

Symmetric Key Cryptography

Lecture 8

MAC

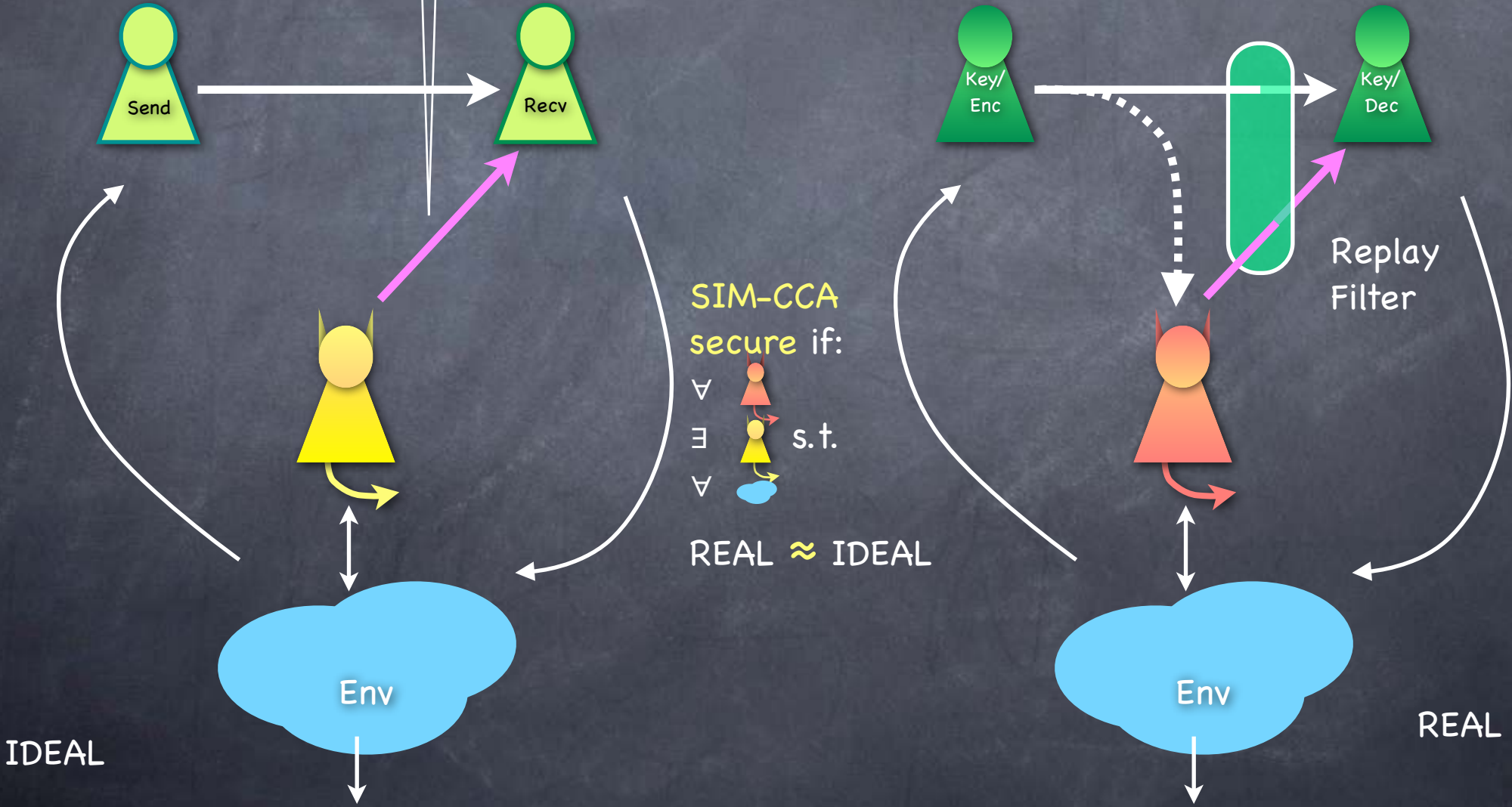
Summary

RECALL

Symmetric-Key Encryption

SIM-CCA Security

Authentication not required. i.e., Adversary allowed to send own messages (possibly "error")



RECALL

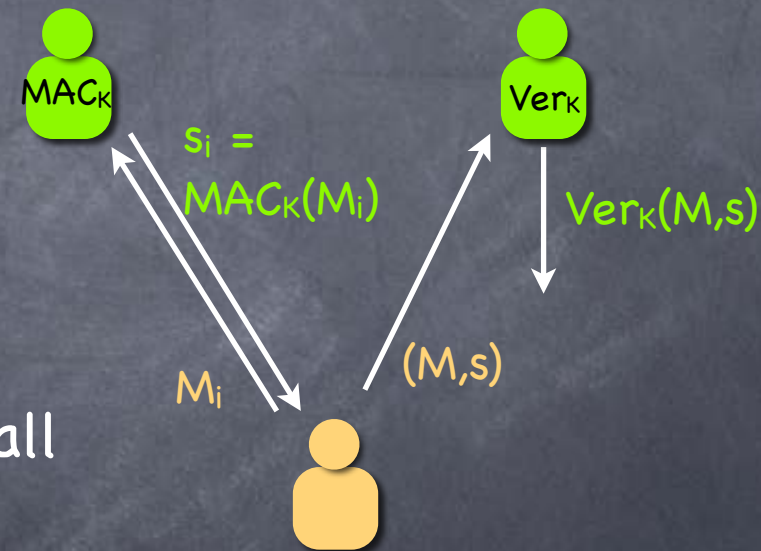
Encryption & Authentication

- CPA secure encryption: Block-cipher/CTR mode construction
- MAC: from a PRF or Block-Cipher
- CCA secure encryption: From CPA secure encryption and MAC. Encrypt-then-MAC. (Gives authentication also.)
- **SKE can be entirely based on Block-Ciphers**
 - A tool that can make things faster: Hash functions (later)

RECALL

Message Authentication Codes

- A single short key shared by Alice and Bob
 - Can sign any (polynomial) number of messages
- A triple (KeyGen, MAC, Verify)
- Correctness: For all K from KeyGen, and all messages M , $\text{Verify}_K(M, \text{MAC}_K(M))=1$
- Security: probability that an adversary can produce (M,s) s.t. $\text{Verify}_K(M,s)=1$ is negligible unless Alice produced an output $s=\text{MAC}_K(M)$



Advantage

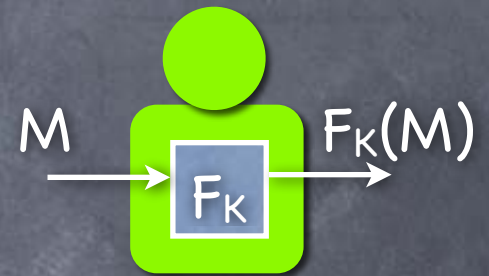
$$= \Pr[\text{Ver}_K(M,s)=1 \text{ and } (M,s) \notin \{(M_i,s_i)\}]$$

RECALL

MAC from PRF

When Each Message is a Single Block

- PRF is a MAC!
 - $MAC_K(M) := F_K(M)$ where F is a PRF
 - $Ver_K(M,S) := 1$ iff $S=F_K(M)$
 - Output length of F_K should be big enough
- If an adversary forges MAC with probability ϵ_{MAC} , then can break PRF with advantage $O(\epsilon_{MAC} - 2^{-m(k)})$ ($m(k)$ being the output length of the PRF) [How?]
 - If random function R used as MAC, then probability of forgery, $\epsilon_{MAC}^* = 2^{-m(k)}$



Recall: Advantage in breaking a PRF $F =$ diff in prob test has of outputting 1, when given F vs. truly random R

RECALL

MAC for Multiple-Block Messages

- A simple solution: "tie the blocks together"
 - Add to each block a random string r (same r for all blocks), total number of blocks, and a sequence number
 - $B_i = (r, t, i, M_i)$
 - $MAC(M) = (r, (MAC(B_i))_{i=1..t})$
 - r prevents mixing blocks from two messages, t prevents dropping blocks and i prevents rearranging
- Inefficient! Tag length increases with message length

CBC-MAC

- PRF domain extension: Chaining the blocks

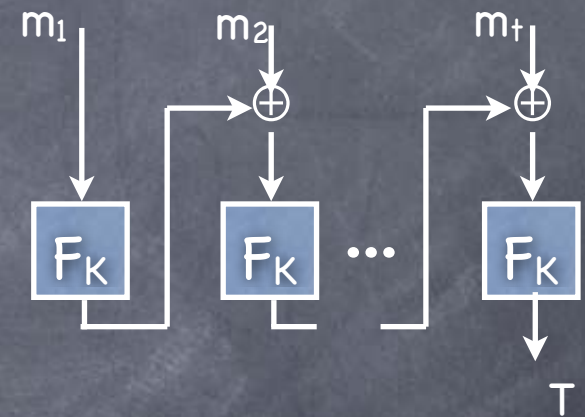
- cf. CBC mode for encryption (which is not a MAC!)

- t-block messages, a single block tag

- Can be shown to be secure

- If restricted to t-block messages (i.e., same length)

- Else **length-extension attacks** possible (by extending a previously signed message)



Patching CBC-MAC

- Patching CBC MAC to handle message of any (polynomial) length but still producing a single block tag (secure if block-cipher is):
 - Derive K as $F_{K'}(t)$, where t is the number of blocks
 - Use first block to specify number of blocks
 - Important that first block is used: if last block, message extension attacks still possible
 - EMAC: Output not the last tag T , but $F_{K'}(T)$, where K' is an independent key (after padding the message to an integral number of blocks). No need to know message length a priori.
 - CMAC: XOR last message block with a key (derived from the original key using the block-cipher). NIST Recommendation. 2005
- **Later**: Hash-based HMAC used in TLS and IPSec IETF Standard. 1997

Authenticated Encryption

- Doing encryption + authentication better
 - Generic composition: encrypt, then MAC MAC-then-encrypt is not necessarily CCA-secure
 - Needs two keys and two passes
- AE aims to do this more efficiently
 - Several constructions based on block-ciphers (modes of operation) provably secure modeling block-cipher as PRP
 - One pass: IAPM, OCB, ... [patented]
 - Two pass: CCM, GCM, SIV, ... [included in NIST standards]
 - AE with Associated Data: Allows unencrypted (but authenticated) parts of the plaintext, for headers etc.

SKE in Practice

Stream Ciphers

- A key should be used for only a single stream
- RC4, eSTREAM portfolio, ...
- In practice, stream ciphers take a key and an "IV" (initialization vector) as inputs
 - Heuristic goal: behave somewhat like a PRF (instead of a PRG) so that it can be used for multi-message encryption
 - But often breaks if used this way
- NIST Standard: For multi-message encryption, use a block-cipher in CTR mode

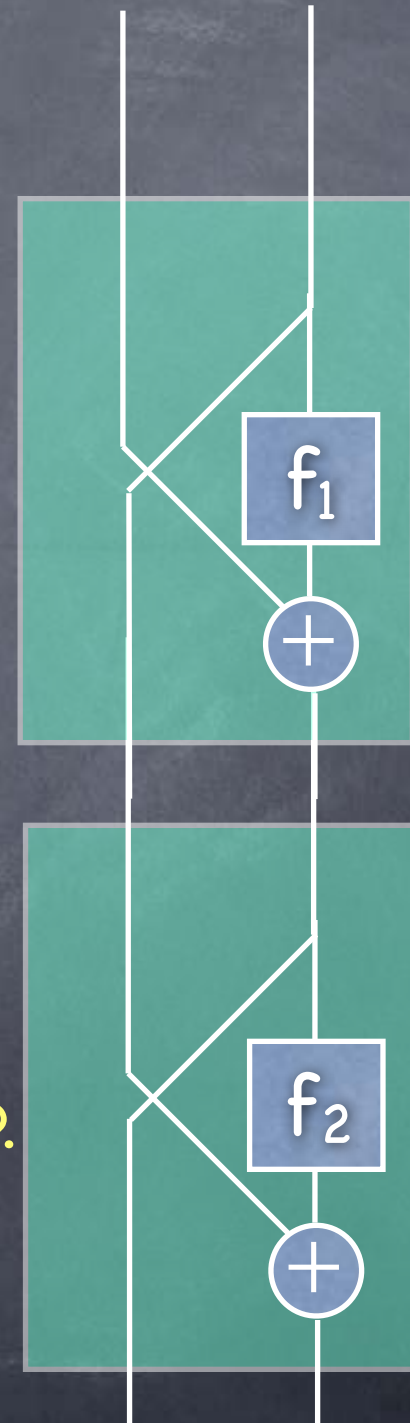
Also used to denote the random nonce chosen for encryption using a block-cipher

Block Ciphers

- DES, 3DES, Blowfish, AES, ...
 - Heuristic constructions
 - Permutations that can be inverted with the key
 - Speed (hardware/software) is of the essence
 - But should withstand known attacks
 - As a PRP (or at least, against key recovery)

Feistel Network

- Building a permutation from a (block) function
 - Let $f: \{0,1\}^m \rightarrow \{0,1\}^m$ be an arbitrary function
 - $F_f: \{0,1\}^{2m} \rightarrow \{0,1\}^{2m}$ defined as $F_f(x,y) = (y, x \oplus f(y))$
 - F_f is a permutation (Why?)
 - Can invert (How?)
 - Given functions f_1, \dots, f_t can build a t -layer Feistel network $F_{f_1 \dots f_t}$
 - Still a permutation from $\{0,1\}^{2m}$ to $\{0,1\}^{2m}$
- **Luby-Rackoff:** A 3-layer Feistel network with PRFs (with independent seeds) as round functions is a PRP. A 4-layer Feistel of PRFs gives a strong PRP.
- Fewer layers do not suffice! [Exercise]



DES Block Cipher

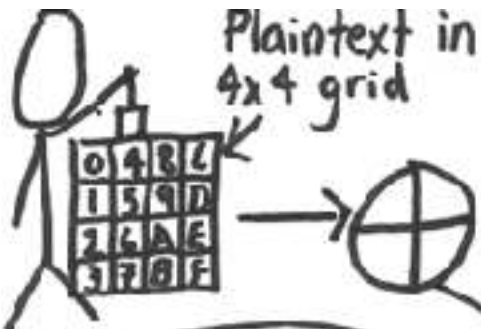
NIST Standard. 1976

- Data Encryption Standard (DES), Triple-DES, DES-X
- DES uses a 16-layer Feistel network (and a few other steps)
 - The round functions are not PRFs, but ad hoc
 - “Confuse and diffuse”
 - Defined for fixed key/block lengths (56 bits and 64 bits); key is used to generate subkeys for round functions
- DES’s key length too short
 - Can now mount brute force key-recovery attacks (e.g. using \$10K hardware, running for under a week, in 2006; now, in under a day)
- DES-X: extra keys to pad input and output
- Triple DES: 3 successive applications of DES (or DES^{-1}) with 3 keys

AES Block Cipher

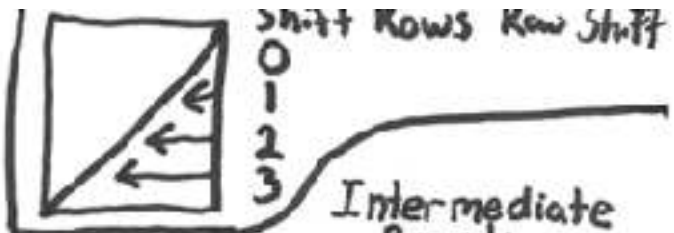
NIST Standard. 2001

- Advanced Encryption Standard (AES)
 - AES-128, AES-192, AES-256 (3 key sizes; block size = 128 bits)
 - Very efficient in software implementations (unlike DES)
 - Uses "Substitute-and-Permute" instead of Feistel networks
 - Has some algebraic structure
 - Operations in a vector space over the field $GF(2^8)$
 - The algebraic structure may lead to "attacks"? Not yet.
 - Some implementations may lead to side-channel attacks (e.g. cache-timing attacks)
 - Widely considered secure, but no "simple" hardness assumption known to imply any sort of security for AES



AES Crib Sheet

(Handy for memorizing)



General Math

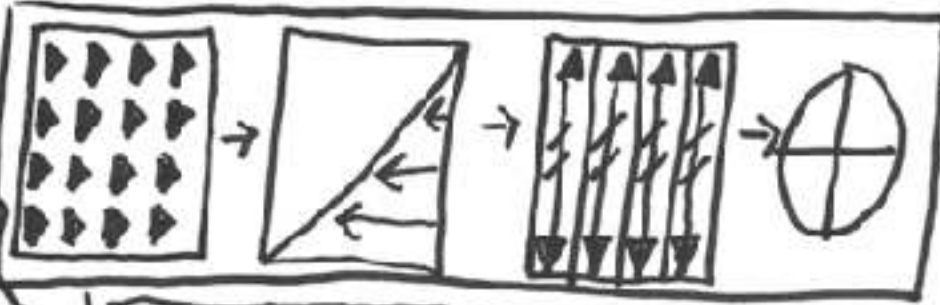
1.1B = AES Polynomial = $x^8 + x^4 + x^3 + x + 1$

Fast Multiply

$x \cdot a(x) = (a \ll 1) \oplus (a_7 = 1) ? 1B : 00$

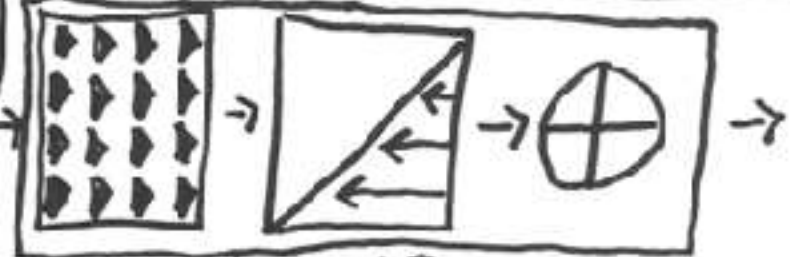
$\log(x \cdot y) = \log(x) + \log(y)$

Use $(x+1) = 03$ for log base



Intermediate Rounds

#	Key
9	128
11	192
13	256



Ciphertext

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

S-Box (SRD)

$SRD[a] = f(g(a))$

$g(a) = a^{-1} \text{ mod } m(x)$

See Think $5^3 \oplus 6^3$

5 is and 3's $[0110\ 0011]^T$

11111000	a_7	01100011
01111100	a_6	01100011
00111110	a_5	01100011
00011111	a_4	01100011
10001111	a_3	01100011
11000111	a_2	01100011
11000111	a_1	01100011
11100011	a_0	01100011

Key Expansion: Round Constants

First Column: 01 02 04 08 ...

S			
0	1	B	K
M	Z	I	E
E	T	Y	

Round Key

K	B3	01	B2
E	6E	00	6E
Y	CB	00	CB
	B7	00	B7

Other Columns:

T	E1	C1
Z	Z1	10
B	66	B4
	F2	CA

Prev Col \oplus Col from Previous round key

Mix Columns: 21132

2	1	1	3
3	2	1	1
1	3	2	1
1	1	3	2

Inverse Mix

EBD9

E	B	D	9
9	E	B	D
D	9	E	B
B	D	9	E

$[a_3]$
 $[a_2]$
 $[a_1]$
 $[a_0]$

Cryptanalysis

- Attacking stream ciphers and block ciphers
 - Typically for key recovery
- Brute force cryptanalysis, using specialized hardware
 - e.g. Attack on DES in 1998
- Several other analytical techniques to speed up attacks
 - Sometimes “theoretical”: on weakened (“reduced round”) constructions, showing improvement over brute-force attack
 - Meet-in-the-middle, linear cryptanalysis, differential cryptanalysis, impossible differential cryptanalysis, boomerang attack, integral cryptanalysis, cube attack, ...

SKE today

- SKE in IPsec, TLS etc. mainly based on AES block-ciphers
 - AES-128, AES-192, AES-256
- A recommended choice: AES Counter-mode + CMAC (or HMAC), encrypt-then-MAC.
 - Gives CCA security, and provides authentication
 - (Standards don't all follow this choice, but still secure)
- Older components/modes still in use
 - Supported by many standards for legacy purposes
 - In many applications (sometimes with modifications)
 - e.g. RC4 in BitTorrent, (older) Skype, PDF