

Logic  
It's so easy even  
computers can  
do it!

Through the  
Looking Glass

# The Looking Glass

- A mirror which shows the negation of every proposition
- Reflection changes **T & F** to **F & T** (resp.)
  - **∨ & ∧** are reflected as **∧ & ∨** (resp.)

Flies(Alice)  
is False

$\neg$  Flies(Alice)  
is True

Flies(Alice) ∨  
Flies(J'wock)  
is True

∨	T	F
T	T	T
F	T	F

?	F	T
F	F	F
T	F	T

$\neg$ Flies(Alice) ?  
 $\neg$ Flies(J'wock)  
is False

∧	F	T
F	F	F
T	F	T

∨	T	F
T	T	T
F	T	F

# The Looking Glass

- A mirror which shows the negation of every <sup>wire</sup> ~~proposition~~
- Reflection changes T & F to F & T (resp.)
  - $\vee$  &  $\wedge$  are reflected as  $\wedge$  &  $\vee$  (resp.)

## De Morgan's Law

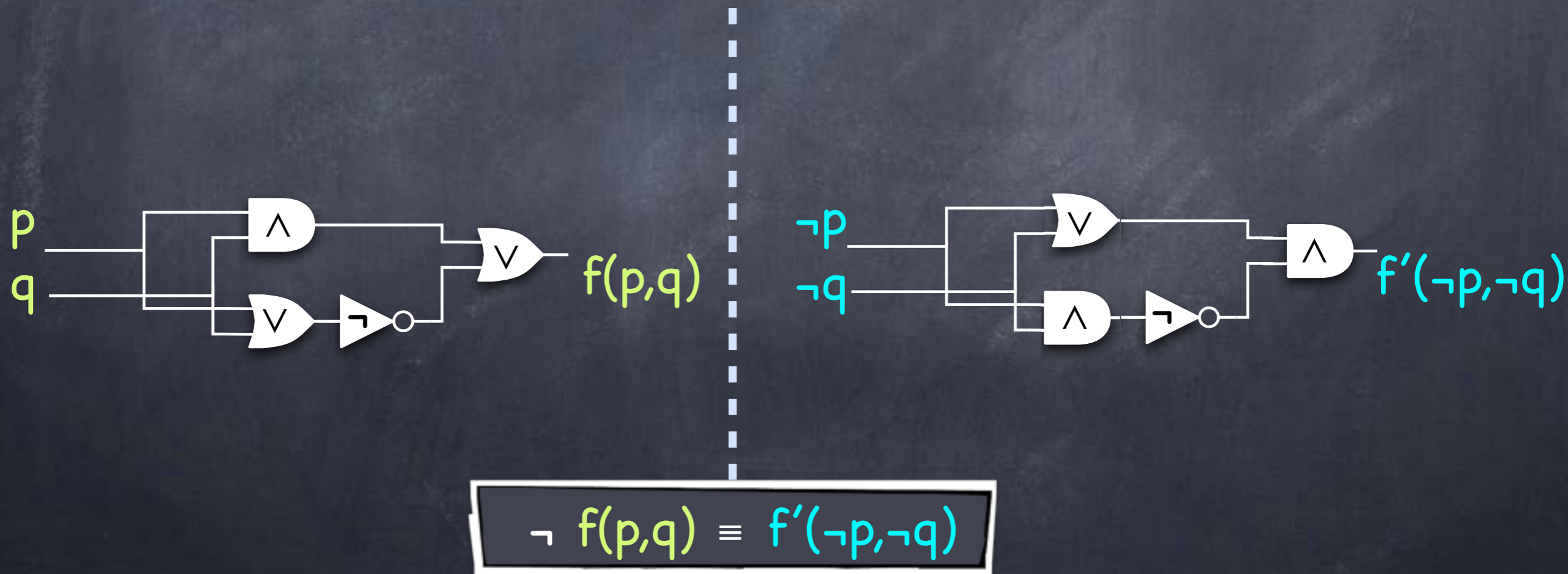
$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$

$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$



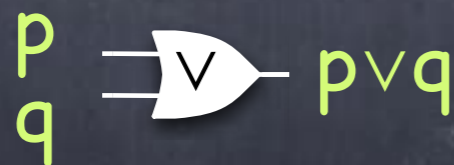
# The Looking Glass

- A mirror which shows the negation of every <sup>wire</sup> ~~proposition~~
- Reflection changes T & F to F & T (resp.)
  - $\vee$  &  $\wedge$  are reflected as  $\wedge$  &  $\vee$  (resp.)



# The Looking Glass

- Reflection changes **T & F** to **F & T** (resp.)
  - **$\vee$  &  $\wedge$**  are reflected as  **$\wedge$  &  $\vee$**  (resp.)
  - **$\forall$  &  $\exists$**  are reflected as  **$\exists$  &  $\forall$**  (resp.)



$\forall x \text{Pred}(x)$

$\exists x \neg \text{Pred}(x)$

$\exists x \text{Pred}(x)$

$\forall x \neg \text{Pred}(x)$

# Two quantifiers

x	y	Likes(x,y)	$\exists y \text{ Likes}(x,y)$ i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

- $\forall x \exists y \text{ Likes}(x,y)$ 
  - Everyone likes someone
- $\forall x \text{ LikesSomeone}(x)$
- True

# Two quantifiers

x	y	Likes(x,y)	$\exists y \text{ Likes}(x,y)$ i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

•  $\forall x \exists y \text{ Likes}(x,y)$

• Everyone likes someone

•  $\forall x \text{ LikesSomeone}(x)$

• True

•  $\exists x \neg (\exists y \text{ Likes}(x,y))$

# Two quantifiers

x	y	Likes(x,y)	$\exists y \text{ Likes}(x,y)$ i.e., LikesSomeone(x)
Alice	Alice	TRUE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	
Jabberwock	Alice	FALSE	TRUE
	Jabberwock	TRUE	
	Flamingo	FALSE	
Flamingo	Alice	FALSE	TRUE
	Jabberwock	FALSE	
	Flamingo	TRUE	

•  $\forall x \exists y \text{ Likes}(x,y)$

• Everyone likes someone

•  $\forall x \text{ LikesSomeone}(x)$

• True

•  $\exists x \forall y \neg \text{ Likes}(x,y)$

• Someone doesn't like anyone

•  $\exists x \text{ DoesntLikeAnyone}(x)$

• False



# Two quantifiers

x	y	Likes(x,y)
Alice	Alice	TRUE
	Jabberwock	FALSE
	Flamingo	TRUE
Jabberwock	Alice	FALSE
	Jabberwock	TRUE
	Flamingo	FALSE
Flamingo	Alice	FALSE
	Jabberwock	FALSE
	Flamingo	TRUE

•  $\exists y \forall x \text{ Likes}(x,y)$



# Two quantifiers

x	y	Likes(x,y)	$\forall x \text{ Likes}(x,y)$ i.e., EveryoneLikes(y)
Alice	Alice	TRUE	FALSE
Jabberwock		FALSE	
Flamingo		FALSE	
Alice	Jabberwock	FALSE	FALSE
Jabberwock		TRUE	
Flamingo		FALSE	
Alice	Flamingo	TRUE	FALSE
Jabberwock		FALSE	
Flamingo		TRUE	

•  $\exists y \forall x \text{ Likes}(x,y)$

• Someone is liked by everyone

• False

•  $\forall y \exists x \neg \text{Likes}(x,y)$

• Everyone is disliked by someone

• True