Symmetric-Key Encryption: constructions

Lecture 5

PRG, Stream Cipher

PRF, Block Cipher

RECALL

Pseudorandomness Generator (PRG)

- Expand a short random **seed** to a "random-looking" string
- First, PRG with fixed stretch: $G_k: \{0,1\}^k \to \{0,1\}^{n(k)}$, $n(k) > k$
- How does one define random-looking?
  - Next-Bit Unpredictability: PPT adversary can't predict i^{th} bit of a sample from its first (i-1) bits (for every $i \in \{1,\ldots,n\}$)
  - A "more satisfactory" definition:
    - PPT adversary can't distinguish between a sample from $\{G_k(x)\}_{x \leftarrow \{0,1\}^k}$ and one from $\{0,1\}^{n(k)}$
    - $\{G_k(x)\}_{x \leftarrow \{0,1\}^k} \approx U_{n(k)}$
  - **The two definitions are equivalent!**

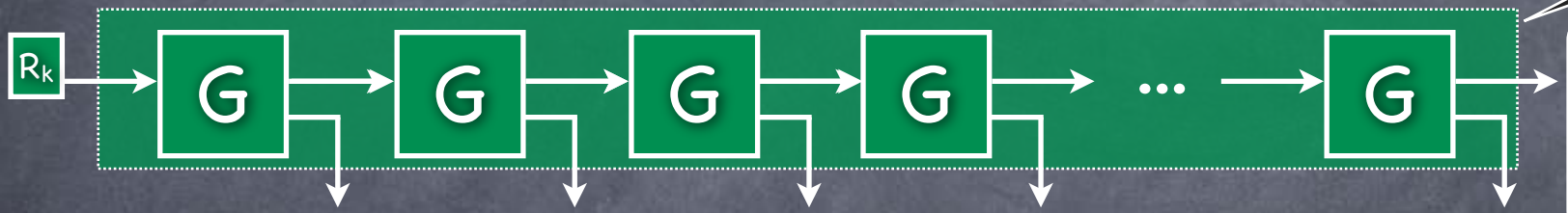
General PRG from 1-Bit Stretch PRG

will build later

- One-bit stretch PRG, G_k: $\{0,1\}^k \rightarrow \{0,1\}^{k+1}$

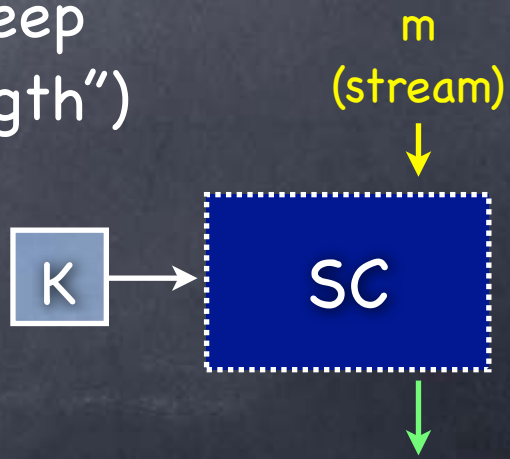
- Increasing the stretch
  - Can use part of the PRG output as a new seed

Why is this a PRG?



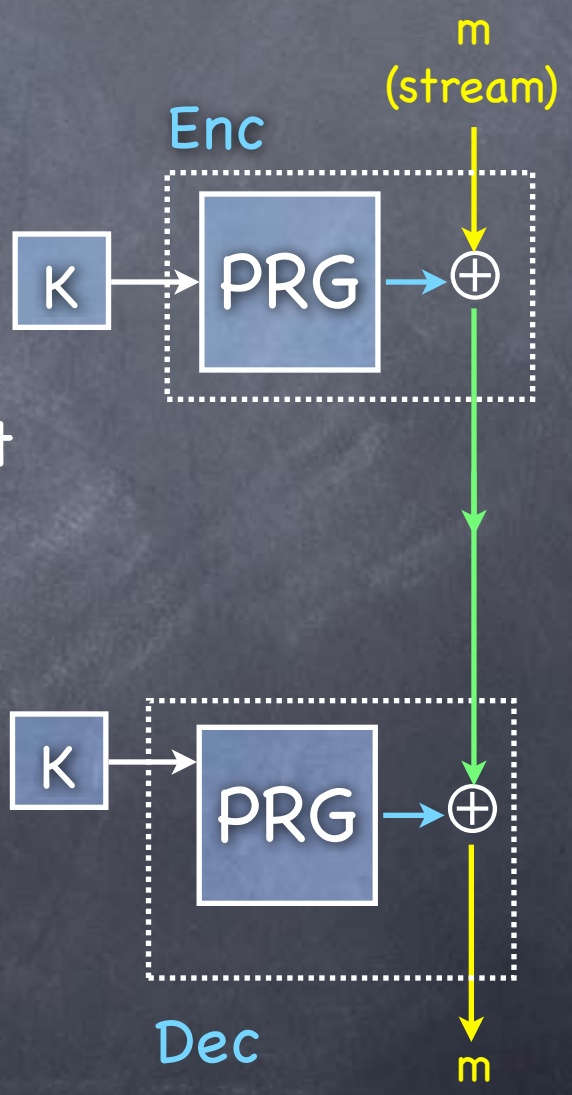
A "hybrid argument"

  - If intermediate seeds are never output, can keep stretching on demand (for any "polynomial length")
  - PRG with variable length output yields a stream cipher



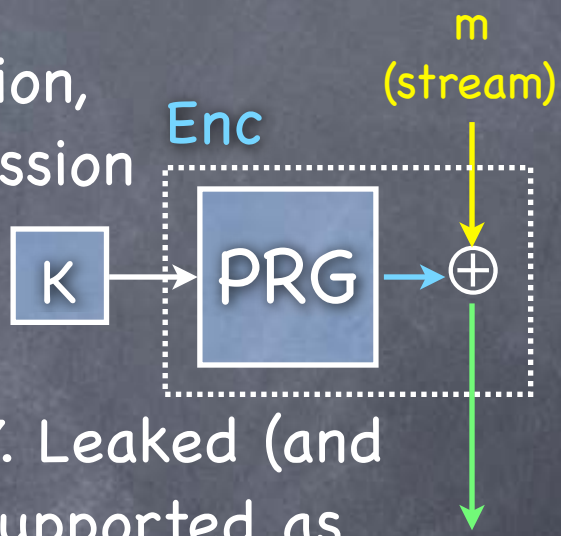
One-time secure SKE with a PRG

- One-time Encryption with a **stream-cipher**:
  - Generate a one-time pad from a short seed
  - Can share just the seed as the key
  - Mask message with the pseudorandom pad
- Decryption is symmetric: plaintext & ciphertext interchanged
- PRG used here can spit out bits on demand, so the message can arrive bit by bit, and the length of the message doesn't have to be a priori fixed
- Security: indistinguishability from using a truly random pad (coming up)



Stream Ciphers

- Stream ciphers in practice
  - Naturally useful for onetime (stream) encryption, in protocols where a key is established per session



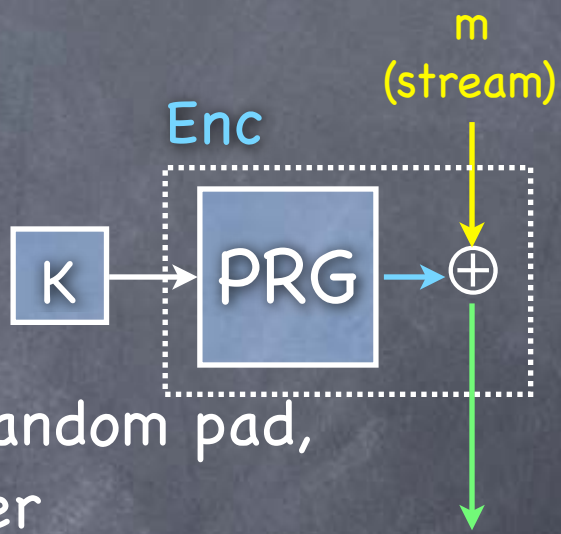
- Many popular candidates:
  - **RC4**: **Obsolete** (but popular). Designed in 1987. Leaked (and broken) in 1994. Still used in BitTorrent, and supported as an option in some protocols.
  - **eSTREAM** portfolio:

Profile 1 (software)	HC-128, Rabbit, Salsa20/12, SOSEMANUK	128 bit keys
Profile 2 (hardware)	Grain, MICKEY, Trivium	80 bit keys

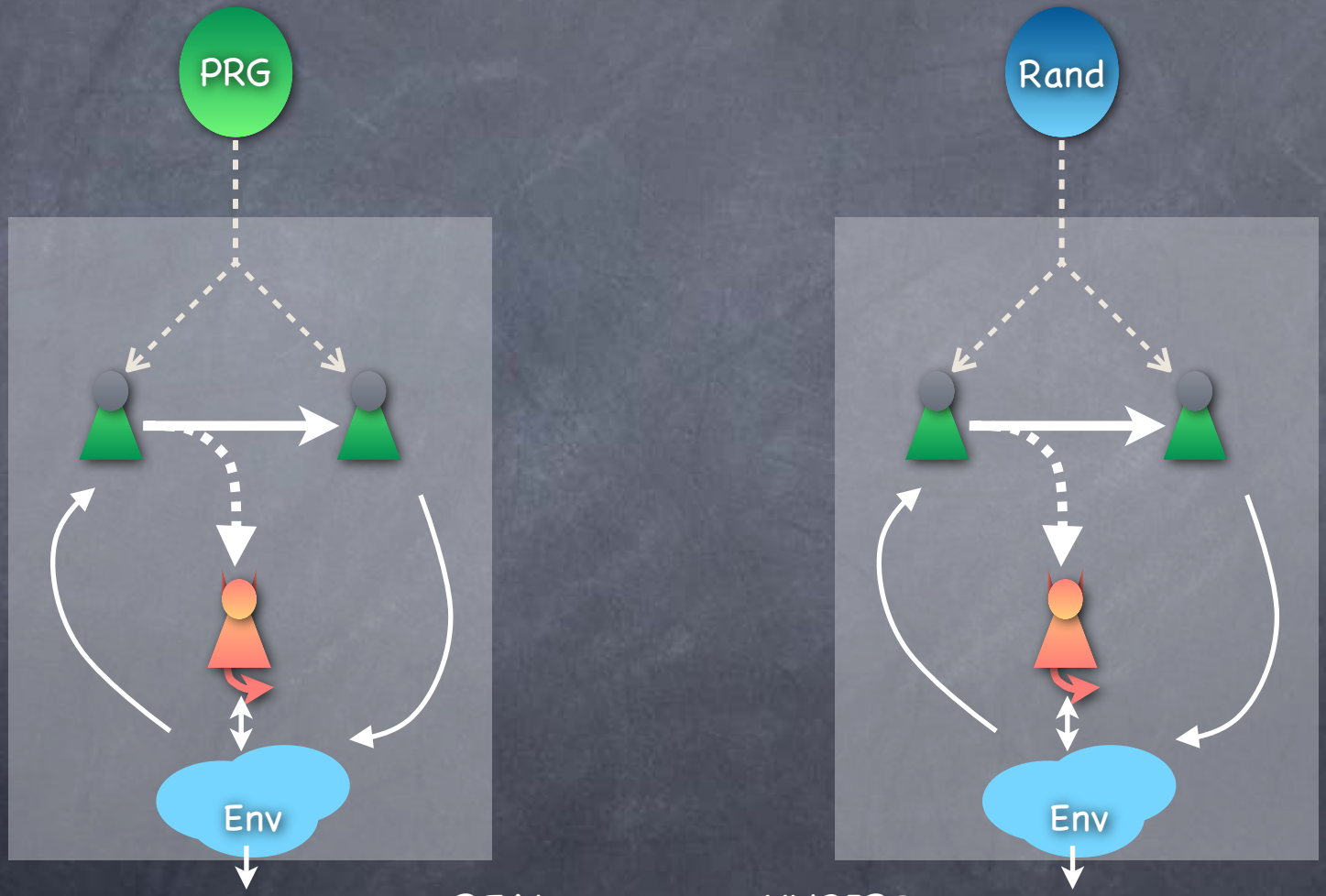
  - **NIST** recommendation: AES in an appropriate mode (later)

One-time secure SKE with a PRG

- In IDEAL experiment, consider simulator that uses a truly random string as the ciphertext
- To show REAL ≈ IDEAL
- Consider an intermediate world, HYBRID:
  - Like REAL, but Enc/Dec use a (long) truly random pad, instead of the output from the stream-cipher
  - HYBRID = IDEAL (recall perfect security of one-time pad)
  - Claim: REAL ≈ HYBRID
    - Consider the experiments as a system that accepts the pad from outside (R′ = PRG(K) for random K, or truly random R) and outputs the environment's output. This system is PPT, and so can't distinguish pseudorandom from random.



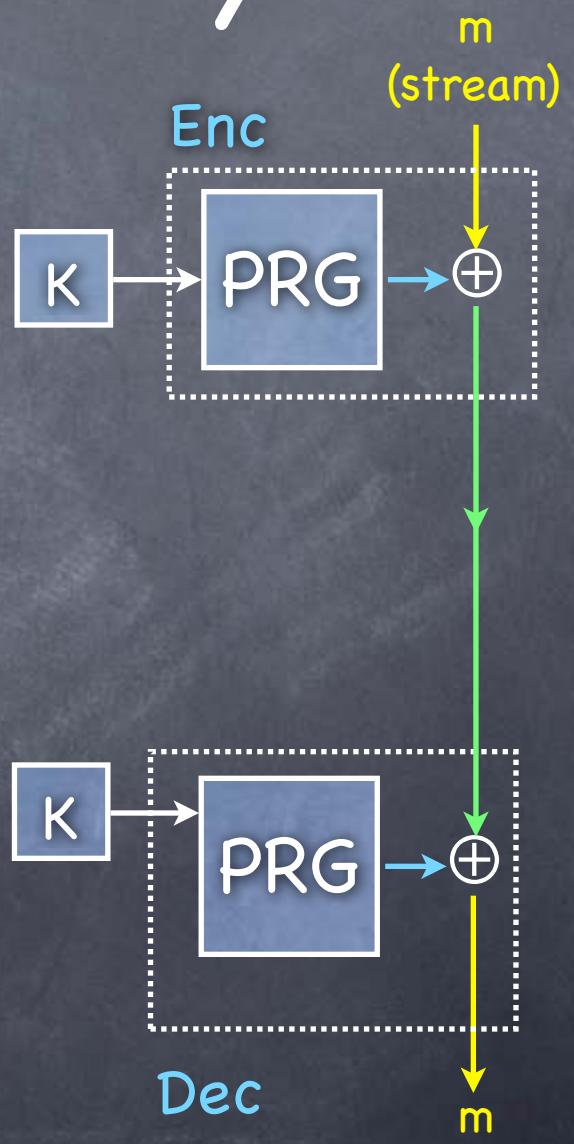
One-time secure SKE with a PRG



REAL ≈ HYBIRD

One-time secure SKE with a PRG: Summary

- G is a PRG if $\{G_k(x)\}_{x \leftarrow \{0,1\}^k} \approx U_{n(k)}$ and G PPT
- A PRG can be used to obtain a one-time CPA-secure SKE
  - Stream cipher: Using a PRG without an a priori bound n(k) on the output length
- Security: The pad produced by the PRG is indistinguishable from a truly random pad
  - Hence the scheme is indistinguishable from the one-time pad scheme (which is one-time CPA secure for fixed length messages)
- Next question: Multiple-message SKE?



Beyond One-Time

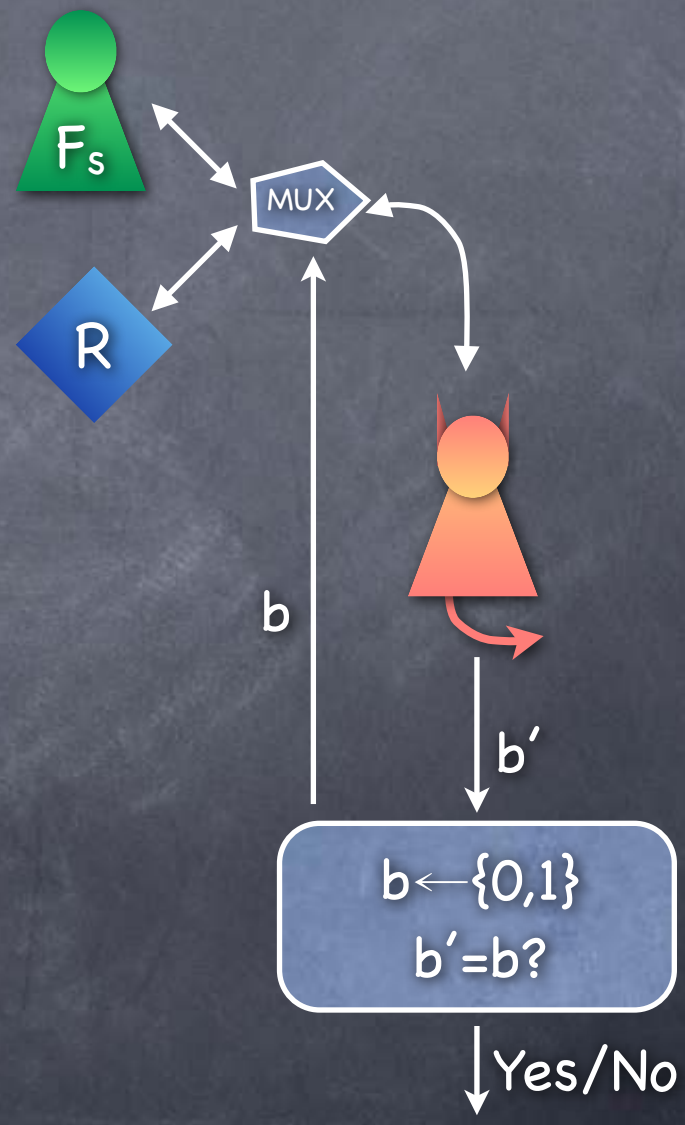
- Need to make sure that the same part of the one-time pad is never reused
  - Sender and receiver will need to maintain state and stay in sync (indicating how much of the pad has already been used)
    - Or only sender maintains the index, but sends it to the receiver. Then receiver will need to run the stream-cipher to get to that index.
    - A PRG with direct access to any part of the output stream?
- Pseudo Random Function (PRF)

Pseudorandom Function (PRF)

- A compact representation of an exponentially long (pseudorandom) string
  - Allows "random-access" (instead of just sequential access)
    - A function F(s;i) outputs the i^{th} block of the pseudorandom string corresponding to seed s
    - Exponentially many blocks (i.e., large domain for i)
- Pseudorandom Function
  - Need to define pseudorandomness for a function (not a string)

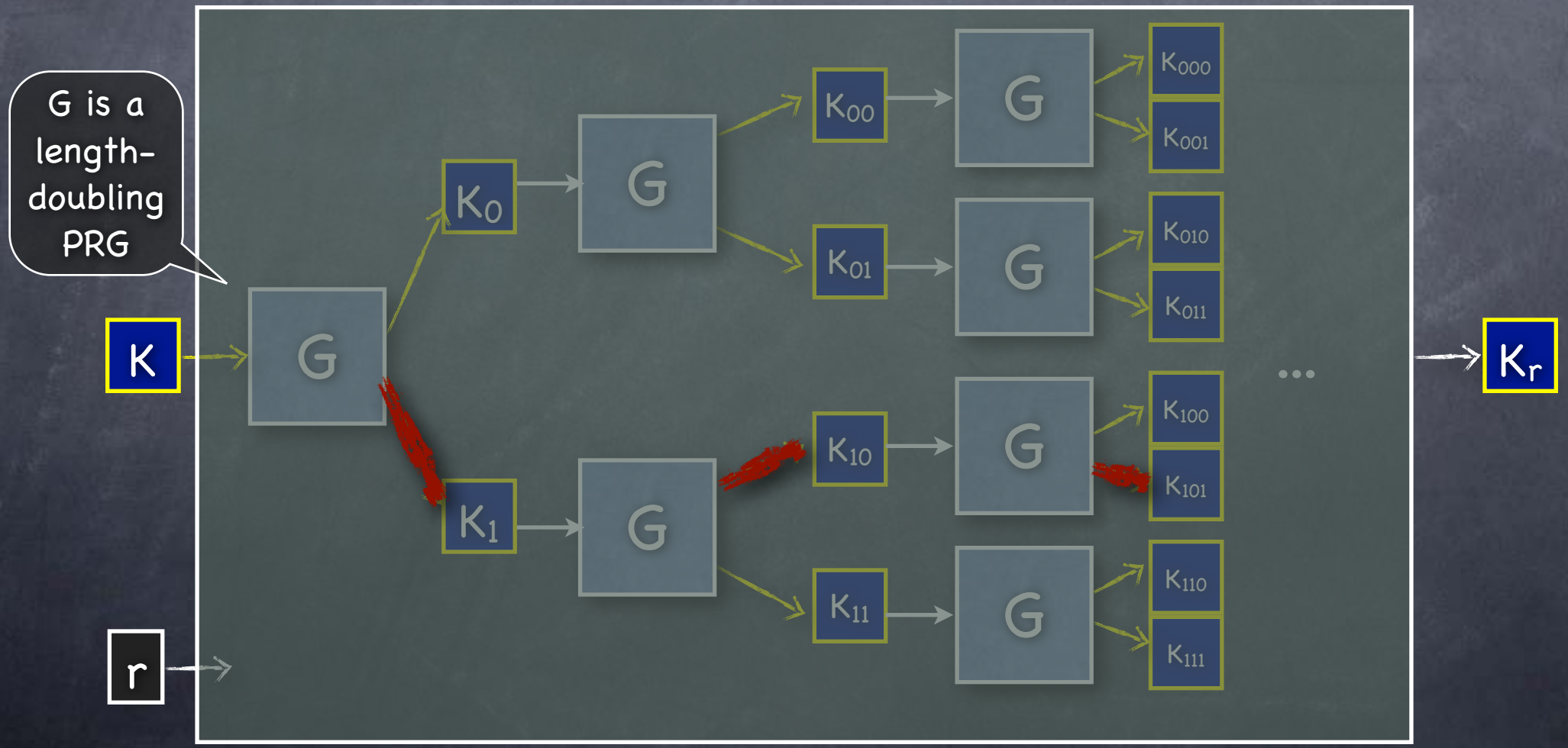
Pseudorandom Function (PRF)

- F: $\{0,1\}^k \times \{0,1\}^{m(k)} \rightarrow \{0,1\}^{n(k)}$ is a PRF if all PPT adversaries have negligible advantage in the PRF experiment
  - Adversary given oracle access to either F with a random seed, or a random function R: $\{0,1\}^{m(k)} \rightarrow \{0,1\}^{n(k)}$. Needs to guess which.
  - Note: Only 2^k seeds for F
    - But $2^{(n2^m)}$ functions R
  - PRF stretches k bits to $n2^m$ bits



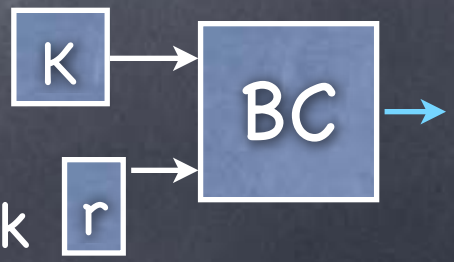
Pseudorandom Function (PRF)

- A PRF can be constructed from any PRG



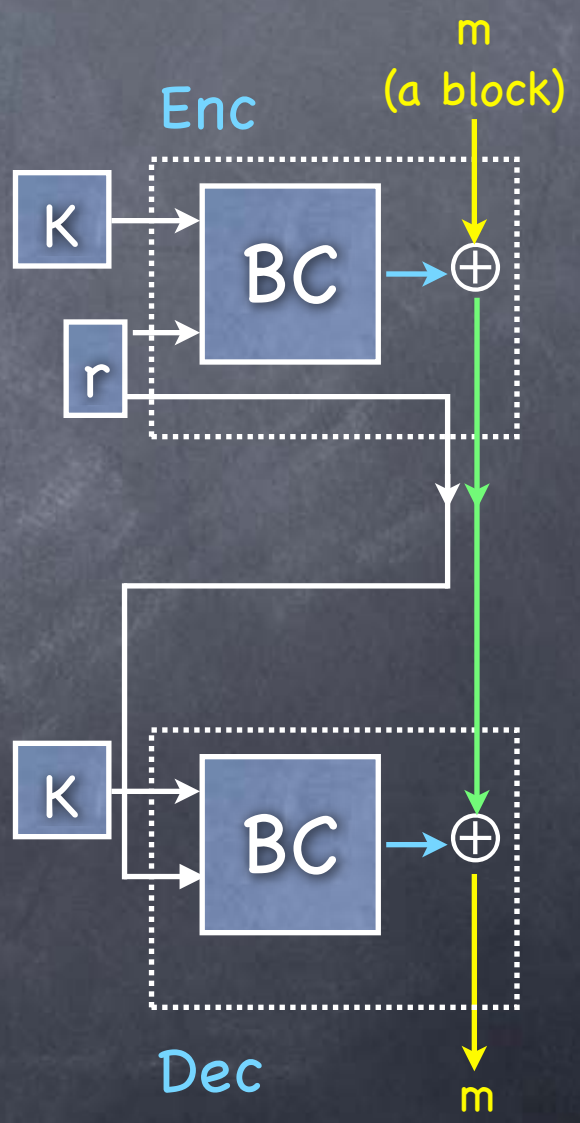
Pseudorandom Function (PRF)

- A PRF can be constructed from any PRG
  - Not blazing fast: needs |r| evaluations of a PRG
  - Faster constructions based on specific number-theoretic computational complexity assumptions
  - Fast heuristic constructions
- PRF in practice: Block Cipher
  - Extra features/requirements:
    - Permutation: input block (r) to output block
    - Key can be used as an inversion trapdoor
    - Pseudorandomness even with access to inversion



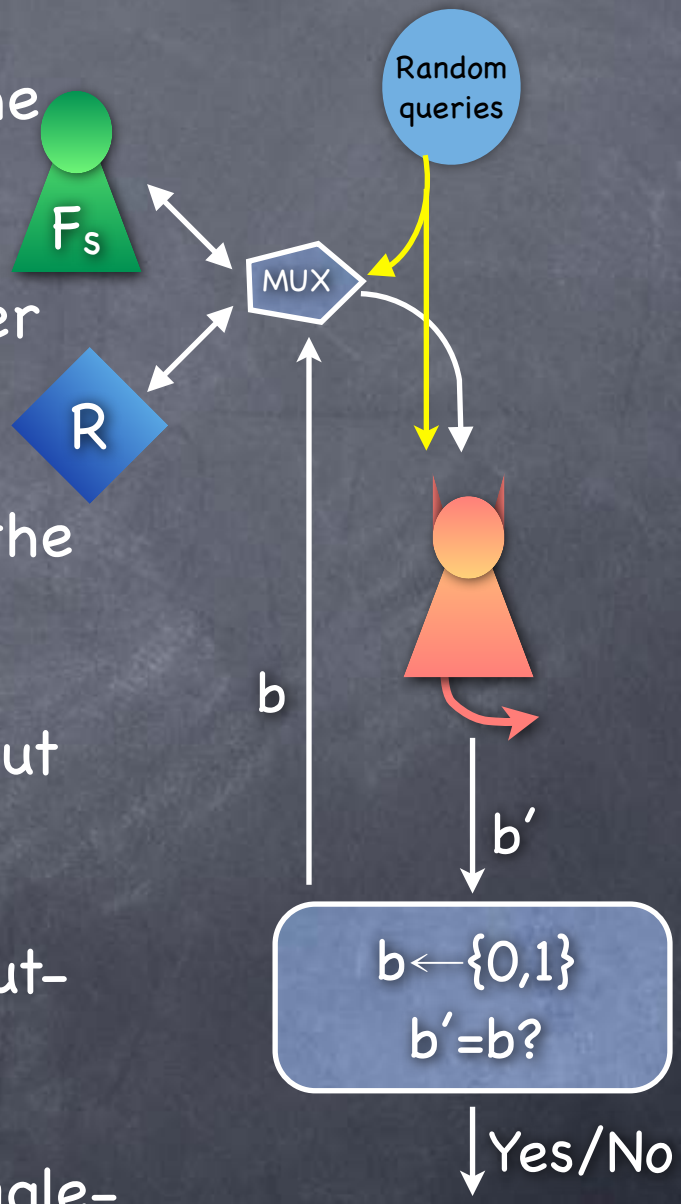
CPA-secure SKE with a PRF (or Block Cipher)

- Suppose Alice and Bob have shared a key (seed) for a block-cipher (or PRF) BC
- For each encryption, Alice will pick a fresh pseudorandom pad, by picking a new value r and setting pad=$BC_K(r)$
- Bob needs to be able to generate the same pad, so Alice sends r (in the clear, as part of the ciphertext) to Bob
- Even if Eve sees r, PRF security guarantees that $BC_K(r)$ is pseudorandom. (In fact, Eve could have picked r, as long as we ensure no r is reused.)
- How to pick a new r?
  - Pick at random!



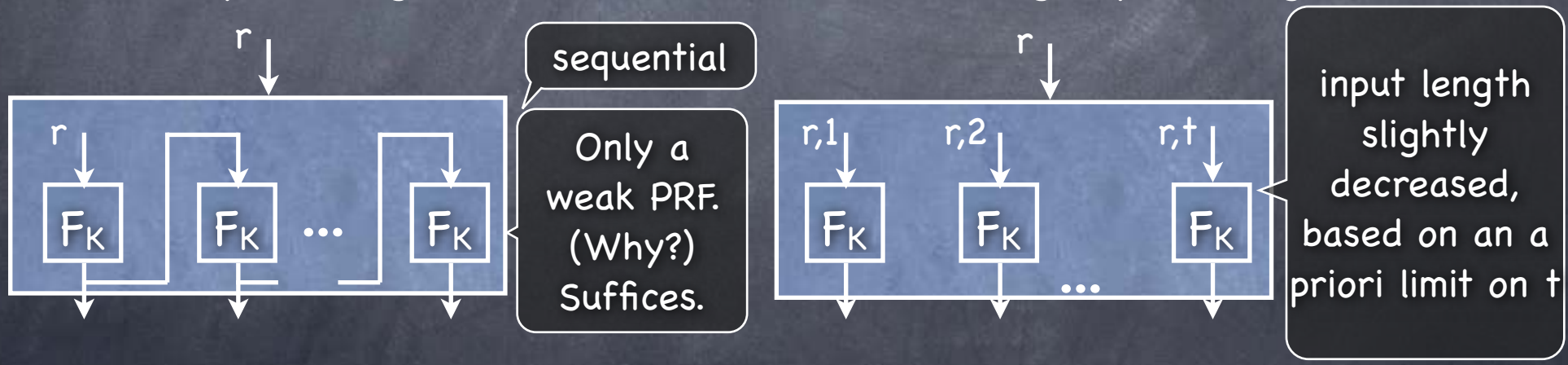
Weak PRF

- Note: CPA-Security relied on the inputs to the PRF being just distinct (not random)
  - But if the input is indeed random, a weaker guarantee on PRF suffices
- Weak PRF: Similar to PRF, but the inputs to the oracle are chosen randomly
  - As before, adversary can see both the input and the output
  - As before, adversary can see as many input-output pairs as it wants
- Weak PRF suffices for CPA-secure SKE of single-block messages



CPA-secure SKE with a Block Cipher

- How to encrypt a long message (multiple blocks)?
  - Chop the message into blocks and independently encrypt each block as before?
  - Works, but ciphertext size is double that of the plaintext (if r is one-block long)
- Extend output length of a PRF (w/o increasing input length)



- Output is indistinguishable from t random blocks, provided all the inputs to F_K remain distinct (because F itself is a PRF)

CPA-secure SKE with a Block Cipher

- Various "modes" of operation of a Block-cipher (i.e., encryption schemes using a block-cipher). All with one block overhead.
  - **Output Feedback (OFB) mode:** Extend the pseudorandom output using the first construction in the previous slide
  - **Counter (CTR) Mode:** Similar idea as in the second construction. But no a priori limit on number of blocks in a message.
    - Security from low likelihood of (r+1,...,r+t) running into (r'+1,...,r'+t')
  - **Cipher Block Chaining (CBC) mode:** Sequential encryption. Decryption uses F_K^{-1}. Ciphertext an integral number of blocks.