

# Symmetric Key Cryptography

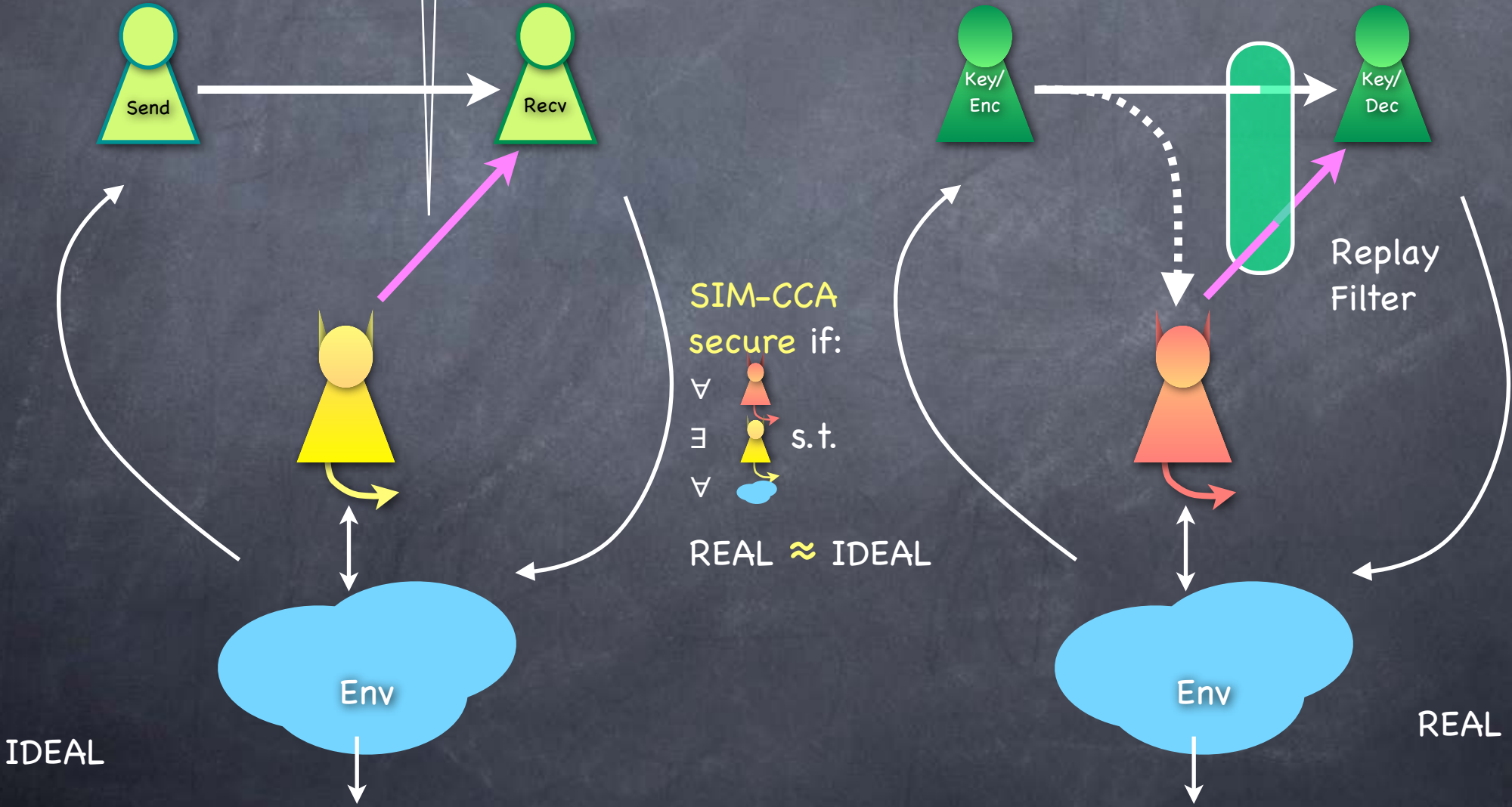
Lecture 8  
Summary

RECALL

# Symmetric-Key Encryption

## SIM-CCA Security

Authentication not required. i.e., Adversary allowed to send own messages (possibly "error")



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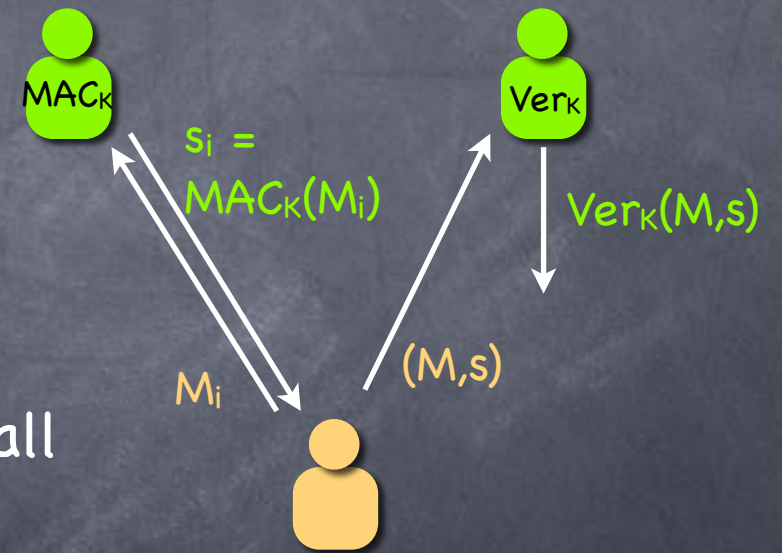
# Encryption & Authentication

- CPA secure encryption: Block-cipher/CTR mode construction
- MAC: from a PRF or Block-Cipher
- CCA secure encryption: From CPA secure encryption and MAC. Encrypt-then-MAC. (Gives authentication also.)
- **SKE can be entirely based on Block-Ciphers**
  - A tool that can make things faster: Hash functions (later)

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# Message Authentication Codes

- A single short key shared by Alice and Bob
  - Can sign any (polynomial) number of messages
- A triple (KeyGen, MAC, Verify)
- Correctness: For all  $K$  from KeyGen, and all messages  $M$ ,  $\text{Verify}_K(M, \text{MAC}_K(M))=1$
- Security: probability that an adversary can produce  $(M,s)$  s.t.  $\text{Verify}_K(M,s)=1$  is negligible unless Alice produced an output  $s=\text{MAC}_K(M)$



Advantage

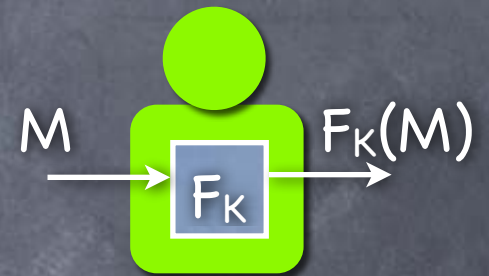
$$= \Pr[ \text{Ver}_K(M,s)=1 \text{ and } (M,s) \notin \{(M_i,s_i)\} ]$$

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# MAC from PRF

## When Each Message is a Single Block

- PRF is a MAC!
  - $MAC_K(M) := F_K(M)$  where  $F$  is a PRF
  - $Ver_K(M,S) := 1$  iff  $S=F_K(M)$
  - Output length of  $F_K$  should be big enough
- If an adversary forges MAC with probability  $\epsilon_{MAC}$ , then can break PRF with advantage  $O(\epsilon_{MAC} - 2^{-m(k)})$  ( $m(k)$  being the output length of the PRF) [How?]
  - If random function  $R$  used as MAC, then probability of forgery,  $\epsilon_{MAC}^* = 2^{-m(k)}$



Recall: Advantage in breaking a PRF  $F =$  diff in prob test has of outputting 1, when given  $F$  vs. truly random  $R$

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# MAC from PRF

## For multi-block messages

- CBC-MAC

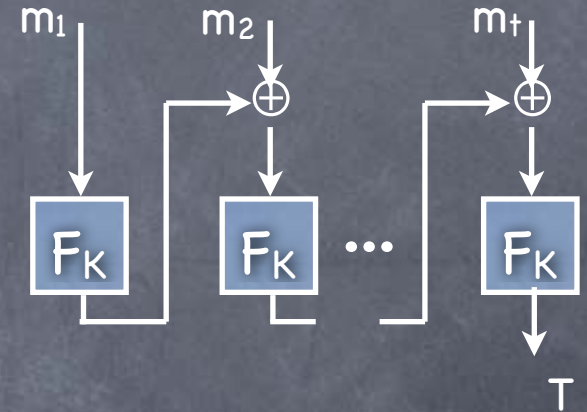
- For fixed number of blocks

- Else **length-extension attacks** possible  
(by extending a previously signed message)

- Many ways to handle variable number of blocks

- e.g., EMAC, CMAC, ...

- Later, HMAC: MAC from a "hash function" (instead of a PRF)



# Authenticated Encryption

- Encryption + authentication (implies CCA secure encryption)
  - Generic composition: encrypt (CPA), then MAC
  - Needs two keys and two passes
- AE aims to do this more efficiently
  - Several constructions based on block-ciphers (modes of operation) provably secure modeling block-cipher as PRP
    - One pass: IAPM, OCB, ... [patented]
    - Two pass: CCM, GCM, ... [included in NIST standards]
  - AE with Associated Data: Allows unencrypted (but authenticated) parts of the plaintext, for headers etc.

MAC-then-encrypt is not necessarily CCA-secure

# SKE in Practice



# Stream Ciphers

- A key should be used for only a single stream
- RC4, eSTREAM portfolio, ...
- In practice, stream ciphers take a key and an “IV” (initialization vector) as inputs
  - Heuristic goal: behave somewhat like a PRF (instead of a PRG) so that it can be used for multi-message encryption
  - But often breaks if used this way
- NIST Standard: For multi-message encryption, use a block-cipher in CTR mode

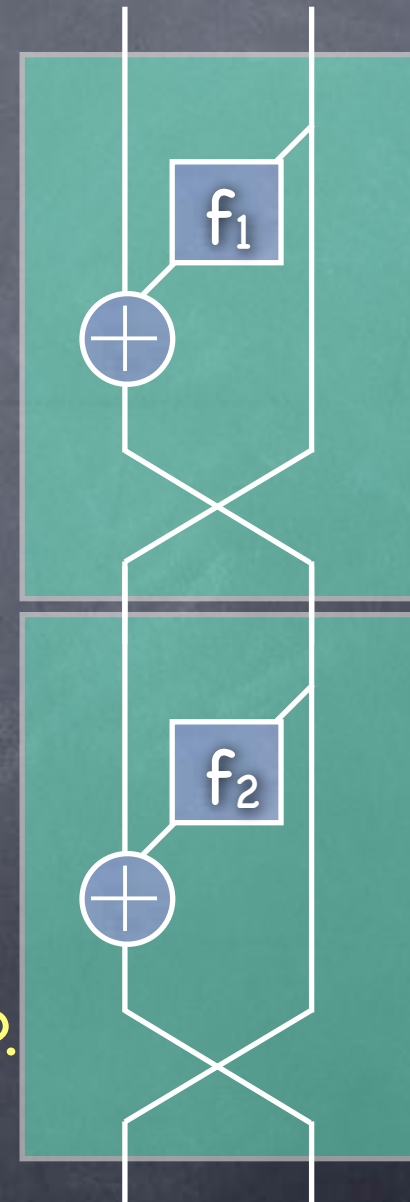
Also used to denote the random nonce chosen for encryption using a block-cipher

# Block Ciphers

- DES, 3DES, Blowfish, AES, ...
  - Heuristic constructions
  - Permutations that can be inverted with the key
  - Speed (hardware/software) is of the essence
  - But should withstand known attacks
    - As a PRP (or at least, against key recovery)

# Feistel Network

- Building a permutation from a (block) function
  - Let  $f: \{0,1\}^m \rightarrow \{0,1\}^m$  be an arbitrary function
  - $F_f: \{0,1\}^{2m} \rightarrow \{0,1\}^{2m}$  defined as  $F_f(x,y) = (y, x \oplus f(y))$ 
    - $F_f$  is a permutation (Why?)
      - Can invert (How?)
        - As efficient as computing  $f$
    - Given functions  $f_1, \dots, f_t$  can build a  $t$ -layer Feistel network  $F_{f_1 \dots f_t}$ 
      - Still a permutation from  $\{0,1\}^{2m}$  to  $\{0,1\}^{2m}$
  - **Luby-Rackoff:** A 3-layer Feistel network with PRFs (with independent seeds) as round functions is a PRP. A 4-layer Feistel of PRFs gives a strong PRP.
  - Fewer layers do not suffice! [Exercise]



# DES Block Cipher

NIST Standard. 1976

- Data Encryption Standard (DES), Triple-DES, DES-X
- DES uses a 16-layer Feistel network (and a few other steps)
  - The round function is not a PRF, but ad hoc
    - “Confuse and diffuse” using “Substitution and Permutation”
  - Defined for fixed key/block lengths (56 bits and 64 bits); key is used to generate subkeys for round functions
- DES’s key length too short
  - Can now mount brute force key-recovery attacks (e.g. using \$10K hardware, running for under a week, in 2006 and by now, in under a day; 1-2 days in general purpose GPUs.)
- DES-X: extra keys to pad input and output
- Triple DES: 3 successive applications of DES (or DES<sup>-1</sup>) with 3 keys

# AES Block Cipher

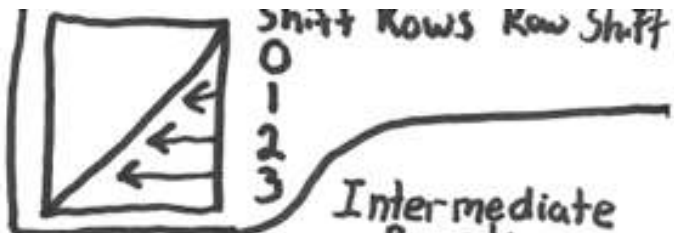
NIST Standard. 2001

- Advanced Encryption Standard (AES)
  - AES-128, AES-192, AES-256 (3 key sizes; block size = 128 bits)
  - Very efficient in software implementations (unlike DES)
  - Like DES, has rounds involving round-keys and “Substitution-and-Permutation”, but not a Feistel network
  - Has some algebraic structure
    - Operations in a vector space over the field  $GF(2^8)$
    - The algebraic structure may lead to “attacks”? Not yet.
  - Some implementations may lead to side-channel attacks (e.g. cache-timing attacks). Countered by using AES instruction set (available in x86, ARM, RISC-V, ...)
  - Widely considered secure, but no “simple” hardness assumption known to imply any sort of security for AES



# AES Crib Sheet

(Handy for memorizing)



General Math

11B = AES Polynomial =  $m(x)$

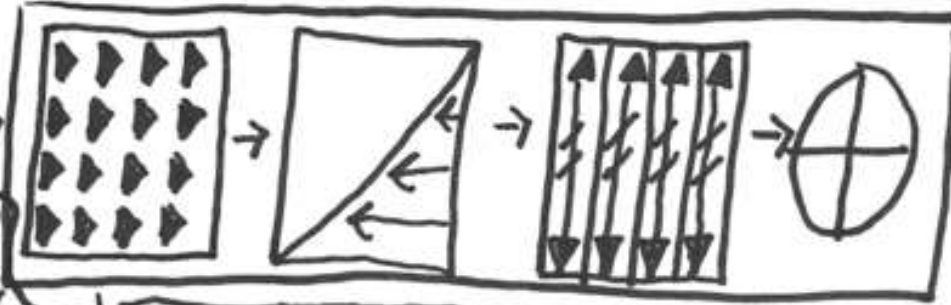
Fast Multiply

$x^8 + x^4 + x^3 + x + 1$

$x \cdot a(x) = (a \ll 1) \oplus (a_7 = 1) ? 1B : 00$

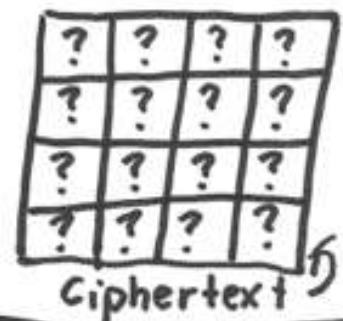
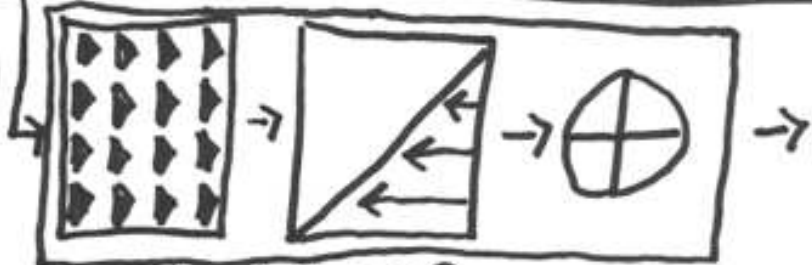
$\log(x \cdot y) = \log(x) + \log(y)$

Use  $(x+1) = 03$  for log base



Intermediate Rounds

#	Key
9	128
11	192
13	256



S-Box (SRD)

$SRD[a] = f(g(a))$

$g(a) = a^{-1} \text{ mod } m(x)$

See a. Think  $5^3 \oplus 6^3$

5 is and 3 0's  $[0110\ 0011]^T$

11	11	000
01	11	100
00	11	110
00	11	111
10	00	111
11	00	111
11	00	011
11	00	011

$a_7$   
 $a_6$   
 $a_5$   
 $a_4$   
 $a_3$   
 $a_2$   
 $a_1$

Key Expansion: Round Constants

First Column: 01 02 04 08 ...

S			
0	1	B	K
M	Z	I	E
E	T	Y	

Round Key 0

Other Columns:

T	E1	C1
Z	Z1	10
8	86	B4
	F2	CA

Prev Col  $\oplus$  Col from Previous round key

Mix Columns:

$21132$

2	1	1	3
3	2	1	1
1	3	2	1
1	1	3	2

$a_3$   
 $a_2$   
 $a_1$   
 $a_0$

Inverse Mix

EBD9

E	B	D	9
9	E	B	D
D	9	E	B
B	D	9	E

$a_3$   
 $a_2$   
 $a_1$   
 $a_0$

# Cryptanalysis

- Attacking stream ciphers and block ciphers
  - Typically for key recovery
- Brute force cryptanalysis, using specialized hardware
  - e.g. Attack on DES in 1998
- Several other analytical techniques to speed up attacks
  - Sometimes “theoretical”: on weakened (“reduced round”) constructions, showing improvement over brute-force attack
  - Meet-in-the-middle, linear cryptanalysis, differential cryptanalysis, impossible differential cryptanalysis, boomerang attack, integral cryptanalysis, cube attack, ...
- Side-channel attacks on implementations

# SKE today

- SKE in IPsec, TLS etc. mainly based on AES block-ciphers
  - AES-128, AES-192, AES-256
- A recommended choice: AES Counter-mode + CMAC (or HMAC), encrypt-then-MAC. Gives CCA security, and provides authentication.
  - Alternately, more optimised AES modes for Authenticated Encryption with Associated Data (e.g., AES-GCM)
- Older components/modes still in use
  - Supported by many standards for legacy purposes
  - In many applications (sometimes with modifications)
    - e.g. RC4 still used in BitTorrent
- API of libraries tend to be “too low-level,” e.g., Block-cipher, instead of Enc, Dec, KeyGen, leading to errors (e.g., use of ECB mode, nonce reuse, key cycles)

