

Problem Set 1 Solutions

Problem 1

Consider a convolutional code with $k = 1$, $n = 3$, and constraint length $l = 3$. Its generators are (111), (110), and (101). A four bit message string is encoded into a 12-bit coded string with this code and transmitted. The decoder at the receiver has obtained the coded string 011001001010. What is the most likely transmitted 4-bit message string?

Solution 1

Consider all possible 4 bit messages. Encode them with the code. Compute the Hamming distance between each such coded string and the received string. The message corresponding to the minimum Hamming distance is the answer. The answer is 1101.

Problem 2

The highest bit rate in the WiFi standards of 802.11a/g uses the QAM64 modulation and a convolutional code of rate $\frac{3}{4}$. Derive the physical layer data rate (in Mbps) obtained by this modulation and coding scheme.

Solution 2

Each 4 microsec symbol has 48 data subcarriers $\rightarrow 48 \cdot 6$ coded bits $\rightarrow 48 \cdot 6 \cdot \frac{3}{4} = 12 \cdot 18$ data bits. Therefore, bit rate comes up to $12 \cdot 18 / 4 = 54$ Mbps.

Problem 3

Consider a sender and receiver communicating over a 20 MHz wireless channel.

- The transmit power of the sender is 100 mW. Convert this value into units of dBm.
- The thermal noise on this channel is measured to be -100 dBm. Convert this value into units of watts.
- What is the SNR at the receiver if the transmitted signal suffers a path loss of 90 dB due to channel propagation effects? (Use the transmit power and noise values from (a) and (b) above.)
- What is the maximum rate at which information can be transmitted between this sender and receiver as per Shannon's capacity formula?
- What are the answers to (c) and (d) above if the sender doubles his transmit power?

Solution 3

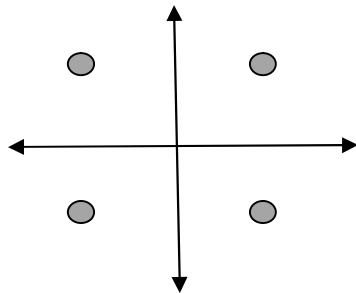
- $10 \cdot \log_{10} (100 \text{ mW} / 1 \text{ mW}) = 20 \text{ dBm}$
- $10 \cdot \log_{10} (x / 1 \text{ mW}) = -100$, $x = 0.1$ pico watt
- SNR at transmitter is $20 \text{ dBm} - (-100 \text{ dBm}) = 120 \text{ dB}$, SNR at receiver = $120 - 90 = 30 \text{ dB}$
- $P_s/P_n = 1000$. $C = 20 \text{ M} \cdot \log_2 (1 + 1000) \sim 199 \text{ Mbps}$.

- e. For a doubling of transmit power, SNR increases by about 3 dB to about 33 dB, capacity increases to about 219 Mbps.

Problem 4

Consider a sender and receiver communicating using the QPSK modulation scheme. The constellation diagram used has valid symbols at the four corners of a unit square as shown below. That is, the valid constellation symbols are $(1,1)$, $(1, -1)$, $(-1, 1)$ and $(-1,-1)$.

- a. How many data bits can be modulated using each constellation symbol in this modulation scheme?
- b. Provide a Gray coding that maps from bits to constellation symbols.
- c. Assume that the receiver demodulates a received symbol to obtain a point (x,y) on the constellation diagram. The receiver must map this point to the closest constellation symbol to extract the most likely transmitted bits. Provide a formula / decision rule that maps (x,y) to the set of likely transmitted bits.



Solution

- a. 2 bits can be sent per symbol.
- b. One possible Gray coding is as follows. $(1,1) \rightarrow$ bits "11", $(1, -1) \rightarrow$ bits "10", $(-1,-1) \rightarrow$ bits "00", $(-1, 1) \rightarrow$ bits "01"
- c. The x and y axes form the decision boundaries. If $x>0$ and $y>0$, map it to constellation point $(1,1)$ and hence to bits "11". If $x>0$ and $y<0$, map it to $(1, -1)$ and hence bits "10". You can derive the other rules similarly.

Problem 5

Please refer to the paper on wireless link measurements from the Roofnet project ("Link-level Measurements from an 802.11b Mesh Network") that is posted on the class website. Refer to Figures 12 and 14, which provide packet delivery rates as a function of SNR, for various transmit bit rates. Figure 12 is derived by connecting the WiFi cards via a channel emulator (i.e., a wire that mimics a wireless channel), which Figure 14 is from real experiments. We will use these curves as a guide to pick the best transmit bit rate between a sender and receiver. Note that the best transmit bit rate is defined for now as the bit rate that maximizes the throughput (which is roughly the packet delivery rate times the bit rate).

- a. Use Figure 12 as your guide and suggest what the best transmit bit rate would be for a sender and receiver that have SNR values of 2,4,6,8, and 10 dB respectively.
- b. Will your choices change if you use Figure 14 instead? What will your new answer be? And why?

Solution

- a. Look up the graph in Figure 12 for packet delivery rates at each bit rate. Pick the bit rate that maximizes throughput ($= \text{packet_delivery_rate} * \text{physical_layer_rate}$). The answers are 2 dB $\rightarrow$ 1 Mbps, 4 dB $\rightarrow$ 5 Mbps, 6 dB $\rightarrow$ 5 Mbps, 8 dB $\rightarrow$ 5 Mbps, 10 dB $\rightarrow$ 11 Mbps
- b. The choice of bit rate for a given SNR should be lower in this case, because real life effects lead to more losses at a given SNR. There is also a range of values of packet delivery rates for a given SNR. So your choice of bit rate will be more conservative. Many answers are possible depending on how you interpret the graphs and how “safe” you want your rate choice to be (given that loss rate has a wide distribution at a given SNR). Your answer will be marked correct as long as you justify your choice.