Lecture 2: Wireless Physical Layer, Channel, and Capacity

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Radio waves and spectrum

- Physical layer of mobile systems – wireless using electromagnetic (EM) waves
- EM spectrum

<table>
<thead>
<tr>
<th>Name</th>
<th>LF (low freq)</th>
<th>VHF (very high)</th>
<th>UHF (ultra high)</th>
<th>SHF (super high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>30Hz – 30 MHz</td>
<td>30 – 300 MHz</td>
<td>300 MHz – 3 GHz</td>
<td>3 – 30 GHz</td>
</tr>
<tr>
<td>Wavelength (speed of light / frequency)</td>
<td>Few km to metres</td>
<td>Few metres</td>
<td>Few hundreds of cm</td>
<td>Few cm</td>
</tr>
<tr>
<td>Used by</td>
<td>Radio, submarines</td>
<td>Analog TV</td>
<td>Digital TV, cellular, WiFi</td>
<td>WiFi, satellites</td>
</tr>
</tbody>
</table>
Radio waves and spectrum (2)

- What dictates choice of spectrum?
  - **Antenna size**: of the order of wavelength, so lower frequencies need costlier antennas
  - **Multiplexing of users**: two users using same frequency bands interfere, so regulations to enable sharing
  - **Propagation characteristics**: lower frequencies propagate better. LF/MF/HF propagate long distances around earth. VHF reflected by ionosphere. Higher frequencies need line of sight (LOS)

- Common bands
  - 800-900, 1800-1900, 2000-2100 MHz – cellular
  - 2.4GHz ISM band – WiFi (802.11b/g/n)
  - Good balance between antenna size and propagation
Antennas

- Convert between electromagnetic waves on a wire to EM waves in air and vice versa
  - Isotropic: equal radiation in all directions, ideal
  - Simple dipole antennas: omni-directional in one plane, figure ‘8’ in other two planes
  - Directional antennas: focused in only one direction
  - Sectorized antennas: combinations of directional antennas
  - Smart antennas, antenna arrays – steer multiple directional antennas to focus signal to a user
- This is the domain of EE, we won’t go into much detail
Communication using radio waves

- Transmit information using a sin/cosine wave of a given frequency (carrier)
- Simple idea: analog modulation of signal amplitude.
- Used in AM radio
- Error prone and not resilient to distortions
Communication using radio waves (2)

- Better idea – **digital modulation**.

- Convert “signal” into stream of bits.
  - Sample signal at fixed intervals, discrete time signals
  - Quantize the signal value into discrete levels. For example, 4 levels can be represented by 2 bits: 00, 01, 10, 11

- Use bits to modulate the amplitude of the carrier. If 1, send the carrier signal. If 0, no signal. This is amplitude shift keying, there are many more modulation techniques.

- Advantage over analog: even if distorted, can easily tell apart 0 and 1
Communication using radio waves (3)

Amplitude Shift Keying
(example of a digital modulation scheme)

Data signal (baseband)
EM wave (carrier)
Transmitted (passband)

Note: I am drawing smooth curves for ease, but always think of them as discrete samples.
Wireless signal propagation

- What happens when you transmit the modulated signal over the air?
  - Signal travels over multiple paths – multiple copies
  - Each copy of the signal suffers different attenuation
    - Path loss (inverse square law)
    - Reflection, refraction, diffraction, shadowing, scattering etc
  - Each copy of the signal may also have its frequency shifted slightly (Doppler shift due to movement)
  - Finally, there is always background thermal noise
  - Various channel models exist to characterize these effects
- Received signal is sum of multiple different copies of the signal + noise
Wireless signal propagation (2)

- Wireless channel is described by channel impulse response “h” – what do you receive when you send one impulse.
- Ideally, you only receive the impulse (with some propagation delay).
- With multipath, you receive multiple copies of the impulse, each scaled by different amounts.

\[
y = h \ast x + n
\]

- Transmitted signal “x”, channel “h”, noise “n”, then received signal is \( y = h \ast x + n \).
- Here “\( \ast \)” is the convolution operator. That is, for each sample in signal, we consider the impulse response, and add up all these components over all samples.
- Need to estimate and compensate for this “h” at receiver – channel equalization.
Wireless signal propagation (3)

- Net effect of the wireless channel
  - Signal is attenuated (weakened) at longer timescales due to path loss etc
  - Variations at smaller timescales due to multipath fading (multiple copies combine constructively or destructively)
  - Finally, background thermal noise
Decoding wireless signals

- Let us revisit our example of digitally modulated signal
- The signal in the “1” bits is distorted due to the channel
- The “0” bits aren’t exactly zero either due to noise
- Net effect: 0 and 1 are hard to distinguish. Possible errors in communication.
- If received signal >> noise, can decode
- If received signal power and noise power are roughly equal, hard to recover the transmitted signal
The concept of SNR

- Signal to noise ratio (SNR) = Power of signal / power of noise

- SNR is measured in logarithmic decibel (dB) units.
  - $\text{SNR(dB)} = 10 \times \log_{10} (\frac{P_s}{P_n})$
  - For example, $\frac{P_s}{P_n} = 1000$, SNR = 30 dB

- Communication systems have a minimum SNR threshold below which they cannot decode correctly

- Path loss and any attenuation can also be measured in dB. E.g., signal transmitted at 50 dB, path loss 30 dB, received $50-30=20$ dB (logarithmic unit, hence subtraction)

- Another unit to measure signal strength: dBm (relative to $P_n=1\text{mW}$). E.g., if signal transmitted at 1000mW, then signal strength = $10\times\log_{10}(1000/1) = 30$ dBm.
How fast can you send?

- SNR decides how easy to decode, and how many errors
- The “symbol duration” (how long you send each 0 and 1) decides the rate at which you can send data bits

How to decide what rate to send at?
- Math gets harder and harder...
Now, the math: Discrete Fourier Transform

- Fourier transforms are a way to represent and deal with wireless signals. Makes math easier and more intuitive.
- Fourier analysis – a way of representing a signal as a sum of sine and cosine curves of various frequencies (approx.)
- Discrete Fourier Transform (DFT): Fourier transform for discrete time signals (which is what we deal with)
- Given N samples of a time series signal, x(n), n=0,...,N-1, DFT computes N coefficients, X(k), k=0,...,N-1, such that, each X(k) represents the weight of a sin/cosine curve of a particular frequency $2\pi k/N$.
- That is, take N sin/cosine curves of N different frequencies ($2\pi k/N$, k=0,...,N-1), scale them by the DFT coefficients X(k) and add them up, you will get the original time series signal x(n), n=0,...,N-1.
- Note that I have used very approximate terminology here to avoid equations. For more detail, please refer to any signal processing textbook.
The time series of a signal is called the **time domain** representation. The DFT coefficients constitute the **frequency domain** representation. They are both equivalent (if you have one, you can get the other).

Let’s look at our example modulation scheme. The time domain and frequency domain graphs are shown here.

Note that carrier has only one component (at its frequency) in DFT.

The transmitted signal is “shifted” to the carrier’s frequency.
So, how fast can you send?

- Let’s revisit question of how fast you can send. The analysis becomes very simple in frequency domain.
- What is the difference between a faster pulse and a slower pulse?
- DFT of faster pulse has a wider frequency “band”, i.e., greater bandwidth than slower pulse.
- Bandwidth is inversely proportional to symbol duration.
- More bandwidth -> more frequencies of EM spectrum used.
So, how fast can you send? (2)

- How fast you can send depends on two factors
  - Your allocated frequency band. For example, 802.11g has 20 MHz channels allocated. So if you send faster than a certain rate, your frequency domain signal will overlap with your neighbor’s signal and lead to data loss. (Note how such an analysis is easy with DFT!)
  - Your hardware capability to sample. More frequencies mean that you must send out more samples. Nyquist’s sampling law says that if your highest frequency in baseband is \( f \), you must sample at the rate of at least \( 2f \). Faster sampling requires faster hardware.
- Once you decide the bandwidth of your signal (based on above two factors), you know how fast your pulses can be
Wireless Channel Capacity

- What is the “speed” of the wireless channel? For example, you have a 100Mbps ethernet link. What is the equivalent for a wireless channel?
- The rate at which you can send information depends on two factors
  - The rate at which you can send symbols, which depends on bandwidth
  - How effectively you can distinguish the 0s and 1s in each symbol, which depends on SNR
Wireless Channel Capacity (2)

- Shannon’s capacity $C = B \times \log_2(1 + \frac{Ps}{Pn})$
- $B =$ bandwidth, $\frac{Ps}{Pn}$ is SNR as a ratio (not dB)
- For example, for a 20 MHz channel, and SNR 30 dB, what is $C$?
  - SNR = 20 dB, so $\frac{Ps}{Pn} = 1000$
  - $C \approx 20\text{M} \times \log_2(1001) \approx 20\text{M} \times 10 \approx 200\text{ Mbps}$
- Shannon’s capacity is an upper bound. That is, if you try to send at a rate $R > C$, you will see lots of errors.
- That said, any scheme that sends at a rate $R < C$ is not bound to succeed, you must build your signal wisely.
- Next lecture, we will see ways of constructing a transmission signal (modulation, channel coding) that will get us close to channel capacity.
Summary

- Wireless physical layer uses electromagnetic waves of a certain frequency spectrum
- The bits in the data signal are used to modulate the carrier to get the transmitted signal
- Wireless channel causes path loss, multipath fading, and other effects that weaken the transmitted signal
- Signal to noise ratio (SNR) determines how easy it is to recover the bits from the received signal
- Bandwidth of the transmission determines how fast you can send bits one after the other
- Shannon gives us a way to calculate the maximum rate at which we can send bits (capacity) as a function of bandwidth and SNR