Symmetric-Key Encryption: constructions

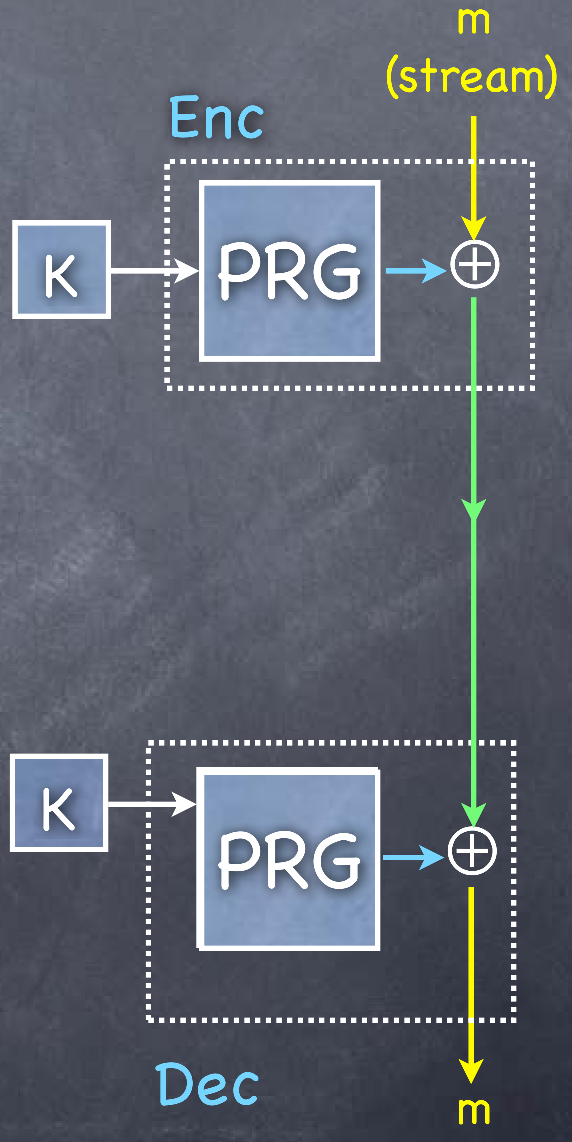
Lecture 5

PRF, Block Cipher

RECALL

PRG

- G is a PRG if $\boxed{\{G_k(x)\}_{x \leftarrow \{0,1\}^k} \approx U_{n(k)}}$ and G PPT
- A PRG can be used to obtain a one-time CPA-secure SKE
  - Stream cipher: PRG without an a priori bound n(k) on the output length
- Security: The pad produced by the PRG is indistinguishable from a truly random pad
  - Hence the scheme is indistinguishable from the one-time pad scheme (which is one-time CPA secure)
- Question: Multiple-message SKE?



Beyond One-Time

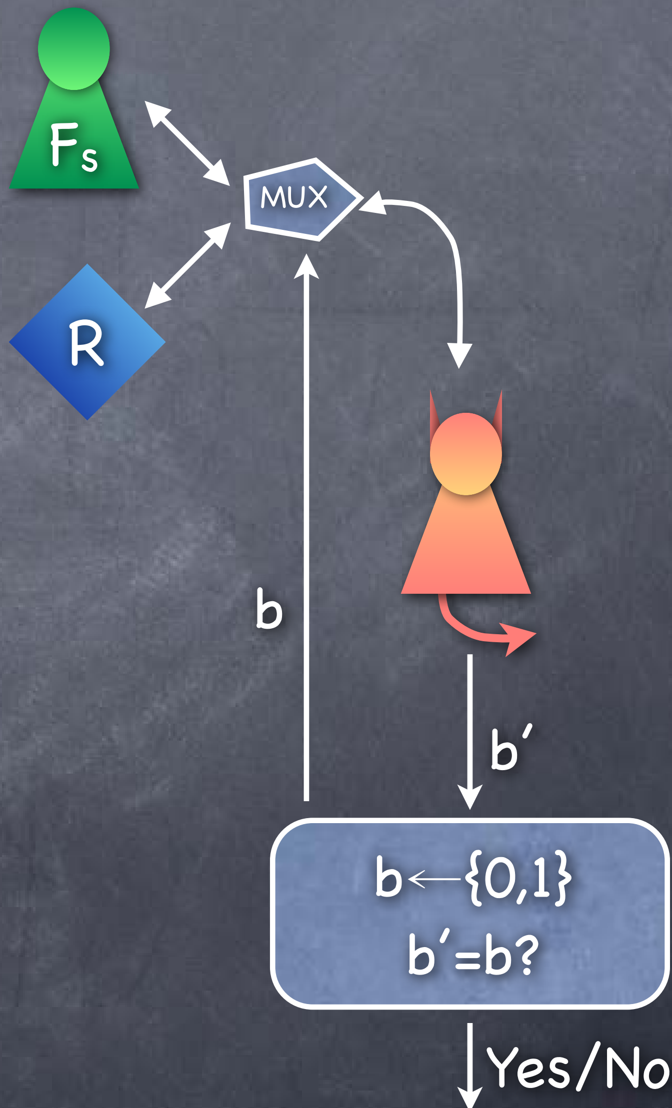
- Need to make sure that the same part of the one-time pad is never reused
  - Sender and receiver will need to maintain state and stay in sync (indicating how much of the pad has already been used)
    - Or only sender maintains the index, but sends it to the receiver. Then receiver will need to run the stream-cipher to get to that index.
    - A PRG with direct access to any part of the output stream?
- Pseudo Random Function (PRF)

Pseudorandom Function (PRF)

- A compact representation of an exponentially long (pseudorandom) string
  - Allows "random-access" (instead of just sequential access)
    - A function F(s;i) outputs the i^{th} block of the pseudorandom string corresponding to seed s
    - Exponentially many blocks (i.e., large domain for i)
- Pseudorandom Function
  - Need to define pseudorandomness for a function (not a string)

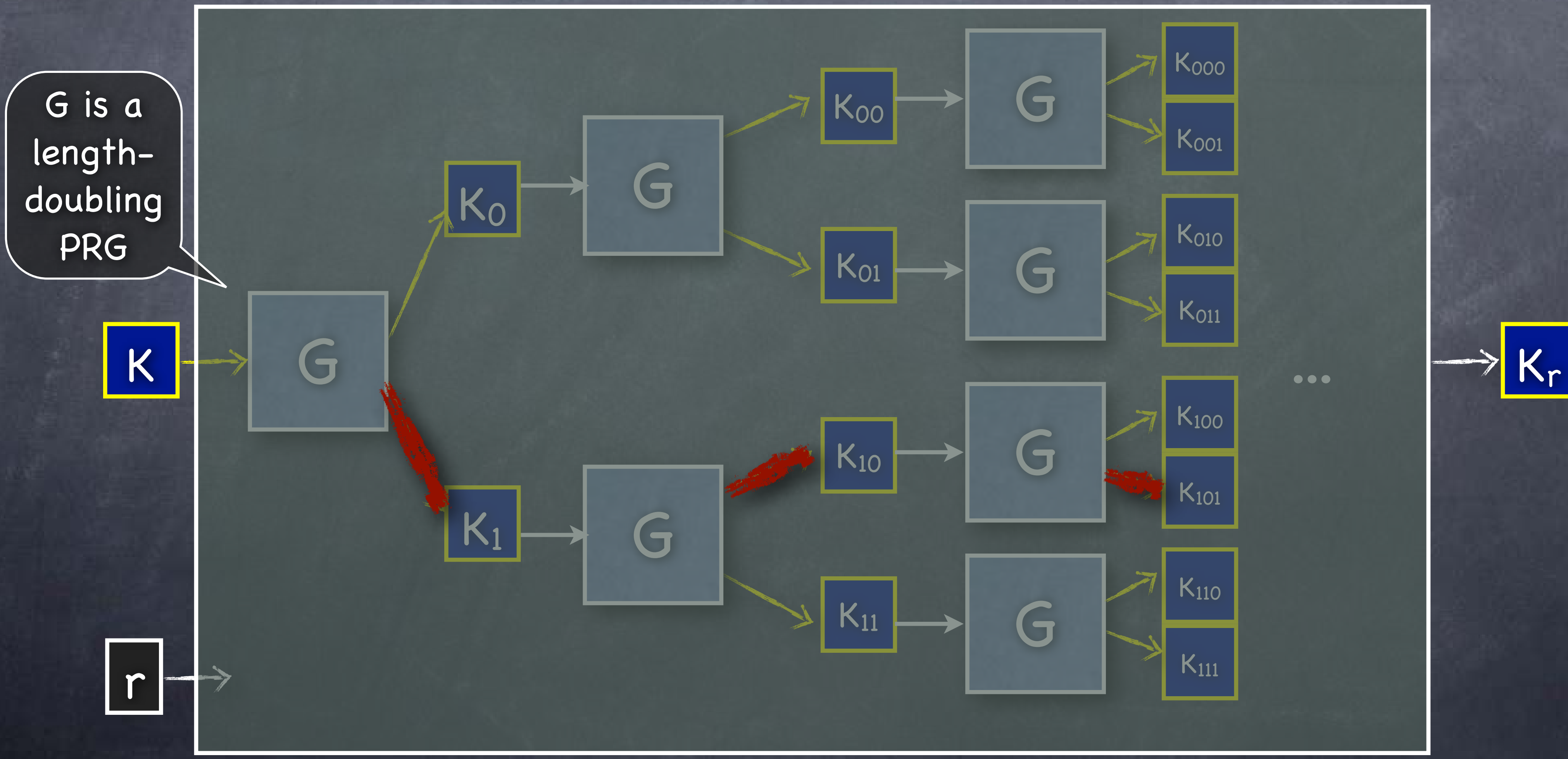
Pseudorandom Function (PRF)

- F: $\{0,1\}^k \times \{0,1\}^{m(k)} \rightarrow \{0,1\}^{n(k)}$ is a PRF if all PPT adversaries have negligible advantage in the PRF experiment
  - Adversary given oracle access to either F with a random seed, or a random function R: $\{0,1\}^{m(k)} \rightarrow \{0,1\}^{n(k)}$. Needs to guess which.
  - Note: Only 2^k seeds for F
    - But $2^{\wedge}(n2^m)$ functions R
  - PRF stretches k bits to $n2^m$ bits



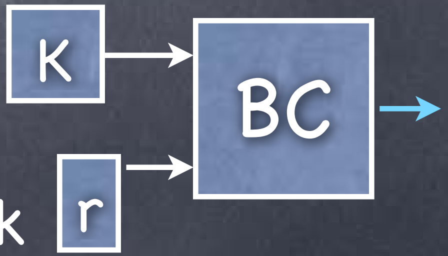
Pseudorandom Function (PRF)

- A PRF can be constructed from any PRG



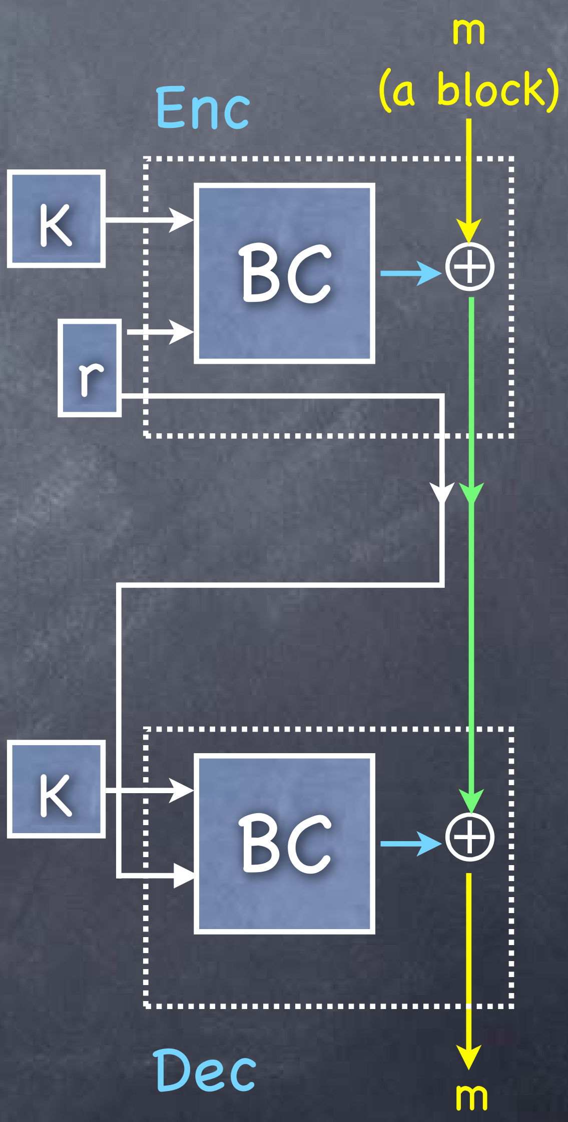
Pseudorandom Function (PRF)

- A PRF can be constructed from any PRG
  - Not blazing fast: needs |r| evaluations of a PRG
  - Faster constructions based on specific number-theoretic computational complexity assumptions
  - Fast heuristic constructions
- PRF in practice: Block Cipher
  - Extra features/requirements:
    - Permutation: input block (r) to output block
    - Key can be used as an inversion trapdoor
    - Pseudorandomness even with access to inversion



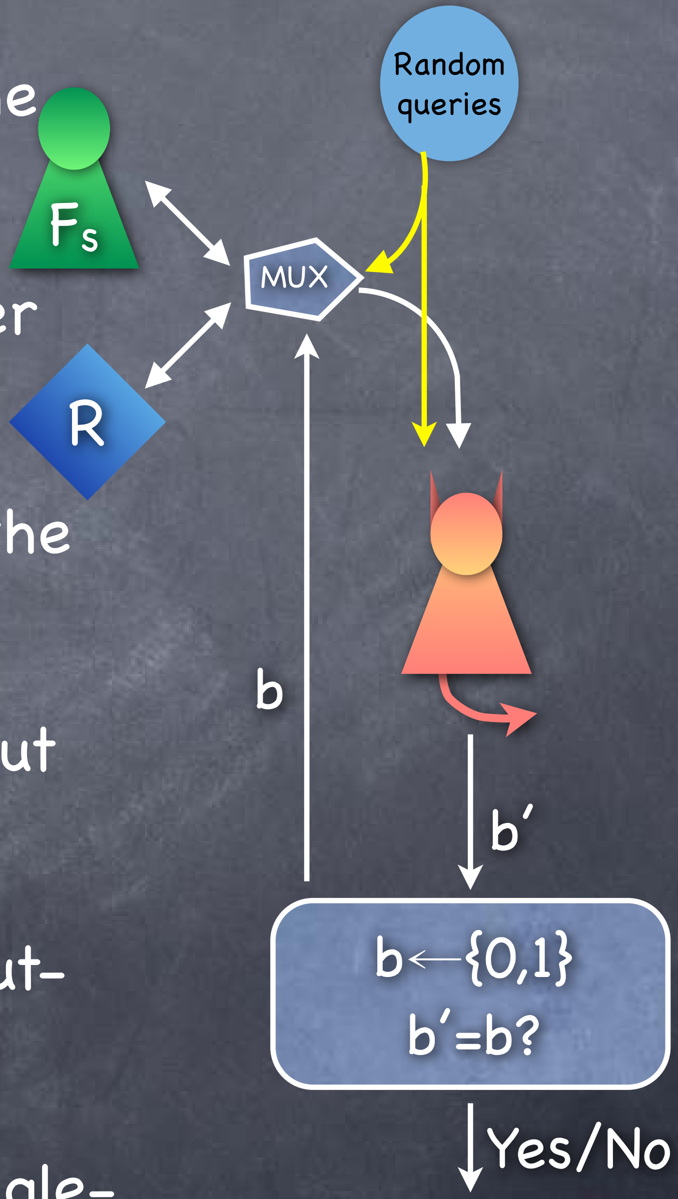
CPA-secure SKE with a PRF (or Block Cipher)

- Suppose Alice and Bob have shared a key (seed) for a block-cipher (or PRF) BC
- For each encryption, Alice will pick a fresh pseudorandom pad, by picking a new value r and setting pad=$BC_K(r)$
- Bob needs to be able to generate the same pad, so Alice sends r (in the clear, as part of the ciphertext) to Bob
- Even if Eve sees r, PRF security guarantees that $BC_K(r)$ is pseudorandom. (In fact, Eve could have picked r, as long as we ensure no r is reused.)
- How to pick a new r?
  - Pick at random!



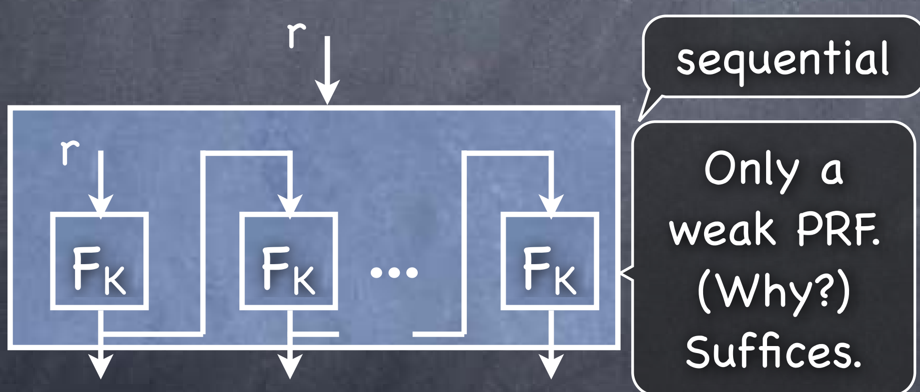
Weak PRF

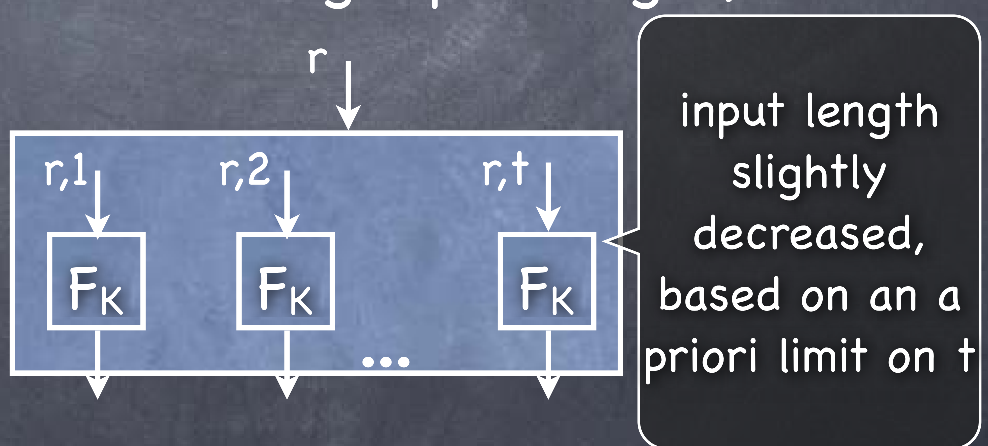
- Note: CPA-Security relied on the inputs to the PRF being just distinct (not random)
  - But if the input is indeed random, a weaker guarantee on PRF suffices
- Weak PRF: Similar to PRF, but the inputs to the oracle are chosen randomly
  - As before, adversary can see both the input and the output
  - As before, adversary can see as many input-output pairs as it wants
- Weak PRF suffices for CPA-secure SKE of <u>single-block messages</u>



CPA-secure SKE with a Block Cipher

- How to encrypt a long message (multiple blocks)?
  - Chop the message into blocks and independently encrypt each block as before?
  - Works, but ciphertext size is double that of the plaintext (if r is one-block long)
- Extend <u>output length</u> of a PRF (w/o increasing input length)

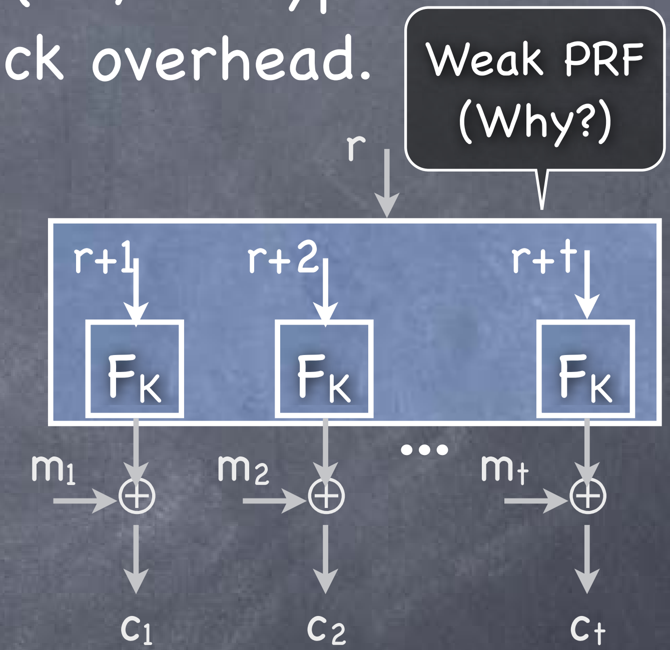




- Output is indistinguishable from t random blocks, provided all the inputs to F_K remain distinct (because F itself is a PRF)

CPA-secure SKE with a Block Cipher

- Various "modes" of operation of a Block-cipher (i.e., encryption schemes using a block-cipher). All with one block overhead.
  - **Output Feedback (OFB) mode:** Extend the pseudorandom output using the first construction in the previous slide
  - **Counter (CTR) Mode:** Similar idea as in the second construction. But no a priori limit on number of blocks in a message.
    - Security from low likelihood of (r+1,…,r+t) running into (r'+1,…,r'+t')



  - **Cipher Block Chaining (CBC) mode:** Sequential encryption. Decryption uses F_K^{-1}. Ciphertext an integral number of blocks.