

Symmetric Key Cryptography

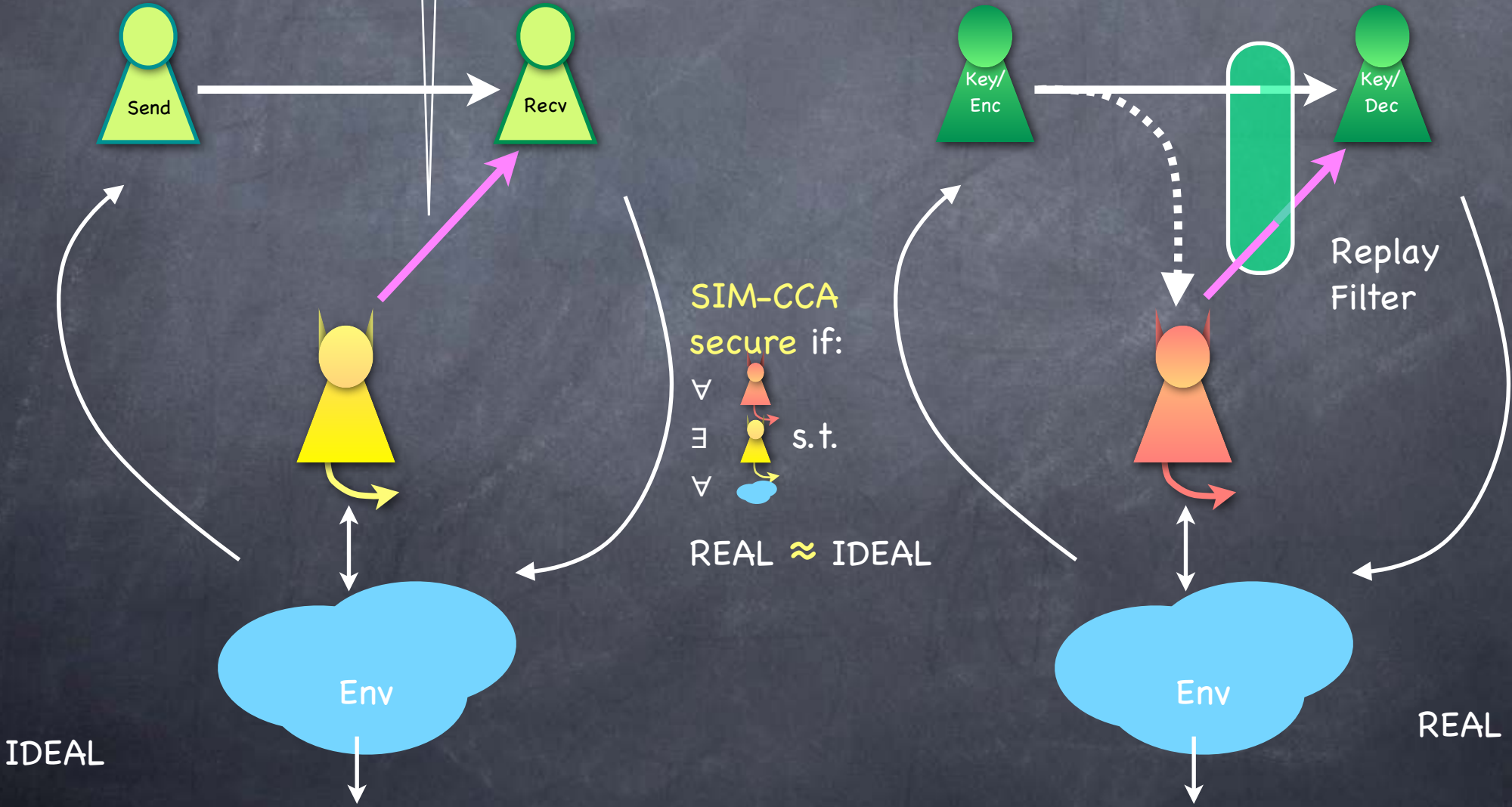
Lecture 8
Summary

RECALL

Symmetric-Key Encryption

SIM-CCA Security

Authentication not required. i.e., Adversary allowed to send own messages (possibly "error")



RECALL

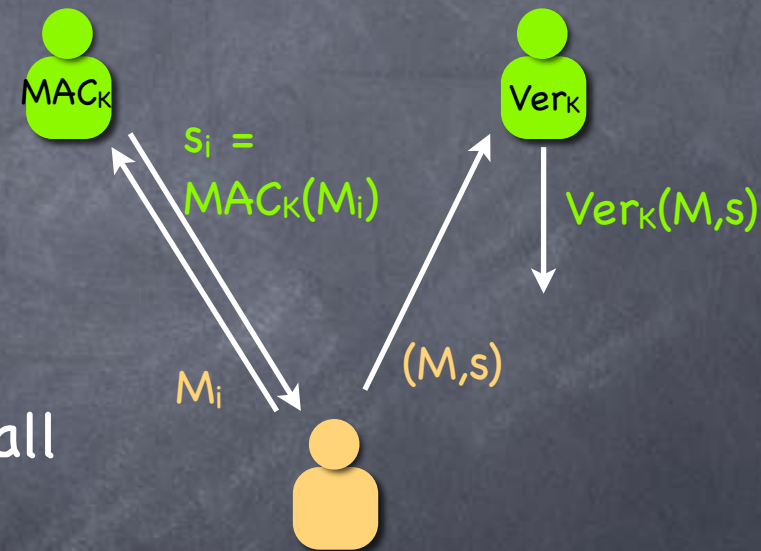
Encryption & Authentication

- CPA secure encryption: Block-cipher/CTR mode construction
- MAC: from a PRF or Block-Cipher
- CCA secure encryption: From CPA secure encryption and MAC. Encrypt-then-MAC. (Gives authentication also.)
- **SKE can be entirely based on Block-Ciphers**
 - A tool that can make things faster: Hash functions (later)

RECALL

Message Authentication Codes

- A single short key shared by Alice and Bob
 - Can sign any (polynomial) number of messages
- A triple (KeyGen, MAC, Verify)
- Correctness: For all K from KeyGen, and all messages M , $\text{Verify}_K(M, \text{MAC}_K(M))=1$
- Security: probability that an adversary can produce (M,s) s.t. $\text{Verify}_K(M,s)=1$ is negligible unless Alice produced an output $s=\text{MAC}_K(M)$



Advantage

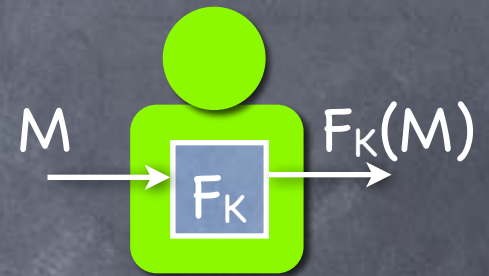
$$= \Pr[\text{Ver}_K(M,s)=1 \text{ and } (M,s) \notin \{(M_i,s_i)\}]$$

RECALL

MAC from PRF

When Each Message is a Single Block

- PRF is a MAC!
 - $MAC_K(M) := F_K(M)$ where F is a PRF
 - $Ver_K(M,S) := 1$ iff $S=F_K(M)$
 - Output length of F_K should be big enough
- If an adversary forges MAC with probability ϵ_{MAC} , then can break PRF with advantage $O(\epsilon_{MAC} - 2^{-m(k)})$ ($m(k)$ being the output length of the PRF) [How?]
 - If random function R used as MAC, then probability of forgery, $\epsilon_{MAC}^* = 2^{-m(k)}$



Recall: Advantage in breaking a PRF $F =$ diff in prob test has of outputting 1, when given F vs. truly random R

RECALL

MAC from PRF

For multi-block messages

- CBC-MAC

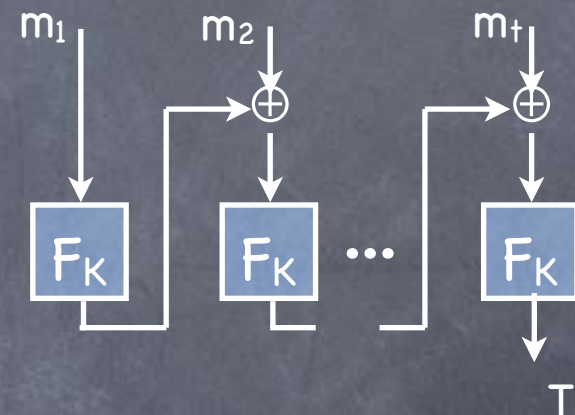
- For fixed number of blocks

- Else **length-extension attacks** possible
(by extending a previously signed message)

- Many ways to handle variable number of blocks

- e.g., EMAC, CMAC, ...

- Later, HMAC: MAC from a "hash function" (instead of a PRF)



Authenticated Encryption

- Encryption + authentication (implies CCA secure encryption)
 - Generic composition: encrypt (CPA), then MAC
 - Needs two keys and two passes
- AE aims to do this more efficiently
 - Several constructions based on block-ciphers (modes of operation) provably secure modeling block-cipher as PRP
 - One pass: IAPM, OCB, ... [patented]
 - Two pass: CCM, GCM, SIV, ... [included in NIST standards]
 - AE with Associated Data: Allows unencrypted (but authenticated) parts of the plaintext, for headers etc.

MAC-then-encrypt is not necessarily CCA-secure

SKE in Practice

Stream Ciphers

- A key should be used for only a single stream
- RC4, eSTREAM portfolio, ...
- In practice, stream ciphers take a key and an "IV" (initialization vector) as inputs
 - Heuristic goal: behave somewhat like a PRF (instead of a PRG) so that it can be used for multi-message encryption
 - But often breaks if used this way
- NIST Standard: For multi-message encryption, use a block-cipher in CTR mode

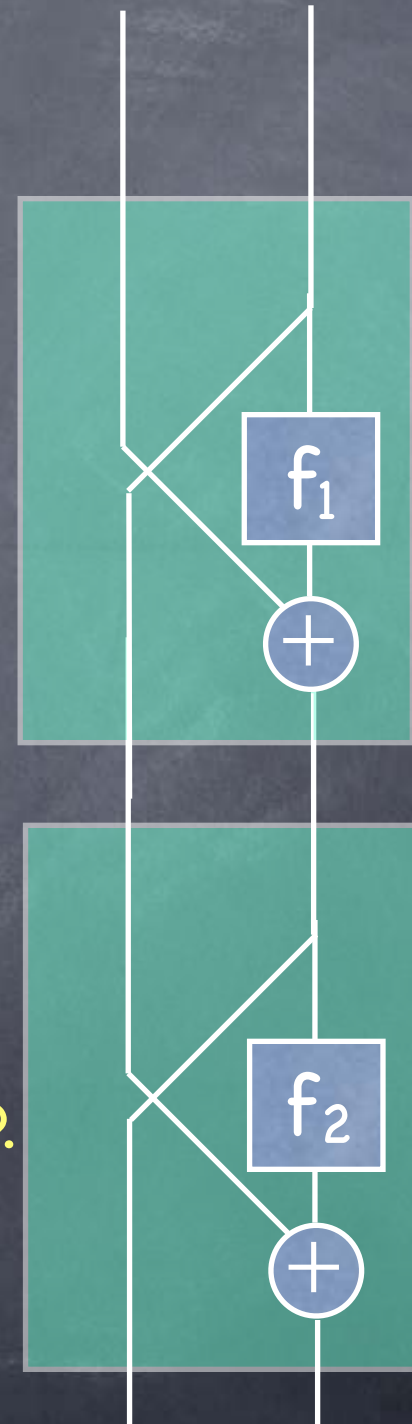
Also used to denote the random nonce chosen for encryption using a block-cipher

Block Ciphers

- DES, 3DES, Blowfish, AES, ...
 - Heuristic constructions
 - Permutations that can be inverted with the key
 - Speed (hardware/software) is of the essence
 - But should withstand known attacks
 - As a PRP (or at least, against key recovery)

Feistel Network

- Building a permutation from a (block) function
 - Let $f: \{0,1\}^m \rightarrow \{0,1\}^m$ be an arbitrary function
 - $F_f: \{0,1\}^{2m} \rightarrow \{0,1\}^{2m}$ defined as $F_f(x,y) = (y, x \oplus f(y))$
 - F_f is a permutation (Why?)
 - Can invert (How?)
 - Given functions f_1, \dots, f_t can build a t -layer Feistel network $F_{f_1 \dots f_t}$
 - Still a permutation from $\{0,1\}^{2m}$ to $\{0,1\}^{2m}$
- **Luby-Rackoff:** A 3-layer Feistel network with PRFs (with independent seeds) as round functions is a PRP. A 4-layer Feistel of PRFs gives a strong PRP.
- Fewer layers do not suffice! [Exercise]



DES Block Cipher

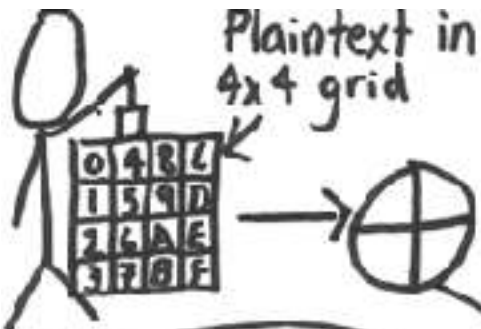
NIST Standard. 1976

- Data Encryption Standard (DES), Triple-DES, DES-X
- DES uses a 16-layer Feistel network (and a few other steps)
 - The round functions are not PRFs, but ad hoc
 - “Confuse and diffuse”
 - Defined for fixed key/block lengths (56 bits and 64 bits); key is used to generate subkeys for round functions
- DES’s key length too short
 - Can now mount brute force key-recovery attacks (e.g. using \$10K hardware, running for under a week, in 2006; now, in under a day)
- DES-X: extra keys to pad input and output
- Triple DES: 3 successive applications of DES (or DES⁻¹) with 3 keys

AES Block Cipher

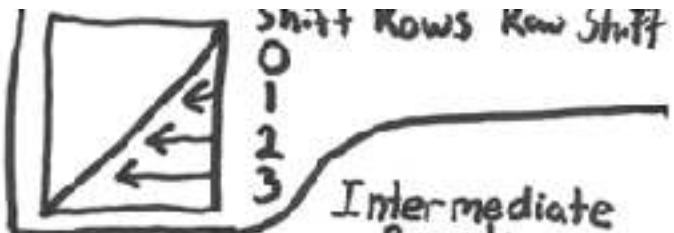
NIST Standard. 2001

- Advanced Encryption Standard (AES)
 - AES-128, AES-192, AES-256 (3 key sizes; block size = 128 bits)
 - Very efficient in software implementations (unlike DES)
 - Uses "Substitute-and-Permute" instead of Feistel networks
 - Has some algebraic structure
 - Operations in a vector space over the field $GF(2^8)$
 - The algebraic structure may lead to "attacks"? Not yet.
 - Some implementations may lead to side-channel attacks (e.g. cache-timing attacks)
 - Widely considered secure, but no "simple" hardness assumption known to imply any sort of security for AES



AES Crib Sheet

(Handy for memorizing)



General Math

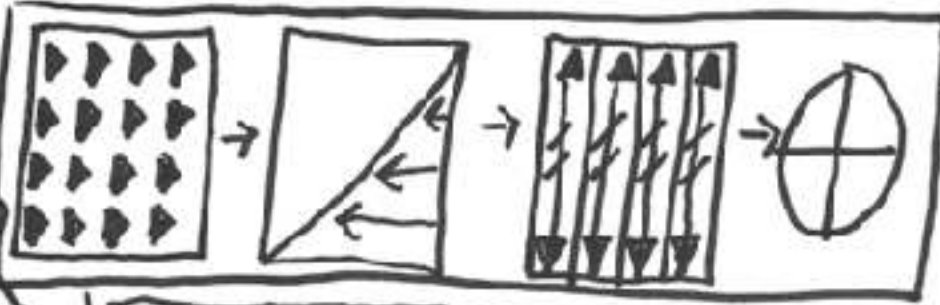
1.1B = AES Polynomial = $x^8 + x^4 + x^3 + x + 1$

Fast Multiply

$x \cdot a(x) = (a \ll 1) \oplus (a_7 = 1) ? 1B : 00$

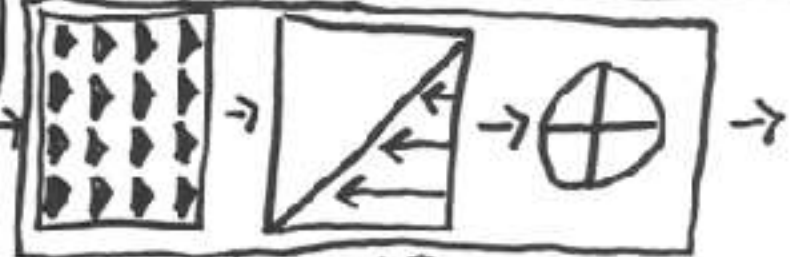
$\log(x \cdot y) = \log(x) + \log(y)$

Use $(x+1) = 03$ for log base



Intermediate Rounds

| # | Key |
|----|-----|
| 9 | 128 |
| 11 | 192 |
| 13 | 256 |



Ciphertext

| | | | |
|---|---|---|---|
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |
| ? | ? | ? | ? |

S-Box (SRD)

$SRD[a] = f(g(a))$

$g(a) = a^{-1} \text{ mod } m(x)$

See Think $5^3 \oplus 6^3$

5 is and 3 0's $[0110\ 0011]^T$

| | | |
|----------|-------|----------|
| 11111000 | a_7 | 01100011 |
| 01111100 | a_6 | 01100011 |
| 00111110 | a_5 | 01100011 |
| 00011111 | a_4 | 01100011 |
| 10001111 | a_3 | 01100011 |
| 11000111 | a_2 | 01100011 |
| 11000111 | a_1 | 01100011 |
| 11100011 | a_0 | 01100011 |

Key Expansion: Round Constants

First Column: 01 02 04 08 ...

| | | | |
|---|---|---|---|
| S | | | |
| 0 | 1 | B | K |
| M | 2 | I | E |
| E | 7 | T | Y |

Round Key

| | | | |
|---|----|----|----|
| K | B3 | 01 | B2 |
| E | 6E | 00 | 6E |
| Y | CB | 00 | CB |
| | B7 | 00 | B7 |

Other Columns:

| | | |
|---|----|----|
| T | E1 | C1 |
| Z | 21 | 10 |
| 8 | 86 | B4 |
| | F2 | CA |

Prev Col \oplus Col from Previous round key

Mix Columns: 21132

| | | | |
|---|---|---|---|
| 2 | 1 | 1 | 3 |
| 3 | 2 | 1 | 1 |
| 1 | 3 | 2 | 1 |
| 1 | 1 | 3 | 2 |

Inverse Mix

| | | | |
|---|---|---|---|
| E | B | D | 9 |
| 9 | E | B | D |
| D | 9 | E | B |
| B | D | 9 | E |

$[a_3]$
 $[a_2]$
 $[a_1]$
 $[a_0]$

Cryptanalysis

- Attacking stream ciphers and block ciphers
 - Typically for key recovery
- Brute force cryptanalysis, using specialized hardware
 - e.g. Attack on DES in 1998
- Several other analytical techniques to speed up attacks
 - Sometimes “theoretical”: on weakened (“reduced round”) constructions, showing improvement over brute-force attack
 - Meet-in-the-middle, linear cryptanalysis, differential cryptanalysis, impossible differential cryptanalysis, boomerang attack, integral cryptanalysis, cube attack, ...

SKE today

- SKE in IPsec, TLS etc. mainly based on AES block-ciphers
 - AES-128, AES-192, AES-256
- A recommended choice: AES Counter-mode + CMAC (or HMAC), encrypt-then-MAC.
 - Gives CCA security, and provides authentication
 - (Standards don't all follow this choice, but still secure)
- Older components/modes still in use
 - Supported by many standards for legacy purposes
 - In many applications (sometimes with modifications)
 - e.g. RC4 in BitTorrent, (older) Skype, PDF