Defining Encryption (ctd.)

Lecture 3

SIM & IND security

Beyond One-Time: CPA security

Computational Indistinguishability

Recall

Onetime Encryption

Perfect Secrecy

- **Perfect secrecy**: $\forall$ m, m′ $\in \mathcal{M}$
  - $\{Enc(m,K)\}_{K \leftarrow KeyGen} = \{Enc(m',K)\}_{K \leftarrow KeyGen}$

Distribution of the ciphertext is defined by the randomness in the key

- In addition, require **correctness**
  - $\forall$ m, K, Dec(Enc(m,K), K) = m
- E.g. One-time pad: $\mathcal{M} = \mathcal{K} = \mathcal{C} = \{0,1\}^n$ and Enc(m,K) = m$\oplus$K, Dec(c,K) = c$\oplus$K
  - More generally $\mathcal{M} = \mathcal{K} = \mathcal{C} = \mathcal{G}$ (a finite group) and Enc(m,K) = m+K, Dec(c,K) = c−K

$\mathcal{M}$ \ $\mathcal{K}$	0	1	2	3
a	x	y	y	z
b	y	x	z	y

Assuming K uniformly drawn from $\mathcal{K}$

Pr[Enc(a,K)=x] = ¼,
Pr[Enc(a,K)=y] = ½,
Pr[Enc(a,K)=z] = ¼

Same for Enc(b,K).

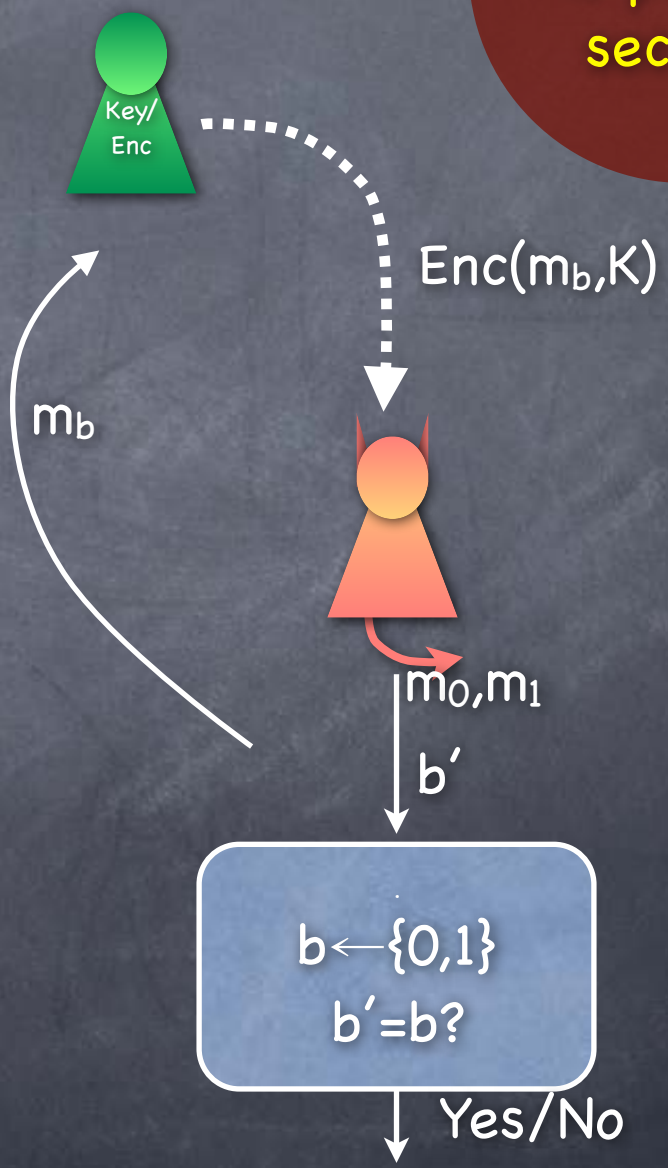
Recall

Onetime Encryption

IND-Onetime Security

Equivalent to perfect secrecy

- IND-Onetime Experiment
  - Experiment picks a random bit b. It also runs KeyGen to get a key K
  - Adversary sends two messages m_0, m_1 to the experiment
  - Experiment replies with Enc(m_b,K)
  - Adversary returns a guess b′
  - Experiments outputs 1 iff b′=b
- IND-Onetime secure if for every adversary, Pr[b′=b] = 1/2



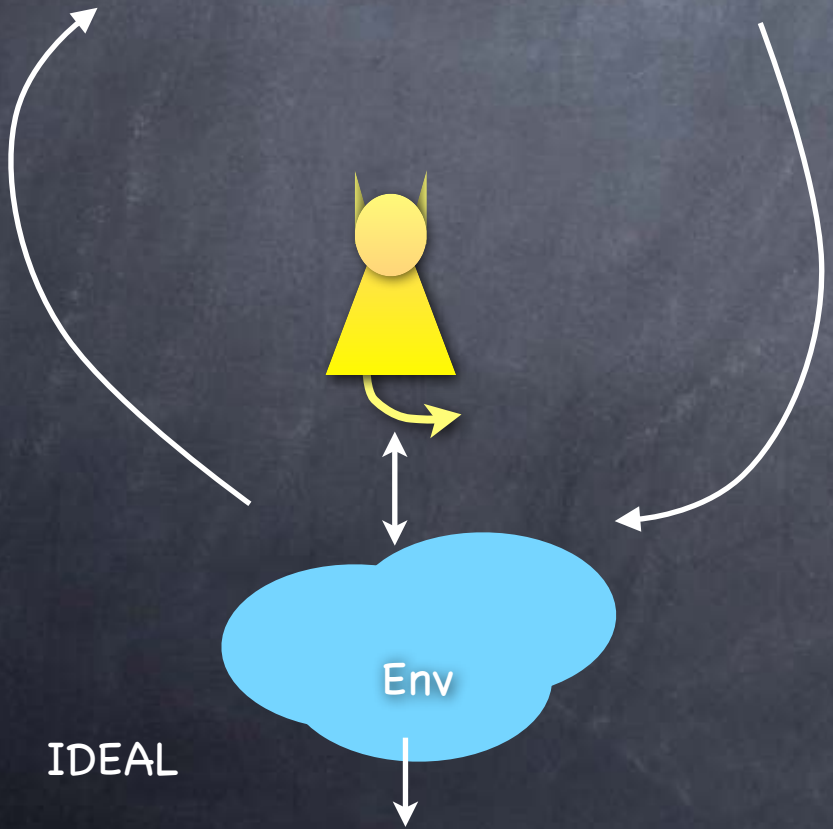
Recall

Onetime Encryption

SIM-Onetime Security

Equivalent to perfect secrecy + correctness

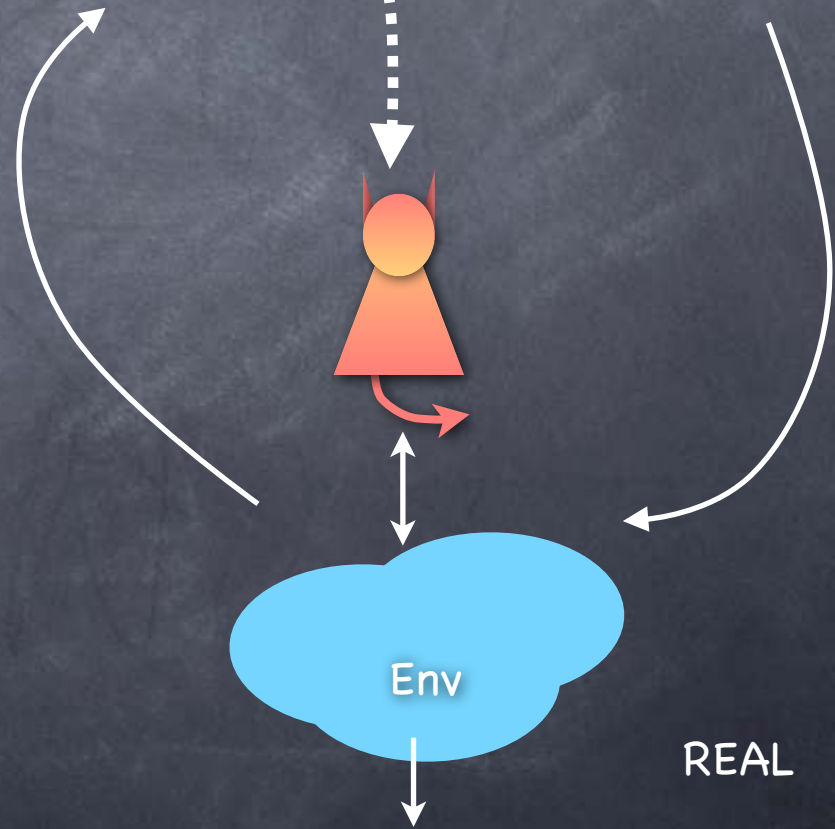
- Class of environments which send only one message



SIM-Onetime secure if:

$\forall$ $\exists$ s.t. $\forall$

IDEAL=REAL



Security of Encryption

- Perfect secrecy is too strong for multiple messages (though too weak in some other respects...)
  - Requires keys as long as the messages
- Relax the requirement by restricting to computationally bounded adversaries (and environments)
- Coming up: Formalizing notions of "computational" security (as opposed to perfect/statistical security)
  - Then, security definitions used for encryption of multiple messages

Symmetric-Key Encryption

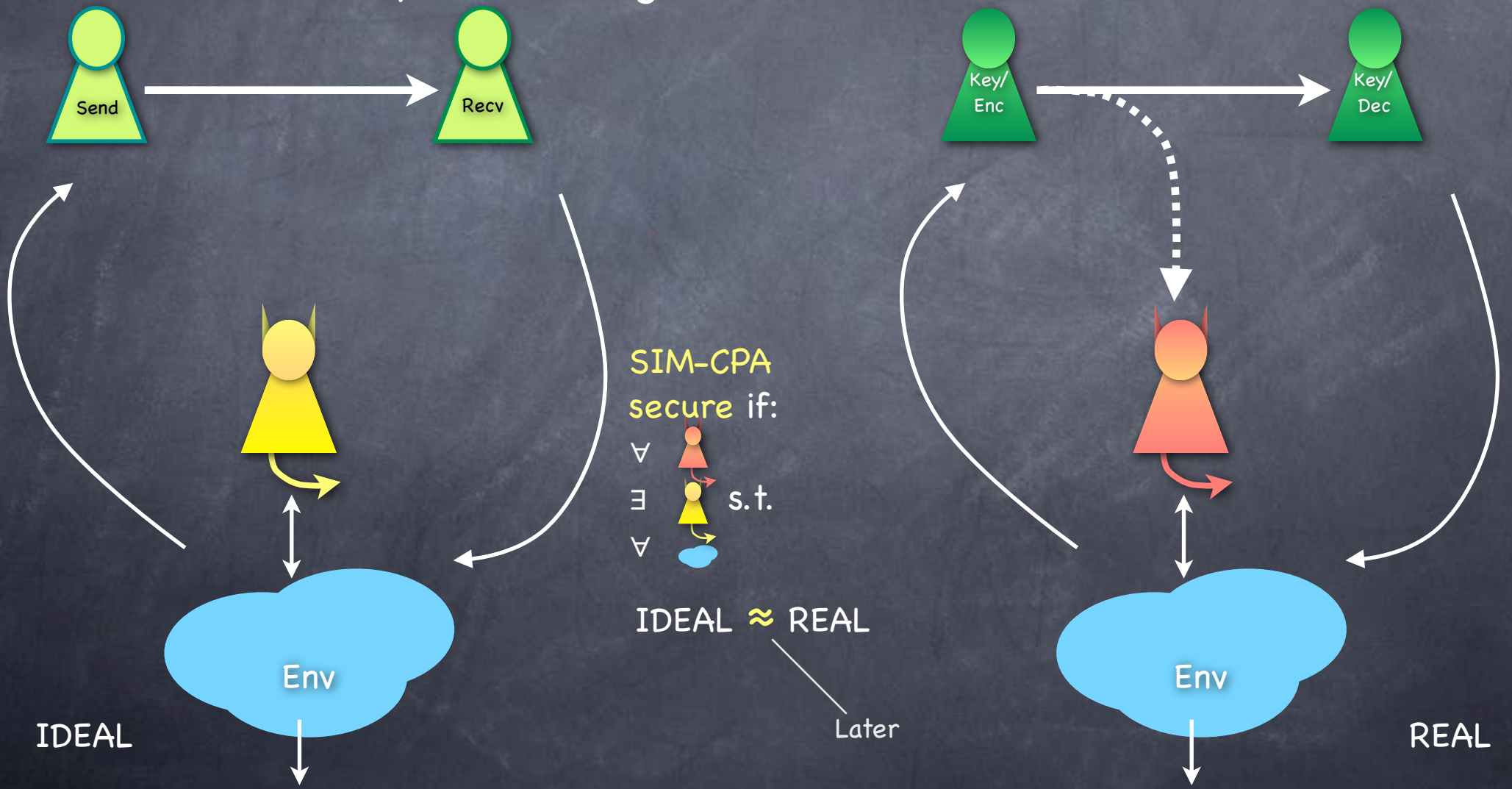
The Syntax

- Shared-key (Private-key) Encryption
  - Key Generation: Randomized
    - $K \leftarrow \mathcal{K}$, uniformly randomly drawn from the key-space (or according to a key-distribution)
  - Encryption: Randomized
    - Enc: $\mathcal{M} \times \mathcal{K} \times \mathcal{R} \rightarrow \mathcal{C}$. During encryption a fresh random string will be chosen uniformly at random from $\mathcal{R}$
  - Decryption: Deterministic
    - Dec: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$

Symmetric-Key Encryption

SIM-CPA Security

- Same as SIM-onetime security, but not restricted to environments which send only one message. All entities "efficient."

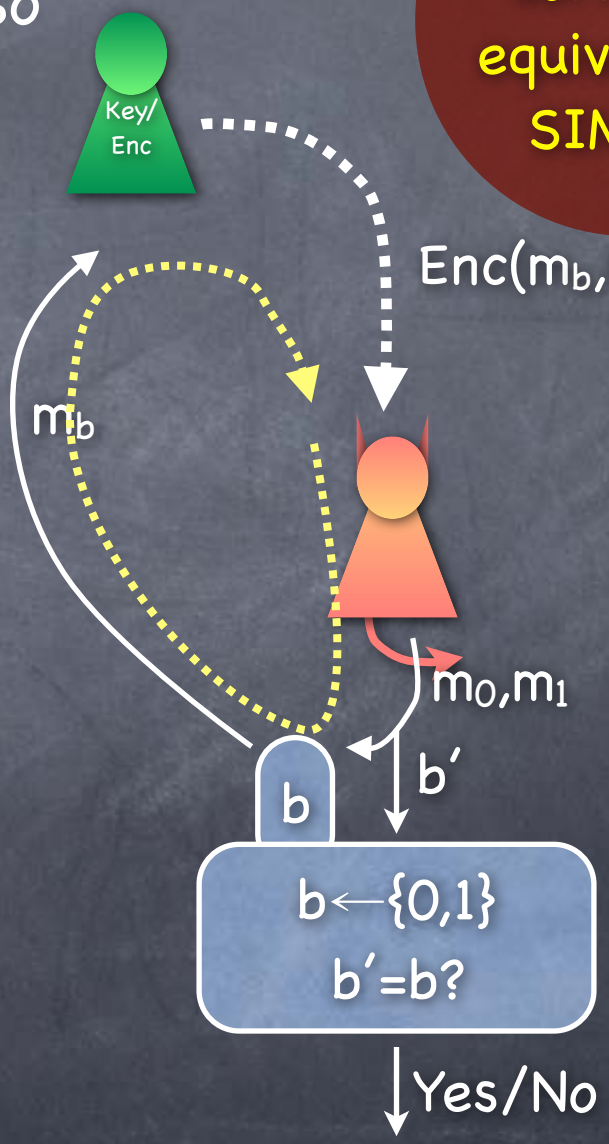


Symmetric-Key Encryption

IND-CPA Security

- Experiment picks a random bit b. It also runs KeyGen to get a key K
  - For as long as Adversary wants
    - Adv sends two messages m_0, m_1 to the experiment
    - Expt returns $Enc(m_b,K)$ to the adversary
  - Adversary returns a guess b′
  - Experiment outputs 1 iff b′=b
- IND-CPA secure if for all "efficient" adversaries $Pr[b'=b] \approx 1/2$

IND-CPA + ~correctness equivalent to SIM-CPA

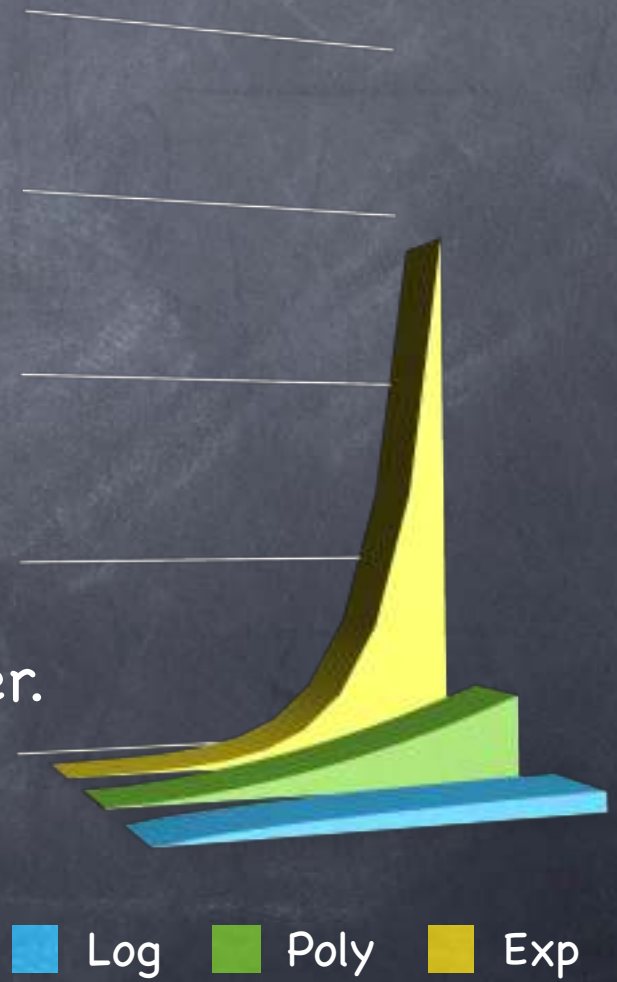


Definitions Summary

- Security definitions:
  - SIM-Onetime = IND-Onetime/Perfect Secrecy + correctness
  - SIM-CPA = IND-CPA + ~correctness: allows using the same key for multiple messages
  - Later: SIM-CCA = IND-CCA + ~correctness: allows active attacks
- Next
  - For multi-message schemes we relaxed the "perfect" simulation requirement
  - But what is $\approx$?

Feasible Computation

- In analyzing complexity of algorithms: Rate at which computational complexity grows with input size
  - e.g. Can do sorting in O(n log n)
- Only the rough rate considered
  - Exact time depends on the technology
  - Real question: Do we scale well? How much more computation will be needed as the instances of the problem get larger.
  - "Polynomial time" (O(n), $O(n^2)$, $O(n^3)$, ...) considered feasible



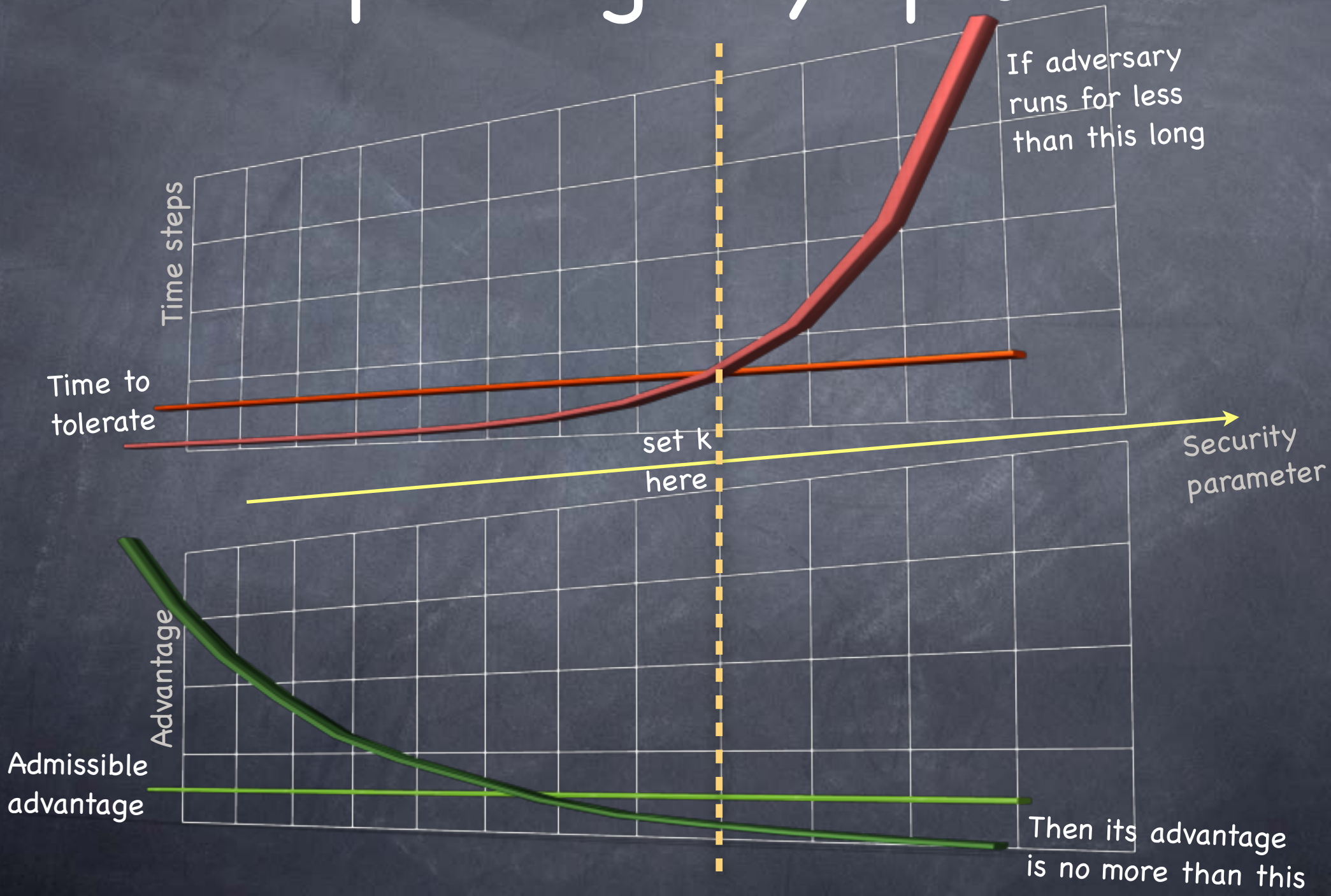
Infeasible Computation

- "Super-Polynomial time" considered infeasible
  - e.g. 2^n, $2^{\sqrt{n}}$, $n^{\log(n)}$
  - i.e., as n grows, quickly becomes "infeasibly large"
- Can we make breaking security infeasible for Eve?
  - What is n (that can grow)?
  - Message size?
    - We need security even if sending only one bit!

Security Parameter

- A parameter that is part of the encryption scheme
  - Not related to message size
  - A knob that can be used to set the security level
  - Will denote by k
- Security guarantees are given asymptotically as a function of the security parameter

Interpreting Asymptotics



Feasible and Negligible

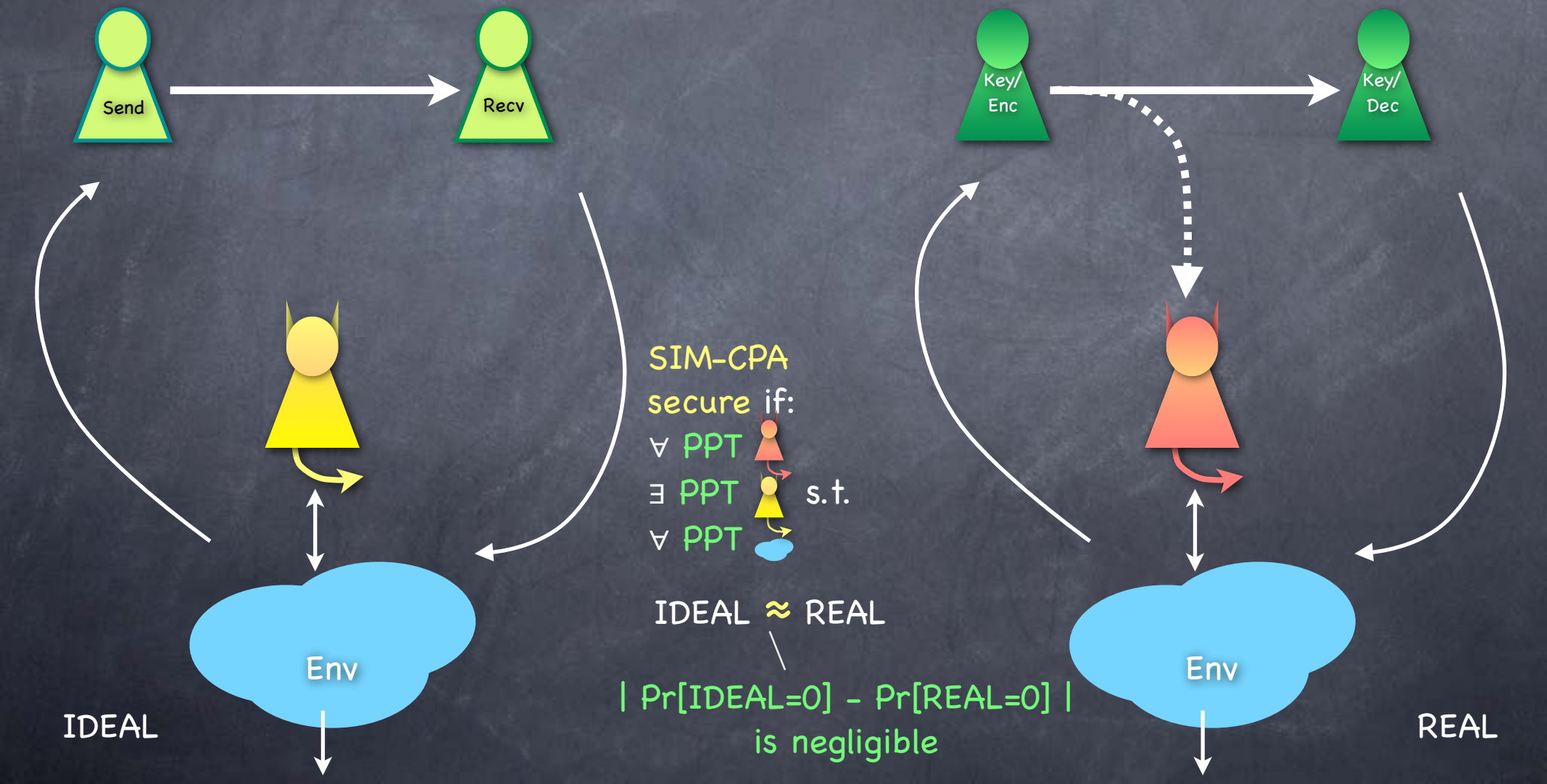
- We want to tolerate Eves who have a running time bounded by some polynomial in k
  - Eve could toss coins: Probabilistic Polynomial-Time (PPT)
  - It is better that we allow Eve high polynomial times too (we'll typically tolerate some super-polynomial time for Eve)
    - But algorithms for Alice/Bob better be very efficient
  - Eve could be non-uniform: a different strategy for each k
- Such an Eve should have only a "negligible" advantage (or, should cause at most a "negligible" difference in the behavior of the environment in the SIM definition)
  - What is negligible?

Negligibly Small

- A negligible quantity: As we turn the knob the quantity should "decrease extremely fast"
  - Negligible: decreases as 1/superpoly(k)
    - i.e., faster than 1/poly(k) for every polynomial
    - e.g.: 2^{-k}, $2^{-\sqrt{k}}$, $k^{-(\log k)}$.
    - Formally: T negligible if $\forall c>0\ \exists k_0\ \forall k>k_0\ \ T(k) < 1/k^c$
  - So that negl(k) × poly(k) = negl'(k)
    - Needed, because Eve can often increase advantage polynomially by spending that much more time/by seeing that many more messages

Symmetric-Key Encryption

SIM-CPA Security



SIM-CPA secure if:

$\forall$ PPT $\exists$ PPT s.t. $\forall$ PPT

IDEAL $\approx$ REAL

$|\ Pr[IDEAL=0] - Pr[REAL=0]\ |$ is negligible

Next

- Constructing (CPA-secure) SKE schemes
  - Pseudorandomness Generator (PRG)
  - One-Way Functions (& OW Permutations)
  - OWP → PRG → (CPA-secure) SKE