfect Secrecy + Correctness ⇒ SIM-Onetime Security



Consider this simulator: Runs adversary internally and lets it talk to the environment directly!
Feeds it encryption of a dummy message

Claim: IDEAL=REAL
(Consider view of

+ for both)



Implicit Details

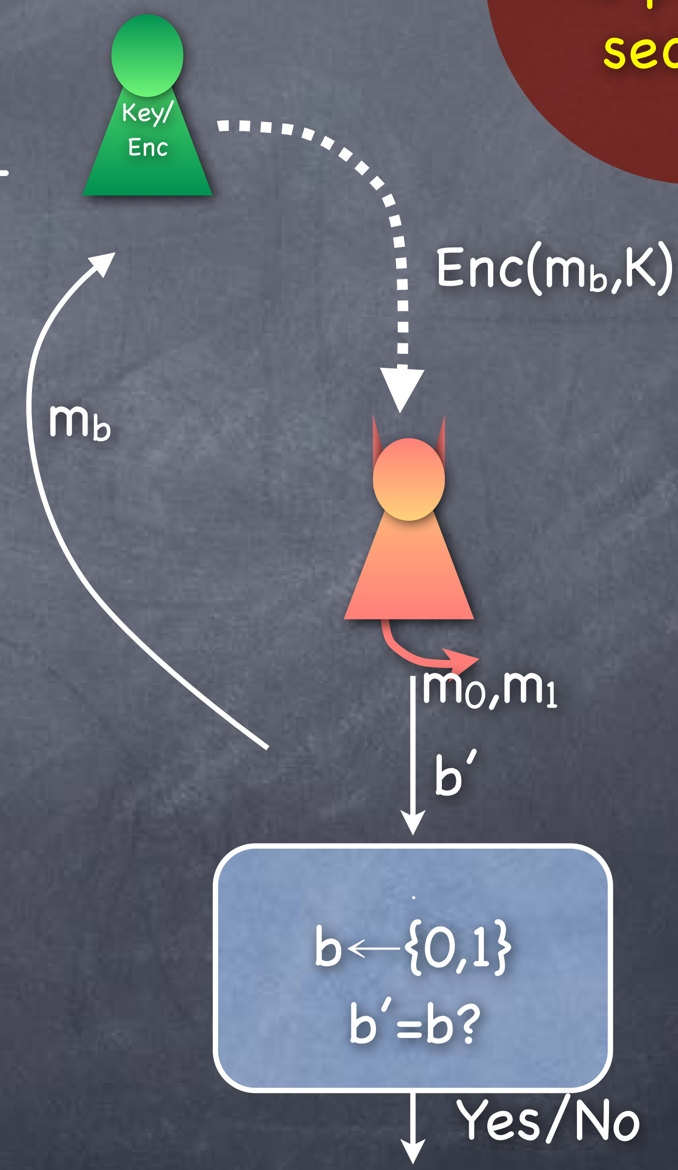
- Random coins used by the encryption scheme is kept private within the programs of the scheme (KeyGen, Enc, Dec)
  - If key is used for anything else (i.e., leaked to the environment) no more guarantees
  - In particular, key can't be the message (no "circularity")
- In REAL, Eve+Env's only inputs are ciphertext and Bob's output
  - In particular no timing attacks
- Message space is finite and known to Eve (and Eve')
  - Alternately, if message length is variable, it is given out to Eve' in IDEAL as well
  - Also, Eve' allowed to learn the fact that a message is sent

Onetime Encryption

IND-Onetime Security

Equivalent to perfect secrecy

- IND-Onetime Experiment
  - Experiment picks a random bit b. It also runs KeyGen to get a key K
  - Adversary sends two messages m_0, m_1 to the experiment
  - Experiment replies with $Enc(m_b,K)$
  - Adversary returns a guess b′
  - Experiments outputs 1 iff b′=b
- IND-Onetime secure if for every adversary, Pr[b′=b] = 1/2



Perspective on Definitions

- "Technical" vs. "Convincing"
- For simple scenarios technical definitions could be convincing
  - e.g. Perfect Secrecy
- IND- definitions tend to be technical: more low-level details, but may not make the big picture clear. Could have "weaknesses"
- SIM- definitions give the big picture, but may not give details of what is involved in satisfying it. Could be "too strong"
- Best of both worlds when they are equivalent:
  use IND- definition while say, proving security of a construction;
  use SIM- definition when low-level details are not important