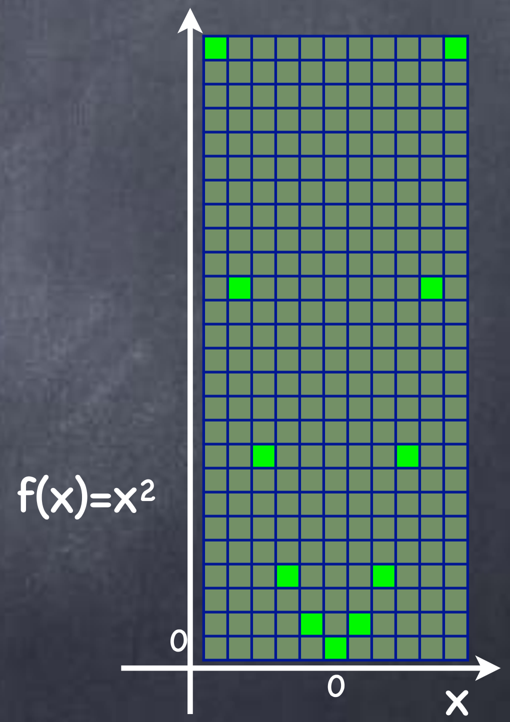
Functions



Functions

- For each element in a universe (domain), a predicate assigns one of two values, True and False.
- "Co-domain" is {True,False}
- Functions: more general co-domains
    - $f : A \to B$
- A function maps each element in the domain to an element in the co-domain
- To specify a function, should specify domain, co-domain and the "table" itself

pair$\in$AIW^2	Likes(pair)
(Alice, Alice)	TRUE
(Alice, Jabberwock)	FALSE
(Alice, Flamingo)	TRUE
(Jabberwock, Alice)	FALSE
(Jabberwock, Jabberwock)	TRUE
(Jabberwock, Flamingo)	FALSE
(Flamingo, Alice)	FALSE
(Flamingo, Jabberwock)	FALSE
(Flamingo, Flamingo)	TRUE

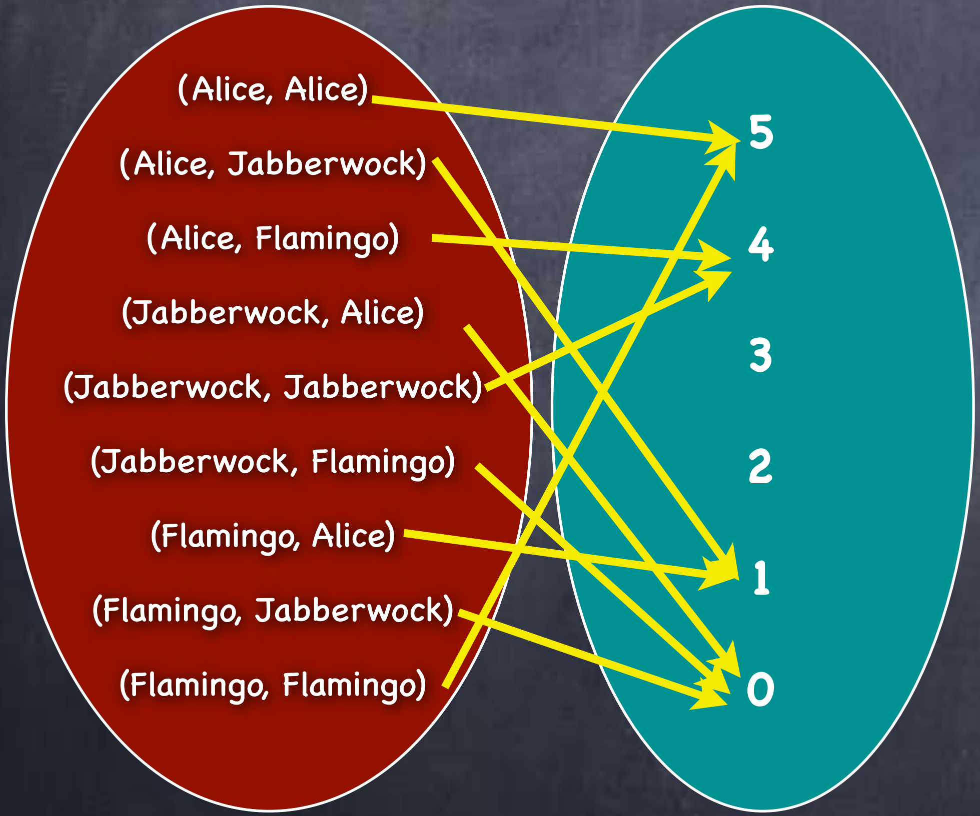
Functions

- eg: Extent of liking, f: $AIW^2 \rightarrow \{0,1,2,3,4,5\}$
  - Note: no empty slot, no slot with more than one entry
  - Not all values from the co-domain need be used
- Image: set of values in the co-domain that do get used
  - For $f:A \rightarrow B$, $Im(f) \subseteq B$ s.t. $Im(f) = \{ y \in B \mid \exists x \in A \ \ f(x) = y \}$

x∈Domain	f(x)∈Co-Domain
(Alice, Alice)	5
(Alice, Jabberwock)	1
(Alice, Flamingo)	4
(Jabberwock, Alice)	0
(Jabberwock, Jabberwock)	4
(Jabberwock, Flamingo)	0
(Flamingo, Alice)	1
(Flamingo, Jabberwock)	0
(Flamingo, Flamingo)	5

Functions

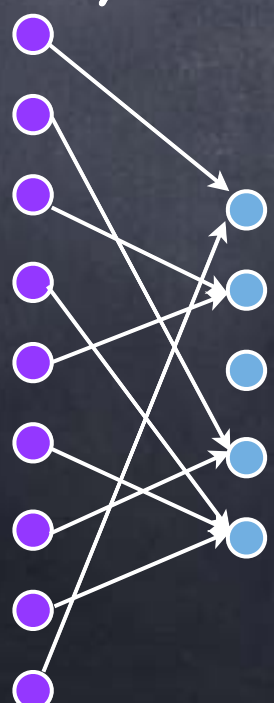
- eg: Extent of liking, f: $AIW^2 \rightarrow \{0,1,2,3,4,5\}$

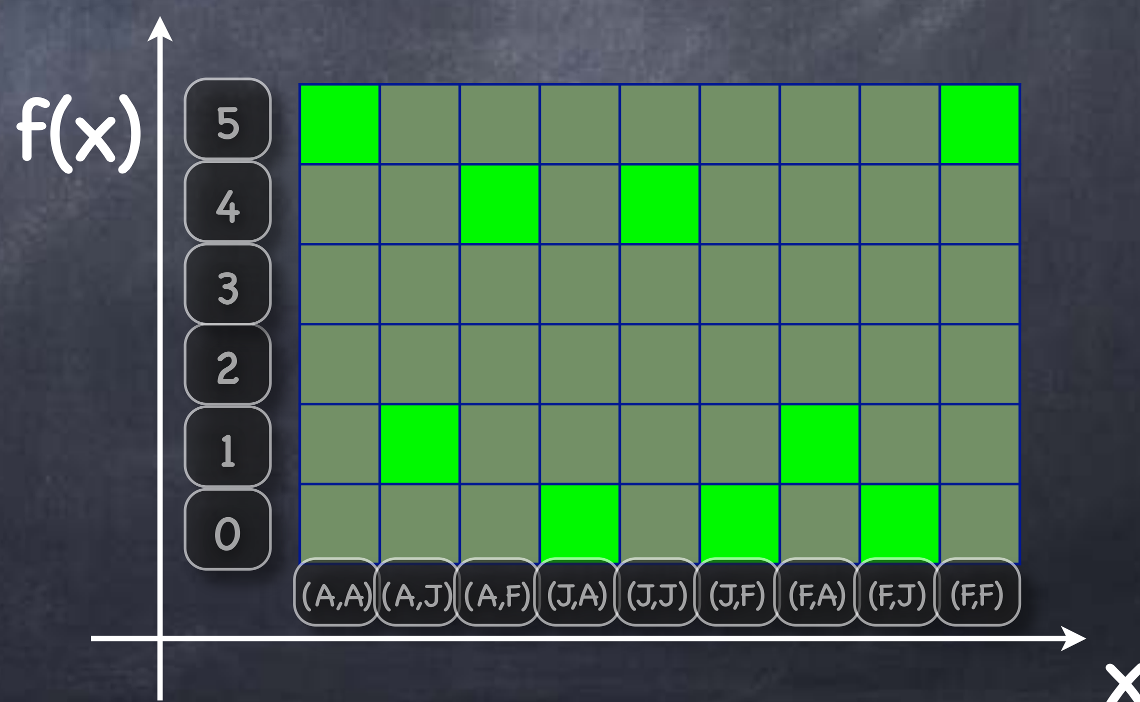


x∈Domain	f(x)∈Co-Domain
(Alice, Alice)	5
(Alice, Jabberwock)	1
(Alice, Flamingo)	4
(Jabberwock, Alice)	0
(Jabberwock, Jabberwock)	4
(Jabberwock, Flamingo)	0
(Flamingo, Alice)	1
(Flamingo, Jabberwock)	0
(Flamingo, Flamingo)	5

Function as a Relation

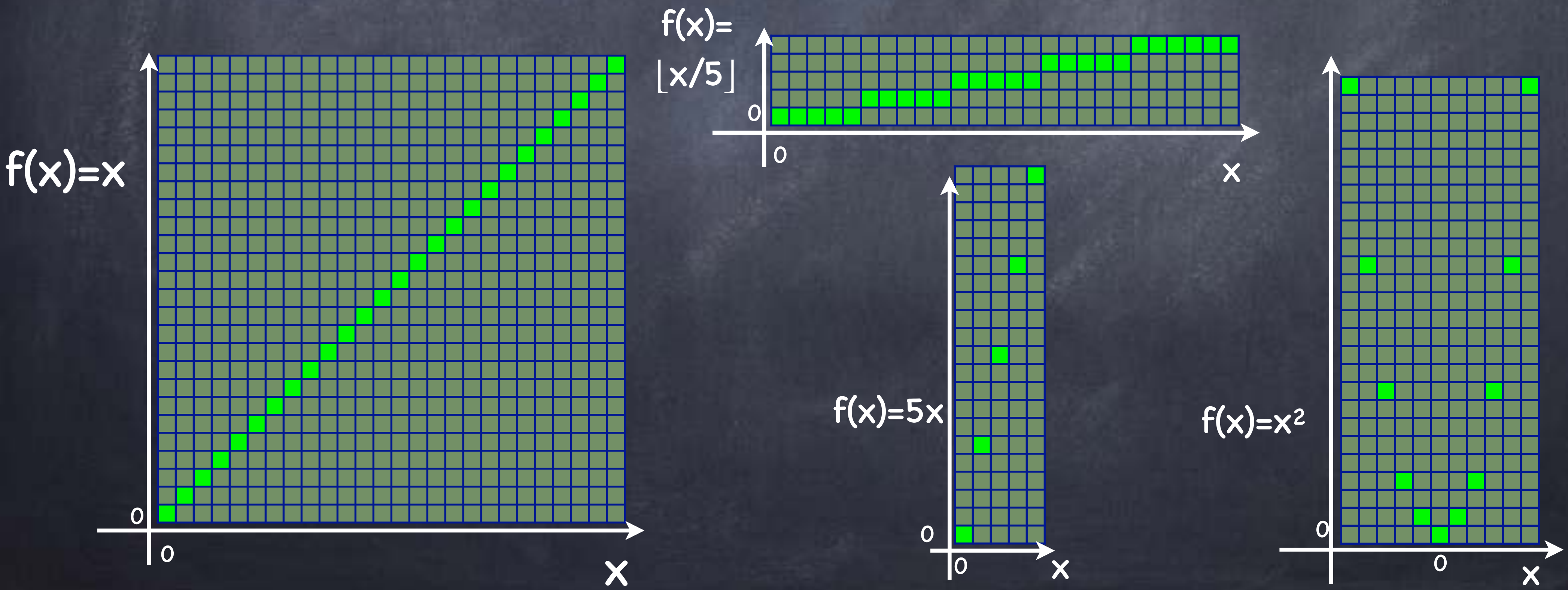
- As a relation between domain & co-domain, $R_f \subseteq$ domain $\times$ co-domain
  $R_f = \{ (x,f(x)) \mid x \in \text{domain} \}$
  - Special property of R_f: every x has a unique y s.t. $(x,y) \in R_f$
- Can be represented using a matrix
  - Convention: domain on the "x-axis", co-domain on the "y-axis"
  - Every column has exactly one cell "switched on"





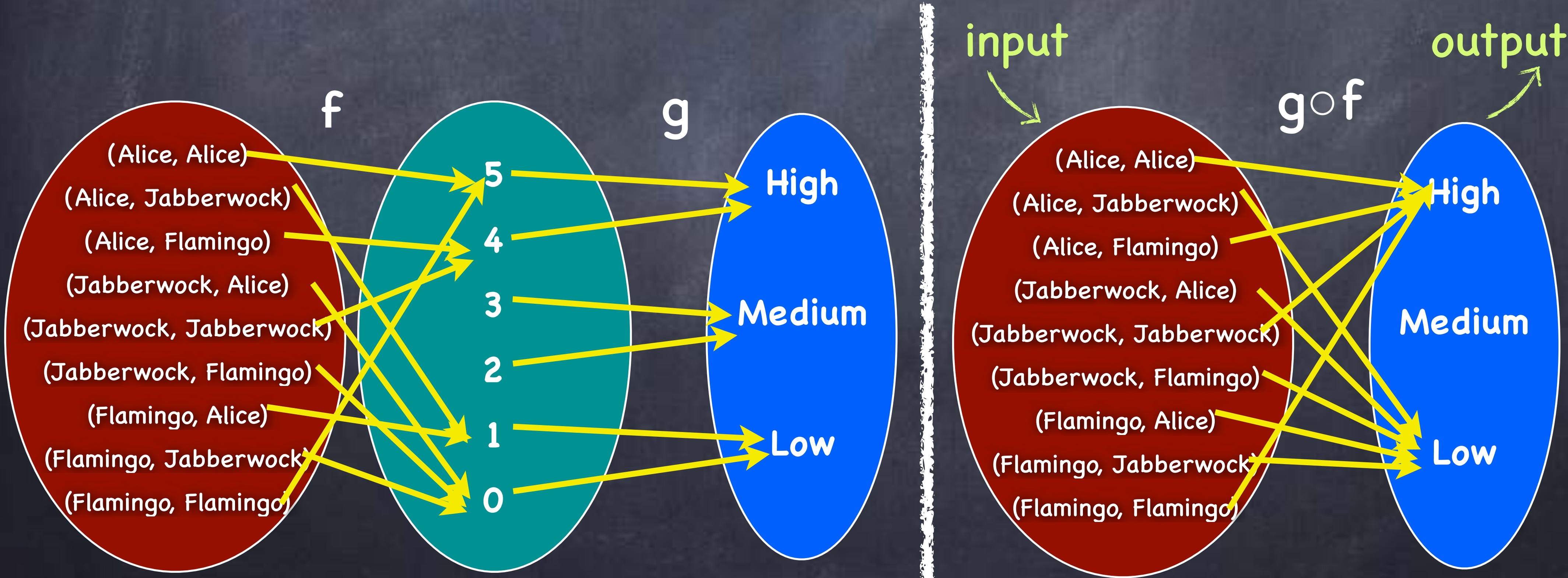
Plotting a Function

- When both domain and co-domain are numerical (or otherwise totally ordered), we often "plot" the function
  - Shows only part of domain/codomain when they are infinite (here f:$\mathbb{Z} \rightarrow \mathbb{Z}$)



Composition

- Composition of functions f and g: $g \circ f$: Domain(f) → Co-domain(g)
  - $g \circ f(x) \triangleq g(f(x))$



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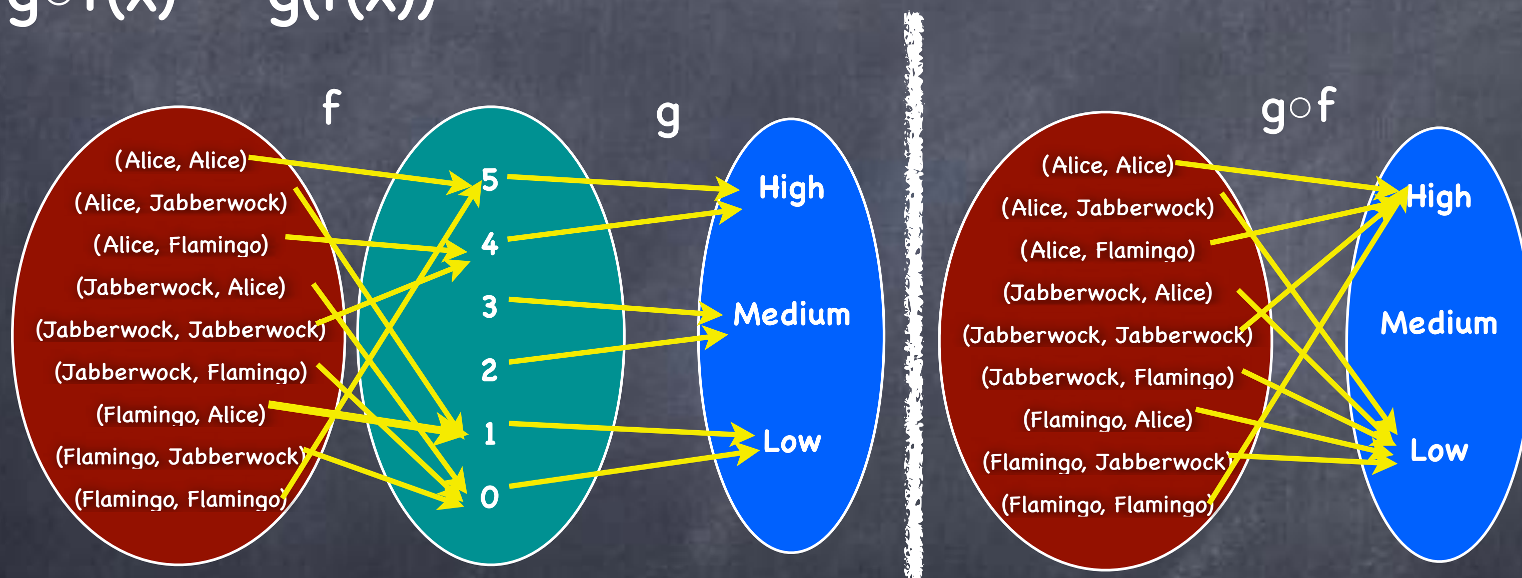

  - Defined only if Im(f) $\subseteq$ Domain(g)
    - Typically, Domain(g) = Co-domain(f)
  - $g \circ f$: Domain(f) $\rightarrow$ Co-domain(g)
  - Im($g \circ f$) $\subseteq$ Im(g)