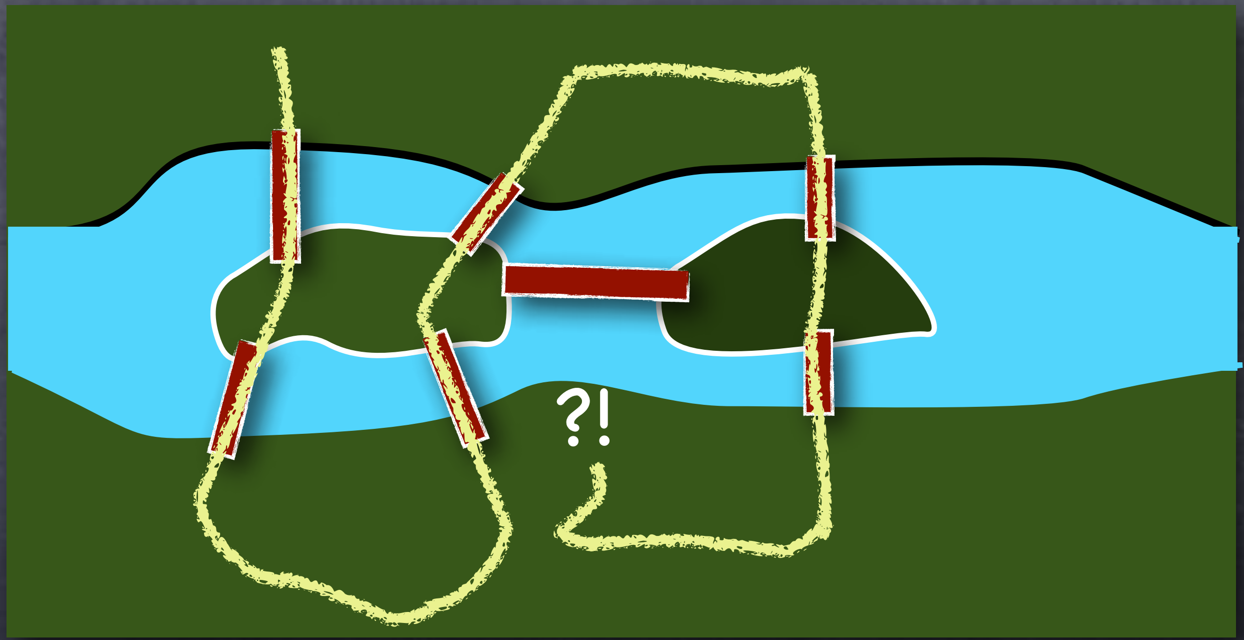
Discrete Structures (Intro to Mind Bending)

Lecture 0

Bridges of Königsberg

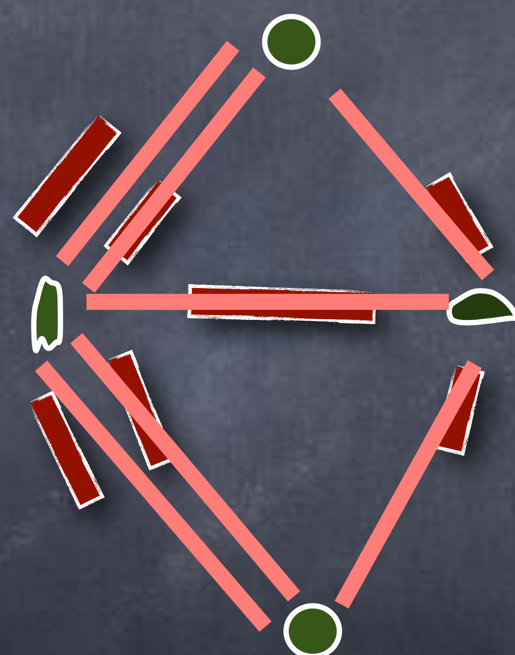
- Cross each bridge exactly once



- Is it impossible? How do we know for sure?

Bridges of Königsberg

- Cross each bridge exactly once



- Is it impossible? How do we know for sure?

Discrete Stuff

- Graphs (maps, friendships, www...)
- Patterns, Symmetry
- Numbers
- Logic, reasoning
- (Discrete) Algorithms
- Digital computers...

Zen and The Mechanics of CS 207

- The art of "Mind Bending"
  - Practice, practice, practice → Mind Expanding
- Don't worry about grades! happiness :-)
  - Your efforts will pay off in this and subsequent courses, and in job interviews, and jobs...
- But do pay heed to the mechanics!

The Mechanics of CS 207

Lecture & Discussion
Explanation, discussion, Q&A

Revision & Assimilation
Read slides/textbook and ponder

Homework/Tutorials
More pondering, wondering

TAs will help you!
Schedule TBA

Exams
Just to incentivize you...

Quizzes: 30%
Midsem: 25%
Endsem: 40%
(5% for in-class quizzes)

In a later course
Re-encountering, relearning
(Thank you 207!)

Even later
By now, second nature!
(Thank you IIT!)

Learning a concept

More Mechanics

- Do watch the course webpage
- We'll use **SAFE**! (Install the app on your own Android device, or we'll give you a device during class)
- Get into Piazza (note down the access code)
- Reference textbook: Kenneth Rosen
- Buy a file/folder for homework problem sets

Pigeonholes & Parties

Suppose you go to a party and there is a game:
How many of your "friends" are at the party?
(Everyone who goes to the party has at least one person there that he/she counts as a friend.)

There will be at least two who have the same number of friends at the party!

But Why?

Pigeonholes & Parties

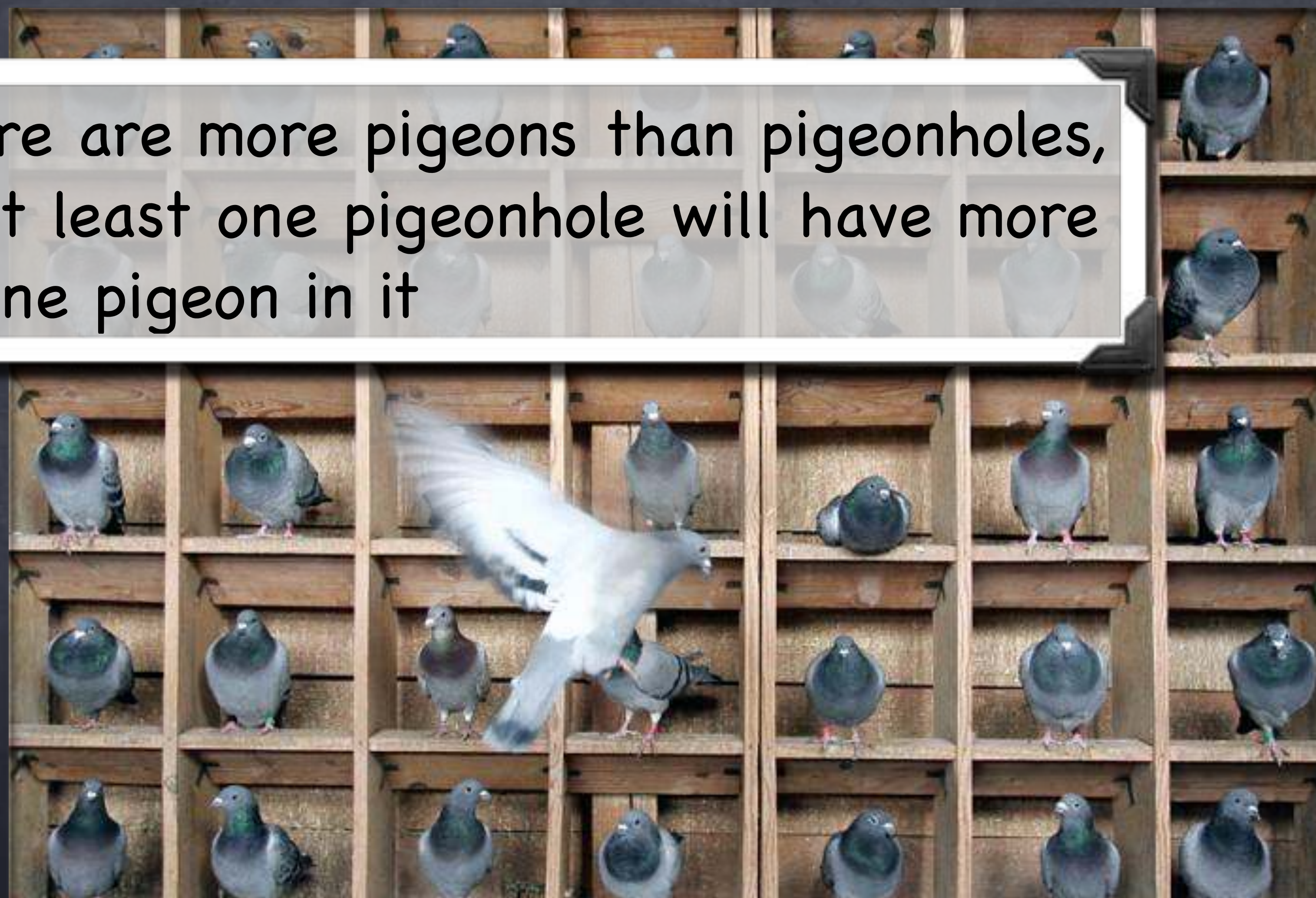
Suppose you go to a party and there is a game:
How many of your "friends" are at the party?
(Everyone who goes to the party has at least one person there that he/she counts as a friend.)

There will be at least two who have the same number of friends at the party!

- If there are 4 people in the party, for each person, the number of friends at the party is 1, 2 or 3.
  - There are 4 of you, and everyone needs to pick a number. There are only 3 numbers to pick from...

The Pigeonhole Principle

If there are more pigeons than pigeonholes, then at least one pigeonhole will have more than one pigeon in it



Pigeonholes & Parties

So, suppose you go to a party and there is a game: How many of your “friends” are at the party?

~~(Everyone who goes to the party has at least one person there that he/she counts as a friend.)~~

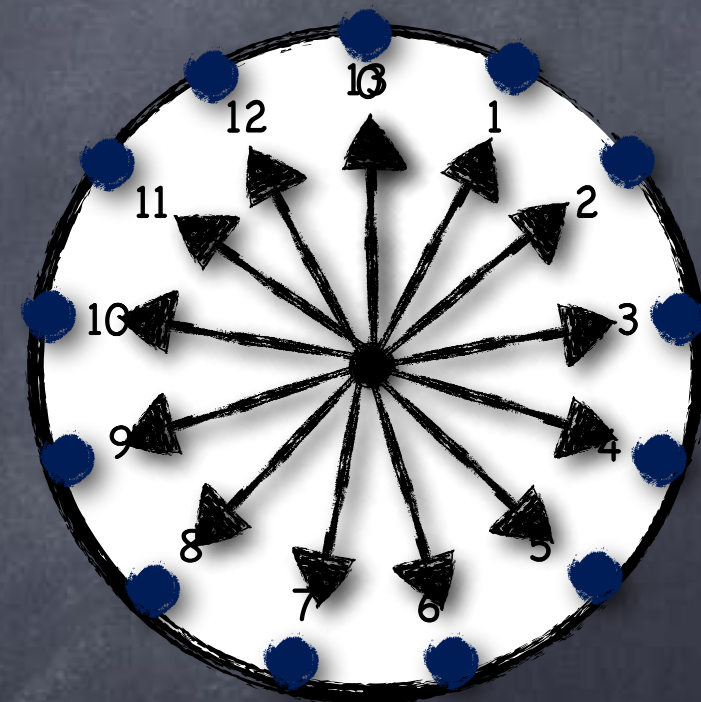
There will be at least two who have the same number of friends at the party!

<u>Point to ponder</u>

Suppose friendships are always reciprocated. Then can you show that the claim holds even if not everyone has a friend at the party?

The Skippy Clock

- Has 13 hours on its dial!
- Needle moves two hours at a time
- Which all numbers will the needle reach?
  - Reaches all of them!



Points to ponder

What if the clock had 12 hours?

What if the needle moved 5 hours at a time?

Topics to be covered

Recursion

Bounding
big-O

Trees

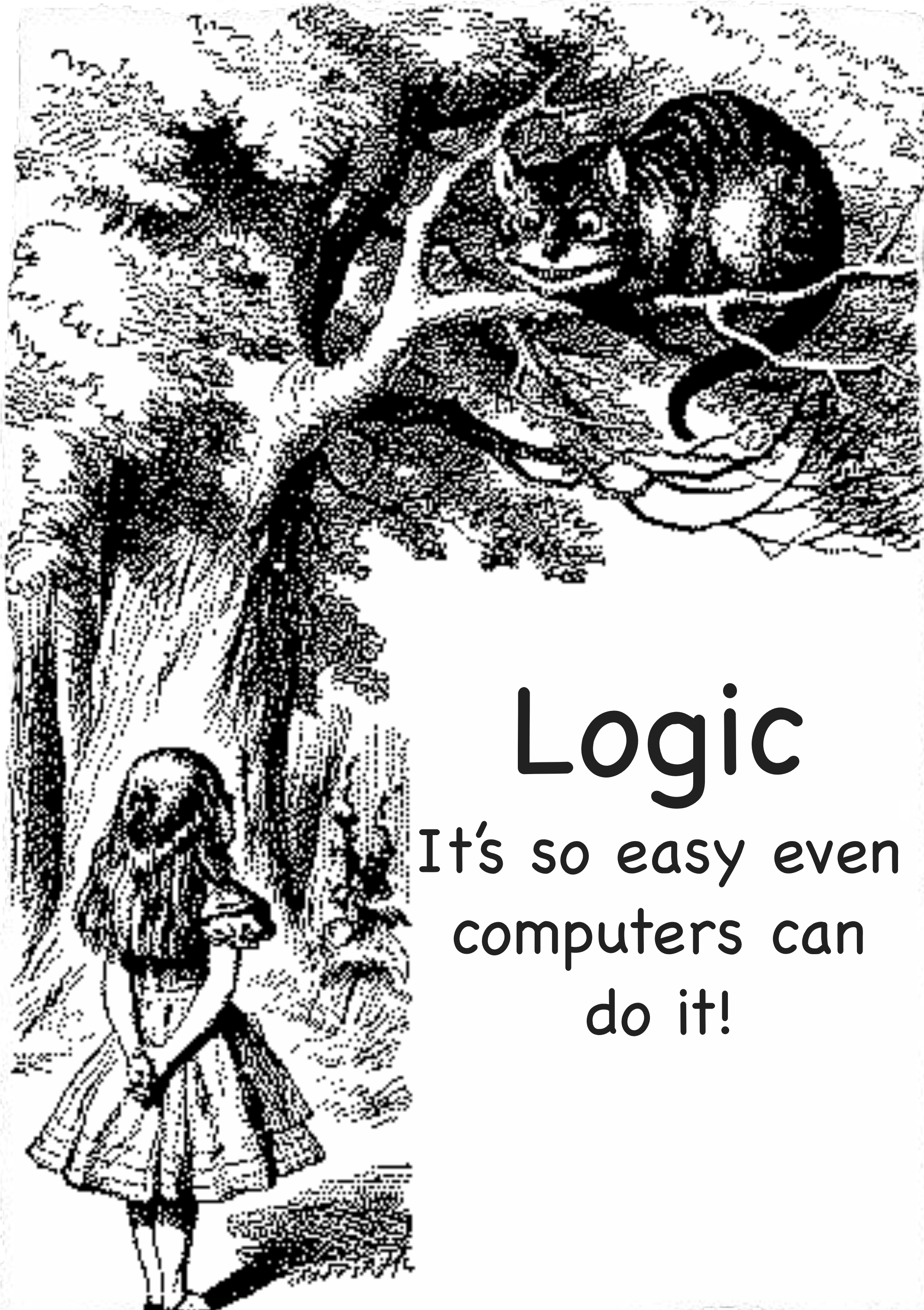
Induction

Counting

Numbers and patterns therein

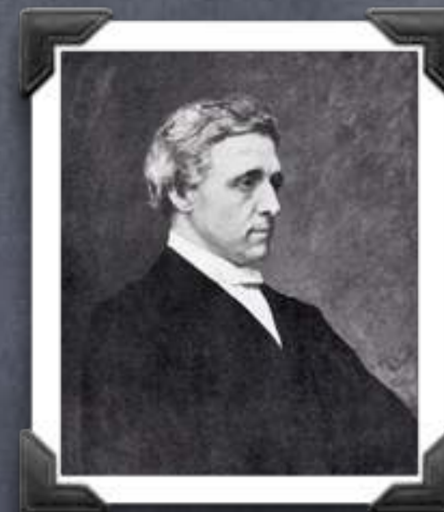
Graphs

Basic tools for expressing ideas
Logic, Proofs,
Sets, Relations, Functions



Logic

It's so easy even computers can do it!



Charles L Dodgson
1832 - 1898

Predicates & Propositions

Logic in action

Expert Systems

- From a repository consisting of "facts" or propositions derive answers to questions posed on the fly
- To automate decision making
- e.g., Prolog: a programming language that can be used to implement such a system

```
mother_child(trude, sally).

father_child(tom, sally).
father_child(tom, erica).
father_child(mike, tom).

sibling(X, Y) ← parent_child(Z, X)
                 ∧ parent_child(Z, Y).

parent_child(X, Y) ← father_child(X, Y).
parent_child(X, Y) ← mother_child(X, Y).
```

```
?- sibling(sally, erica).
Yes
```

The Pointless Game

- Alice and Bob sit down to play a new board game, where they take turns to make "moves" that they can choose (no dice/randomness)

- The rules of the game guarantee that
  - The game can't go on for ever
  - There are no ties — Alice or Bob will win when the game terminates
- Alice and Bob (smart as they are) decide that there is no point in playing the game, because they already know who is going to win it!

But how?

Predicates

- Fix a "domain of discourse" (e.g. set of characters in AIW)
- Predicate: a function that assigns a value of TRUE or FALSE to each element in the domain
  - If you apply a predicate to an element you get a **proposition**. It can be (has truth value) TRUE or FALSE

e.g.: Pink(Flamingo)

	Winged?	Flies?	Pink?
Alice	FALSE	FALSE	FALSE
Jabberwock	TRUE	TRUE	FALSE
Flamingo	TRUE	TRUE	TRUE

Propositional Calculus

cal·cu·lus /ˈkalkyələs/

1. The branch of mathematics that deals with the finding and properti
derivatives and integrals of functions, by methods originall
2. A particular method or system of calculation or rea

Synonyms: stone - calculation - reckoning - com

Unary operator

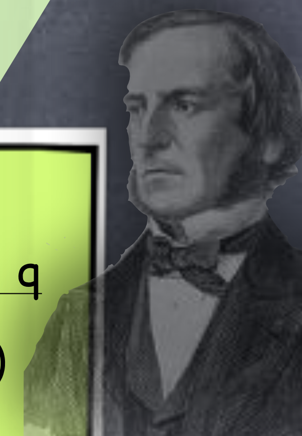
Binary operators

not p
Symbol: $\neg$ p
Negates the truth value.
e.g.: ¬Flies(Alice) has value True

p **or** q
Symbol: p $\vee$ q
True if and only if at least one of p and q is true
e.g.: ¬Flies(Alice) ∨ Pink(Jab'wock) has value True

p **and** q
Symbol: p $\wedge$ q
True if and only if both of p and q are true
e.g.: ¬Flies(Alice) ∧ Pink(Jab'wock) has value False

if p **then** q
Symbol: p $\rightarrow$ q
$(\neg p) \vee (p \wedge q)$
Same as $\neg p \vee q$
e.g.: Flies(Alice) → Pink(Jab'wock) has value True



George Boole
1815 - 1864

∨	T	F
T	T	T
F	T	F

∧	T	F
T	T	F
F	F	F

→	T	F
T	T	F
F	T	T

Operator Gallery

(T)	T	F
T	T	T
F	T	T

(F)	T	F
T	F	F
F	F	F

(1)	T	F
T	T	T
F	F	F

(2)	T	F
T	T	F
F	T	F

¬(1)	T	F
T	F	F
F	T	T

¬(2)	T	F
T	F	T
F	F	T

XOR $\neq$

$\oplus$	T	F
T	F	T
F	T	F

IFF =

$\leftrightarrow$	T	F
T	T	F
F	F	T

OR

$\vee$	T	F
T	T	T
F	T	F

NAND

$\uparrow$	T	F
T	F	T
F	T	T

NOR

$\downarrow$	T	F
T	F	F
F	F	T

AND

$\wedge$	T	F
T	T	F
F	F	F

IMPLIES

$\rightarrow$	T	F
T	T	F
F	T	T

IMPLIED-BY

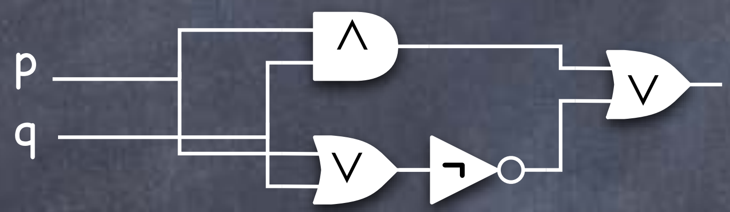
$\leftarrow$	T	F
T	T	T
F	F	T

$\nrightarrow$	T	F
T	F	T
F	F	F

$\nleftarrow$	T	F
T	F	F
F	T	F

Formulas

- A recipe for creating a new proposition from given propositions
  - e.g. $f(p,q) \triangleq (p \wedge q) \vee \neg(p \vee q)$



p	q	f
F	F	T
T	T	T

f	T	F
T	T	F
F	F	T

- Can also use "logic circuits" instead of formulas
- Different formulas can be equivalent to each other
  - e.g., $g(p,q) \triangleq \neg(p \oplus q)$. Then $f \equiv g$.
- A formula on two variables is equivalent to a binary operator

Another Example

- $g(p,q,r) \triangleq (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$
  - "Majority operator"
- $h(p,q,r) \triangleq (p \wedge (q \vee r)) \vee (q \wedge r)$
- $g \equiv h$

$((\neg p) \wedge q) \wedge r$

p	q	r	g	h
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
T	F	F	F	F
F	T	T	T	T
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

Quantified Propositions

(First-Order) Predicate Calculus

x	Winged(x)	Flies(x)	Pink(x)
Alice	FALSE	FALSE	FALSE
Jabberwock	TRUE	TRUE	FALSE
Flamingo	TRUE	TRUE	TRUE

$\in$ AIW

- All characters in AIW are winged. (False!) $\boxed{\forall x \text{ Winged}(x)}$
  - For every character x in AIW, Winged(x) holds
- Some character in AIW is winged. (True) $\boxed{\exists x \text{ Winged}(x)}$
  - There exists a character x in AIW, such that Winged(x) holds

Quantified Propositions

(First-Order) Predicate Calculus

x	Winged(x)	Flies(x)	Pink(x)
Alice	FALSE	FALSE	FALSE
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Flamingo	TRUE	TRUE	TRUE

- Quantifiers: To what "extent" does a predicate evaluate to TRUE in the domain of discourse
  - Universal quantifier, ∀
  - Existential quantifier, ∃

$\boxed{\forall x \text{ Winged}(x)}$

$\boxed{\exists x \text{ Winged}(x)}$

Quantified Propositions

(First-Order) Predicate Calculus

x	Winged(x)	Flies(x)	Pink(x)
Alice	FALSE	FALSE	FALSE
Jabberwock	TRUE	TRUE	FALSE
Flamingo	TRUE	TRUE	TRUE

- Could write $\forall x$ Winged(x) as:
  Winged(Alice) $\wedge$ Winged(J'wock) $\wedge$ Winged(Flamingo)
- And $\exists x$ Winged(x) as:
  Winged(Alice) $\vee$ Winged(J'wock) $\vee$ Winged(Flamingo)
  - But need to list the entire domain (works only if finite)

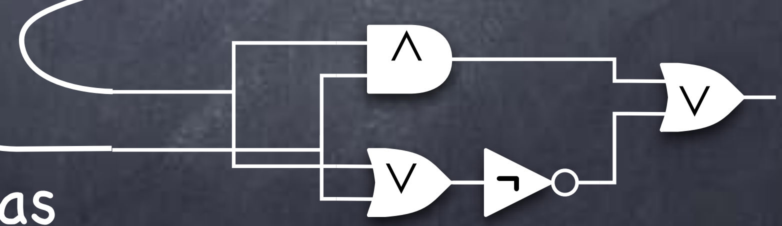
Examples

x	Winged(x)	Flies(x)	Pink(x)	Winged(x)→ ¬Flies(x)
Alice	FALSE	FALSE	FALSE	TRUE
Jabberwock	TRUE	TRUE	FALSE	FALSE
Flamingo	TRUE	TRUE	TRUE	FALSE

- $\forall x \;\; \text{Winged}(x) \leftrightarrow \text{Flies}(x)$ is True
- $\exists x \;\; \text{Winged}(x) \rightarrow \neg\text{Flies}(x)$ is True
- $\forall x \;\; \text{Pink}(x) \rightarrow \text{Flies}(x)$ is True

Today

	Winged?	Flies?	Pink?
Alice	FALSE	FALSE	FALSE
Jabberwock	TRUE	TRUE	FALSE
Flamingo	TRUE	TRUE	TRUE



- Propositions from predicates
- Propositions from formulas
  - Equivalence among formulas
- Quantified propositions
- Next: Manipulating propositions