Robot Locomotion

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Partial slide courtesy by C. Stachniss, W. Burgard and D. Fox

Locomotion of Wheeled Robots

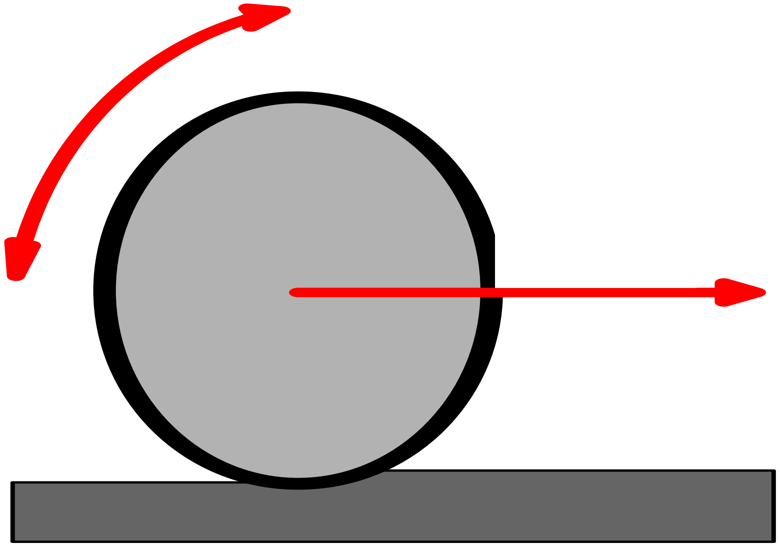
- Locomotion is the “power of motion from place to place”
- Relevant for state estimation

Typical drives

- Differential drive (Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- Mecanum wheels

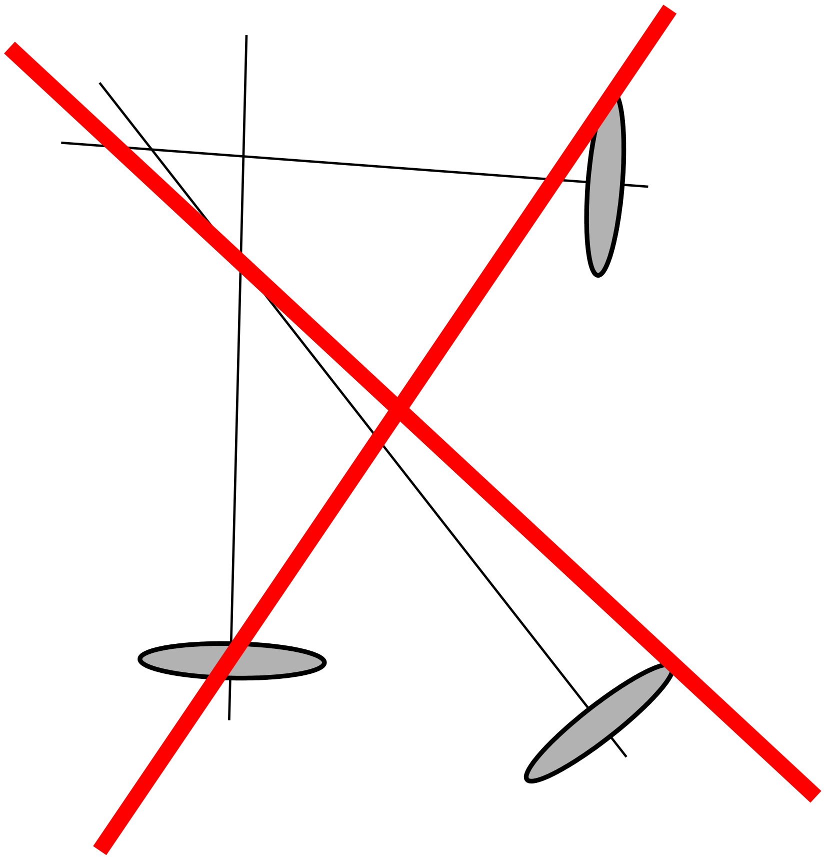
Locomotion of Wheeled Robots

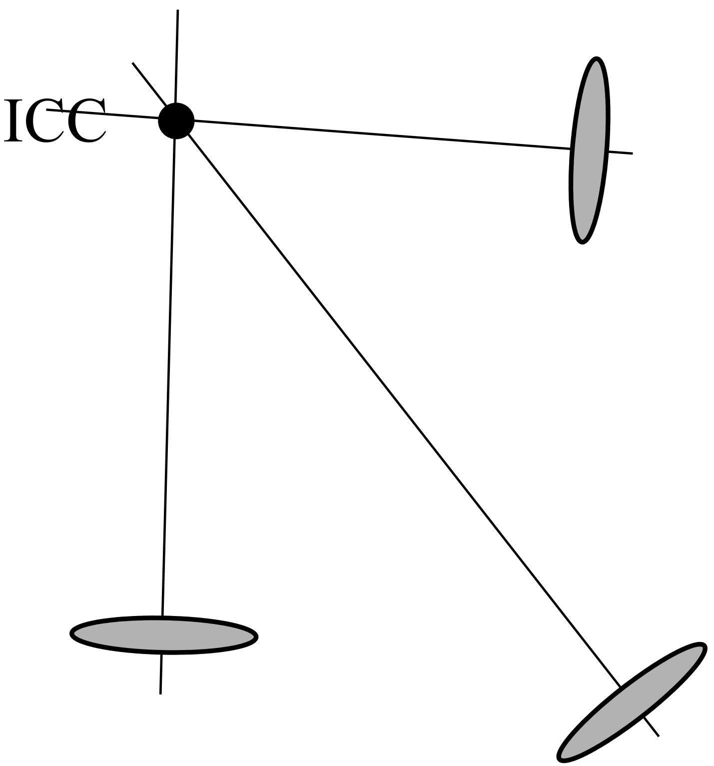
Wheels are turning…



Instantaneous Center of Curvature

For rolling motion to occur, each wheel has to move along its y-axis





Basic Laws of Physics to Describe Wheeled Motion

$$s = v\,t$$

$$\theta = \omega\,t$$

$$v = \omega\,r$$

Differential Drive Robot

Differential Drive Robot

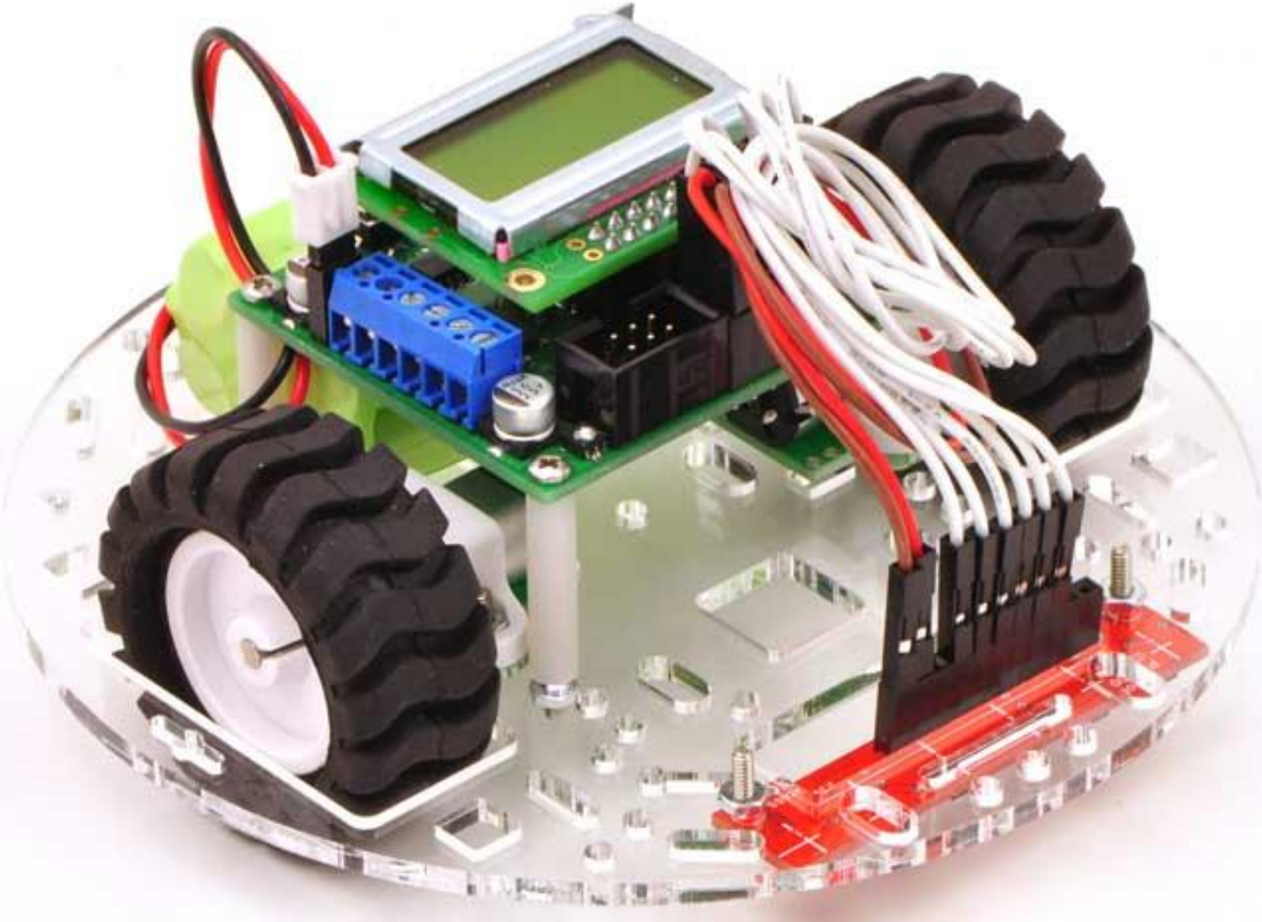
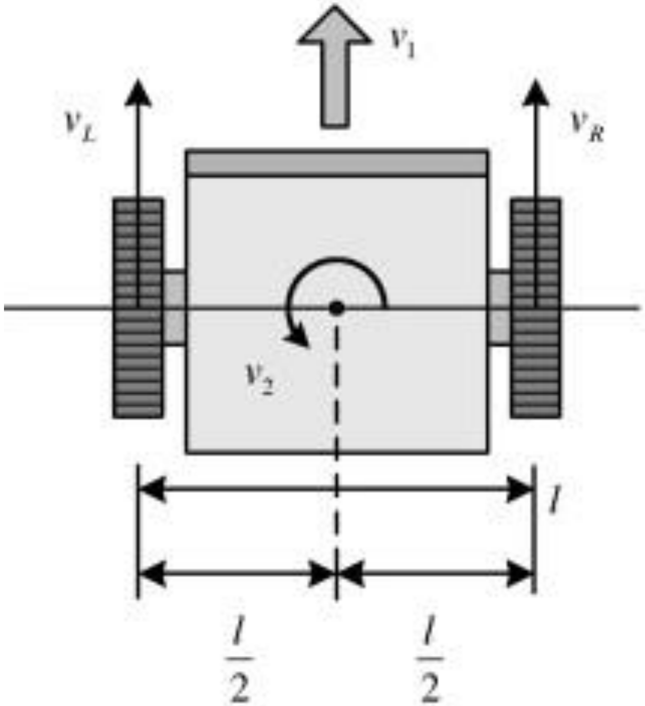


Image Courtesy: ZigoBot

Differential Drive Robot



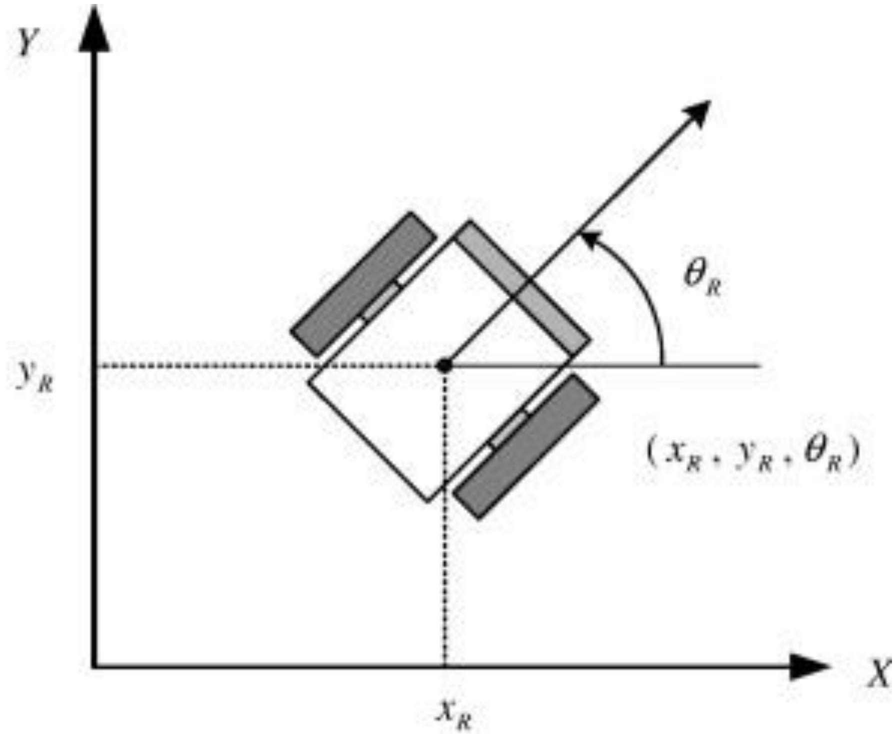
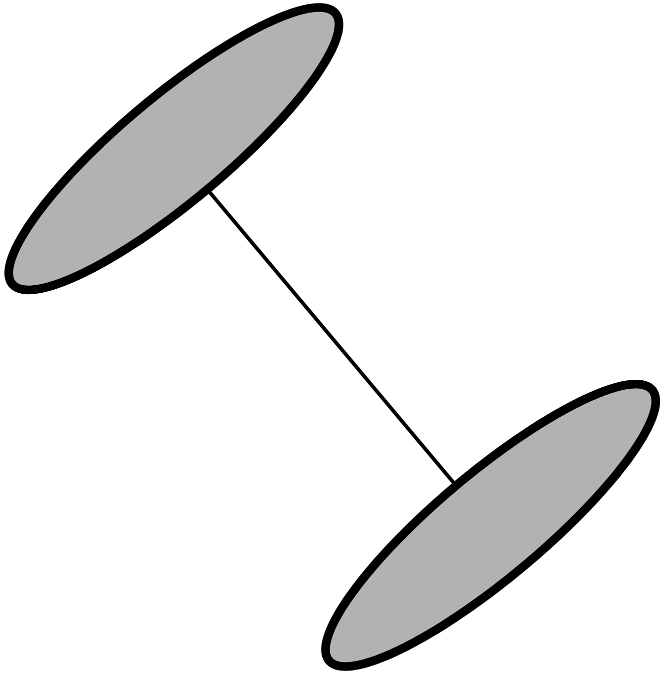
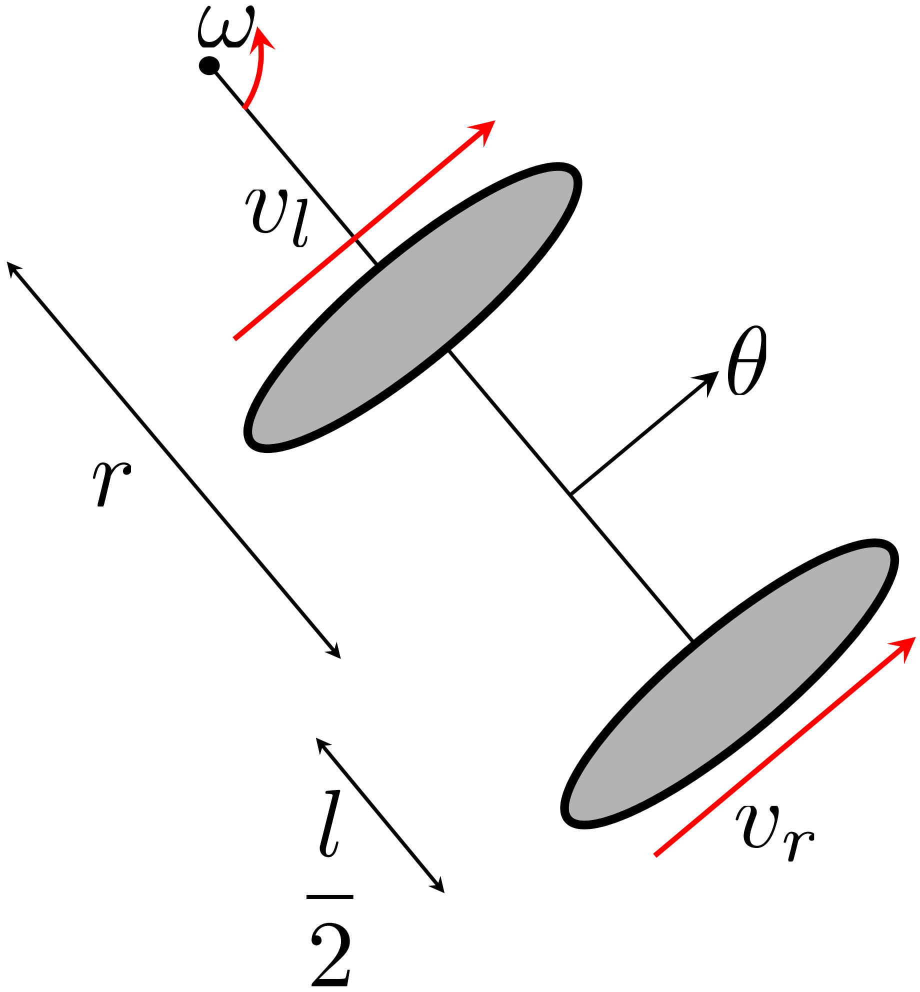


Image Courtesy: Han et al.

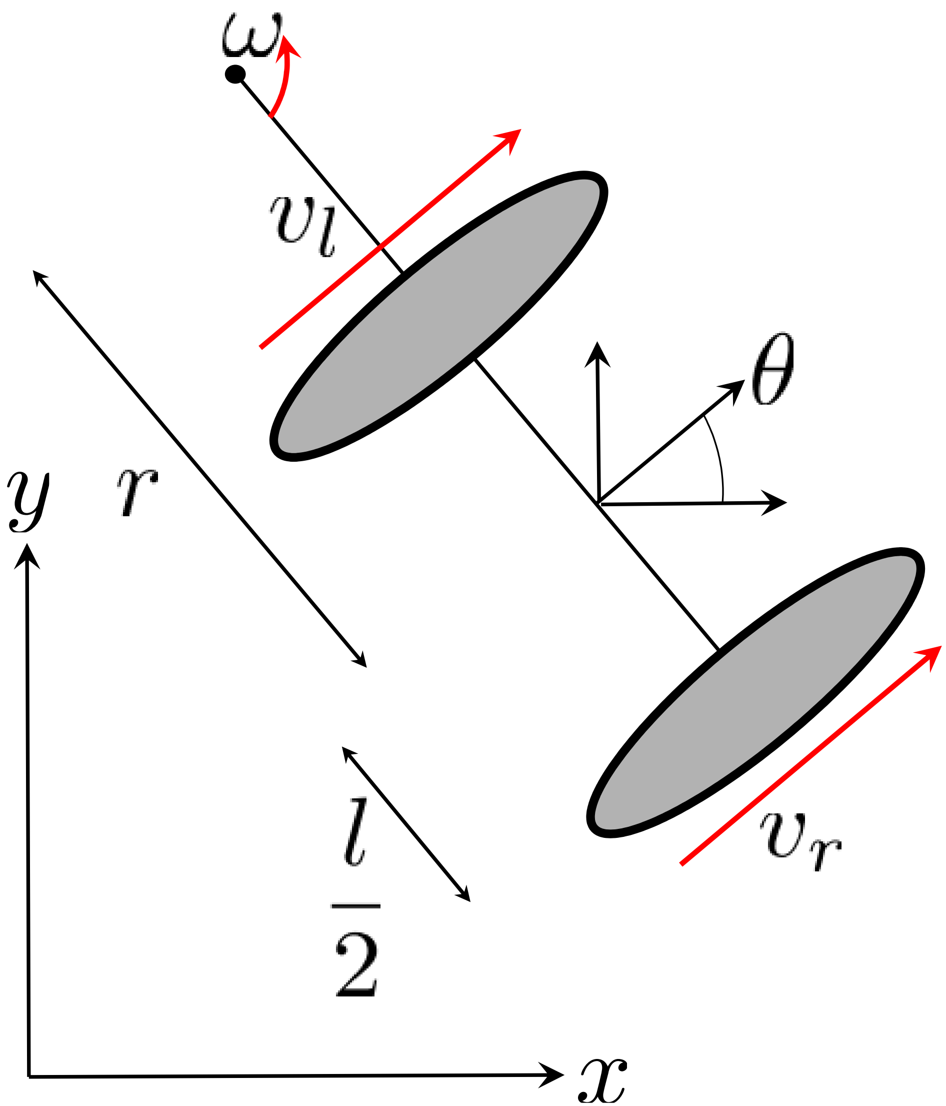
Differential Drive



Differential Drive



Differential Drive



$$v_l = \omega \left(r - l/2\right)$$
$$v_r = \omega \left(r + l/2\right)$$

Differential Drive

$$v_l = \omega \left(r - l/2 \right)$$

$$v_r = \omega \left(r + l/2 \right)$$

Step 1 $$\omega = \frac{v_r}{r + l/2} = \frac{v_l}{r - l/2}$$

Differential Drive

Step 1 $$\omega = \frac{v_r}{r + l/2} = \frac{v_l}{r - l/2}$$

$$v_l\,(r + l/2) = v_r\,(r - l/2)$$

$$v_l\,r + \frac{v_l\,l}{2} = v_r\,r - \frac{v_r\,l}{2}$$

$$r(v_r - v_l) = \frac{l}{2}(v_r + v_l)$$

$$\boxed{r = \frac{l}{2}\,\frac{(v_r + v_l)}{(v_r - v_l)}}$$

Differential Drive

$$v_l = \omega \left(r - l/2 \right)$$

$$v_r = \omega \left(r + l/2 \right)$$

Step 2 $\left(v_r + v_l \right) = \omega \left(r + l/2 + r - l/2 \right)$

Differential Drive

Step 2 $(v_r + v_l) = \omega\,(r + l/2 + r - l/2)$

$$(v_r + v_l) = \omega\, 2\, r$$

$$\boxed{\omega = \frac{(v_r + v_l)}{2\, r}}$$

Differential Drive

Step 3: Combine both results

$$\omega = \frac{(v_r + v_l)}{2\,r} \qquad r = \frac{l}{2}\,\frac{(v_r + v_l)}{(v_r - v_l)}$$

Differential Drive

Step 3: Combine both results

$$\omega = \frac{(v_r + v_l)}{2\,r} \qquad r = \frac{l}{2}\,\frac{(v_r + v_l)}{(v_r - v_l)}$$

$$\omega = \frac{(v_r + v_l)}{2\,\frac{l}{2}\,\frac{(v_r+v_l)}{(v_r-v_l)}}$$

$$\omega = \frac{(v_r + v_l)(v_r - v_l)}{l\,(v_r + v_l)}$$

$$\boxed{\omega = \frac{(v_r - v_l)}{l}}$$

Differential Drive

Step 4: Exploit $v = \omega\, r$

$$\omega = \frac{(v_r - v_l)}{l} \qquad r = \frac{l}{2}\frac{(v_r + v_l)}{(v_r - v_l)}$$

Differential Drive

Step 4: Exploit $v = \omega\, r$

$$\omega = \frac{(v_r - v_l)}{l} \qquad r = \frac{l}{2}\,\frac{(v_r + v_l)}{(v_r - v_l)}$$

$$v = \frac{(v_r - v_l)}{l}\,\frac{l}{2}\,\frac{(v_r + v_l)}{(v_r - v_l)}$$

$$\boxed{v = \frac{(v_r + v_l)}{2}}$$

Differential Drive

In sum, we obtain

$$v = \frac{(v_r + v_l)}{2}$$

translation velocity

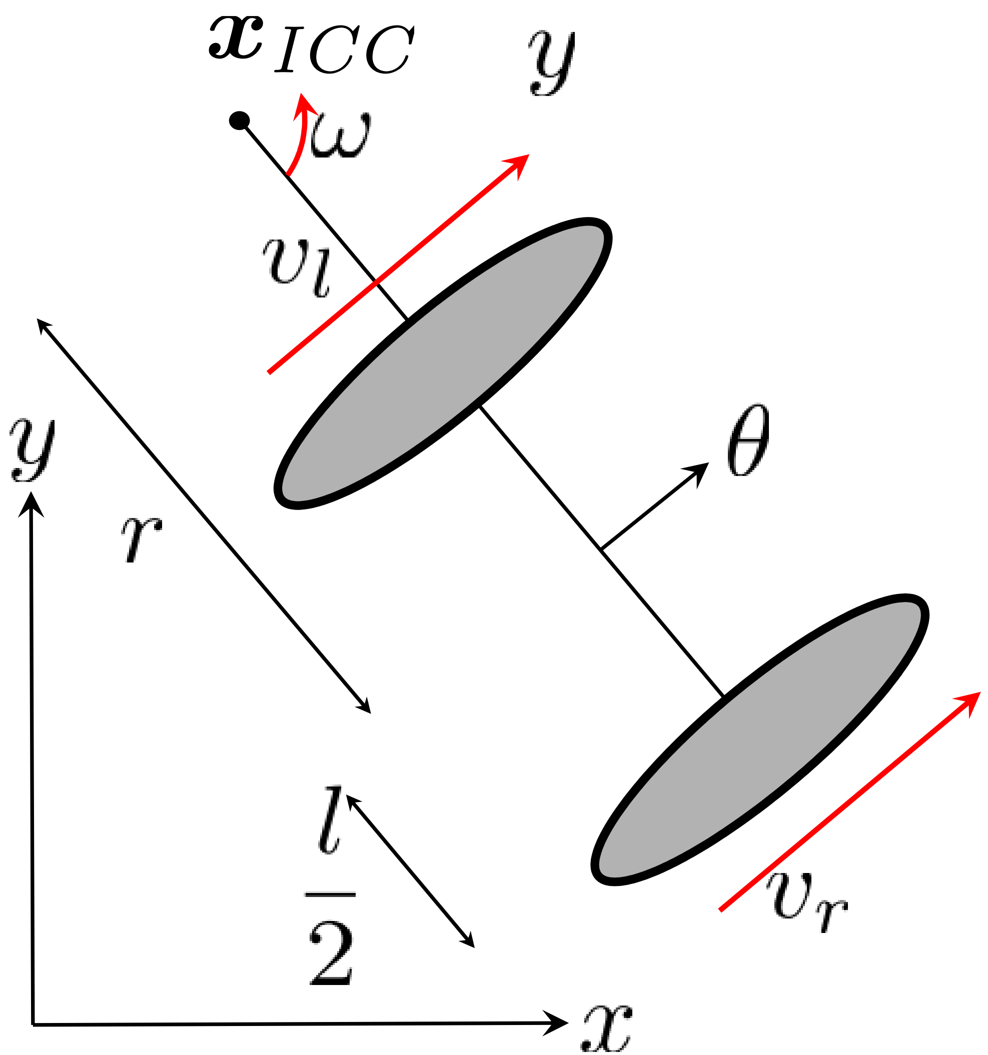
$$\omega = \frac{(v_r - v_l)}{l}$$

rotational velocity

$$r = \frac{l}{2} \frac{(v_r + v_l)}{(v_r - v_l)}$$

radius of the curve

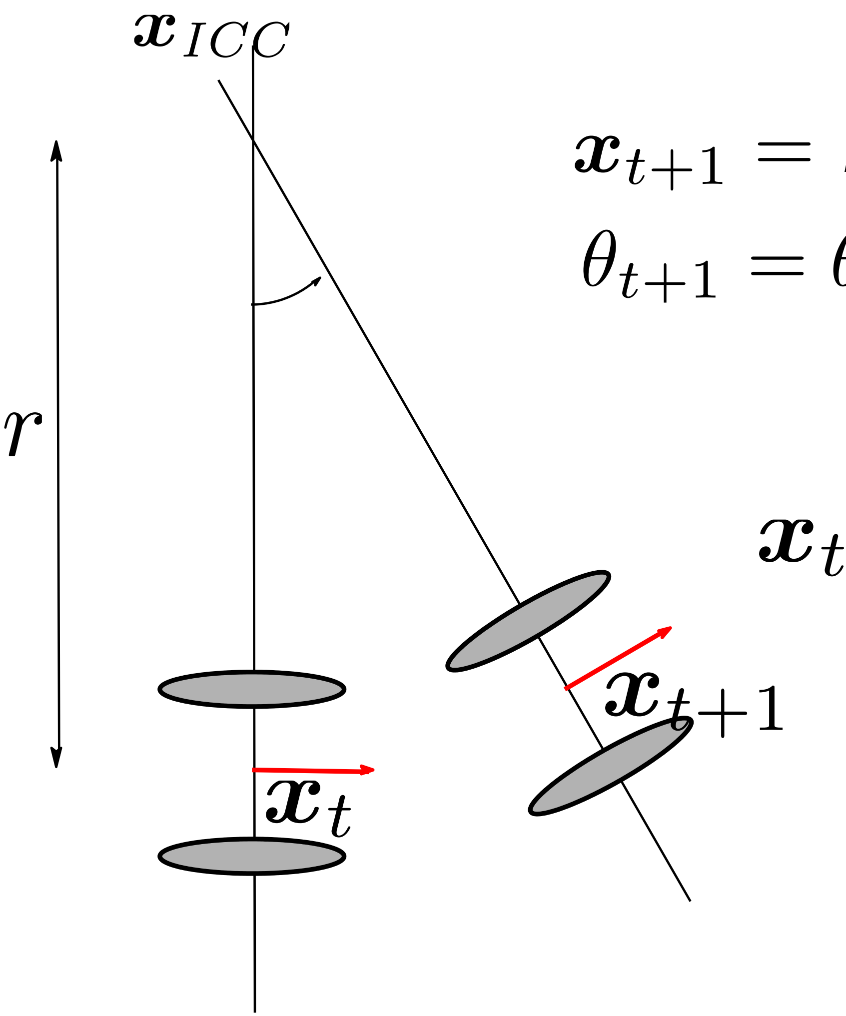
ICC – Instantaneous Center of Curvature



$$r = \frac{l}{2}\frac{(v_r + v_l)}{(v_r - v_l)}$$

$$\boxed{\boldsymbol{x}_{ICC} = \begin{bmatrix} x - r\sin\theta \\ y + r\cos\theta \end{bmatrix}}$$

Forward Kinematics



$$\boldsymbol{x}_{t+1} = R(\omega\,\Delta t)(\boldsymbol{x}_t - \boldsymbol{x}_{ICC}) + \boldsymbol{x}_{ICC}$$

$$\theta_{t+1} = \theta_t + \omega\,\Delta t$$

$$x_t = \int_0^t v(t')\cos\theta_{t'}\,dt'$$

$$y_t = \int_0^t v(t')\cos\theta_{t'}\,dt'$$

$$\theta_t = \int_0^t \omega(t')\,dt'$$

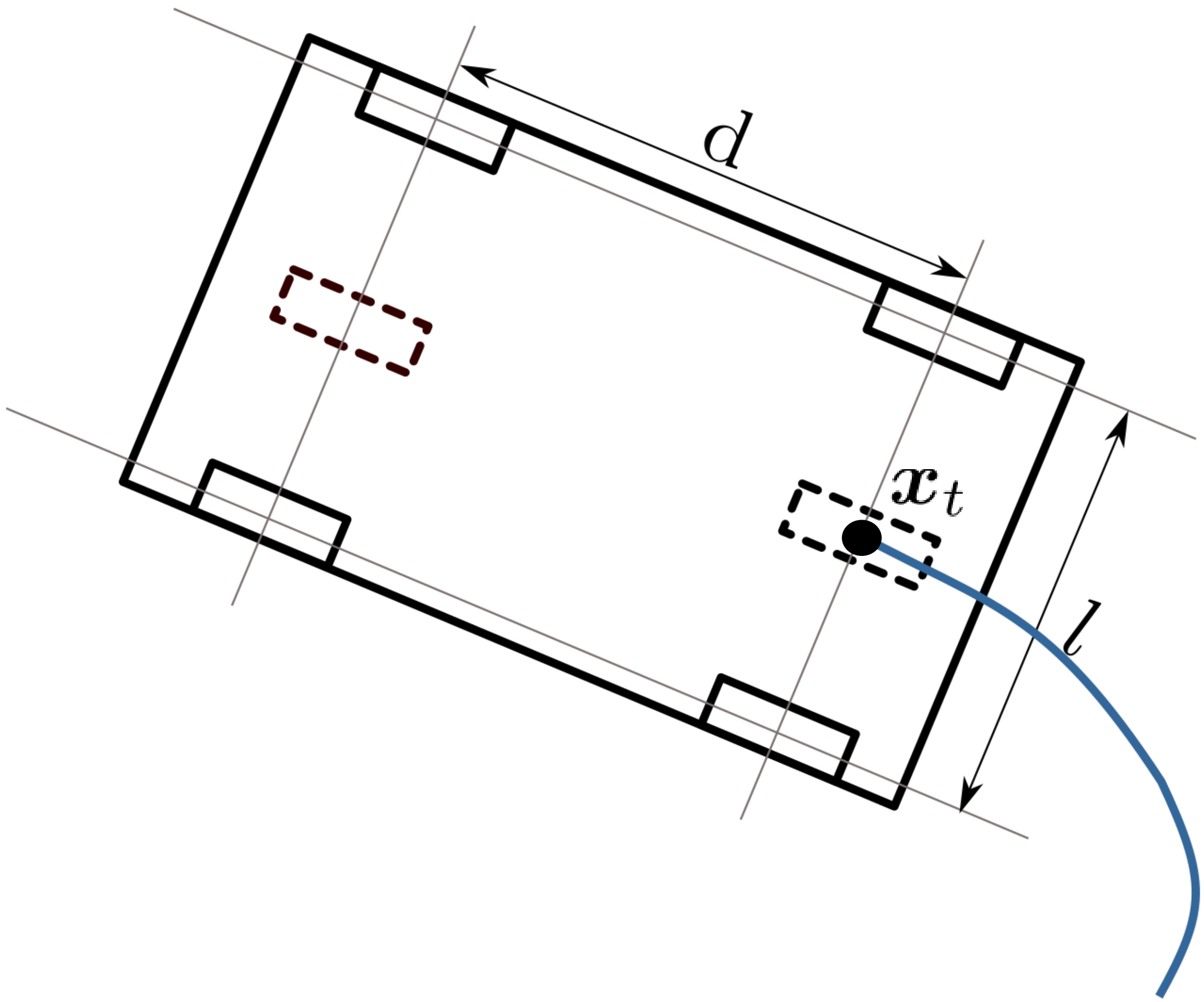
How to Steer this Robot?

Image Courtesy: Clearpath

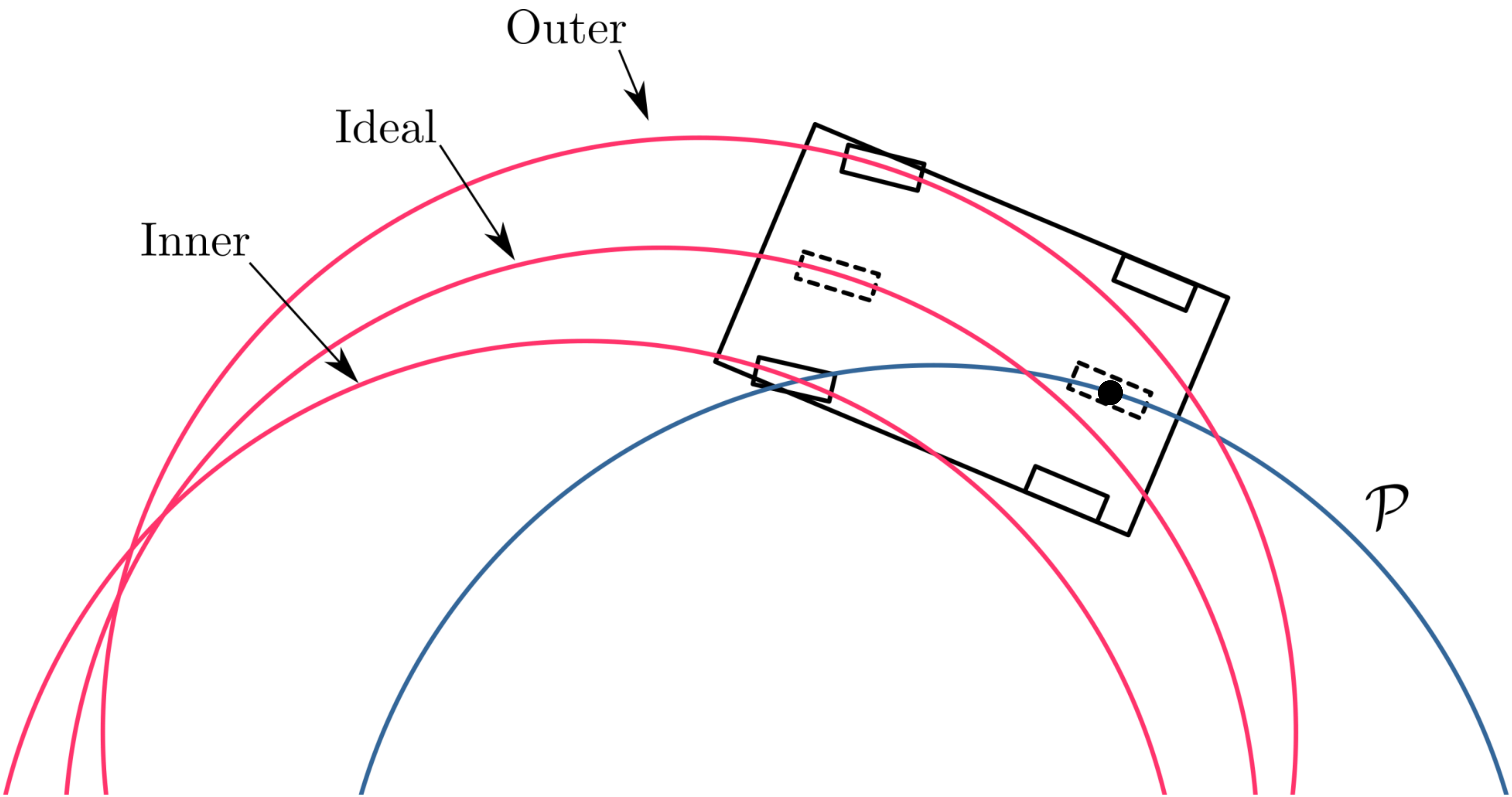
Modeling a Car: Ackermann Drive

Image Courtesy: AID

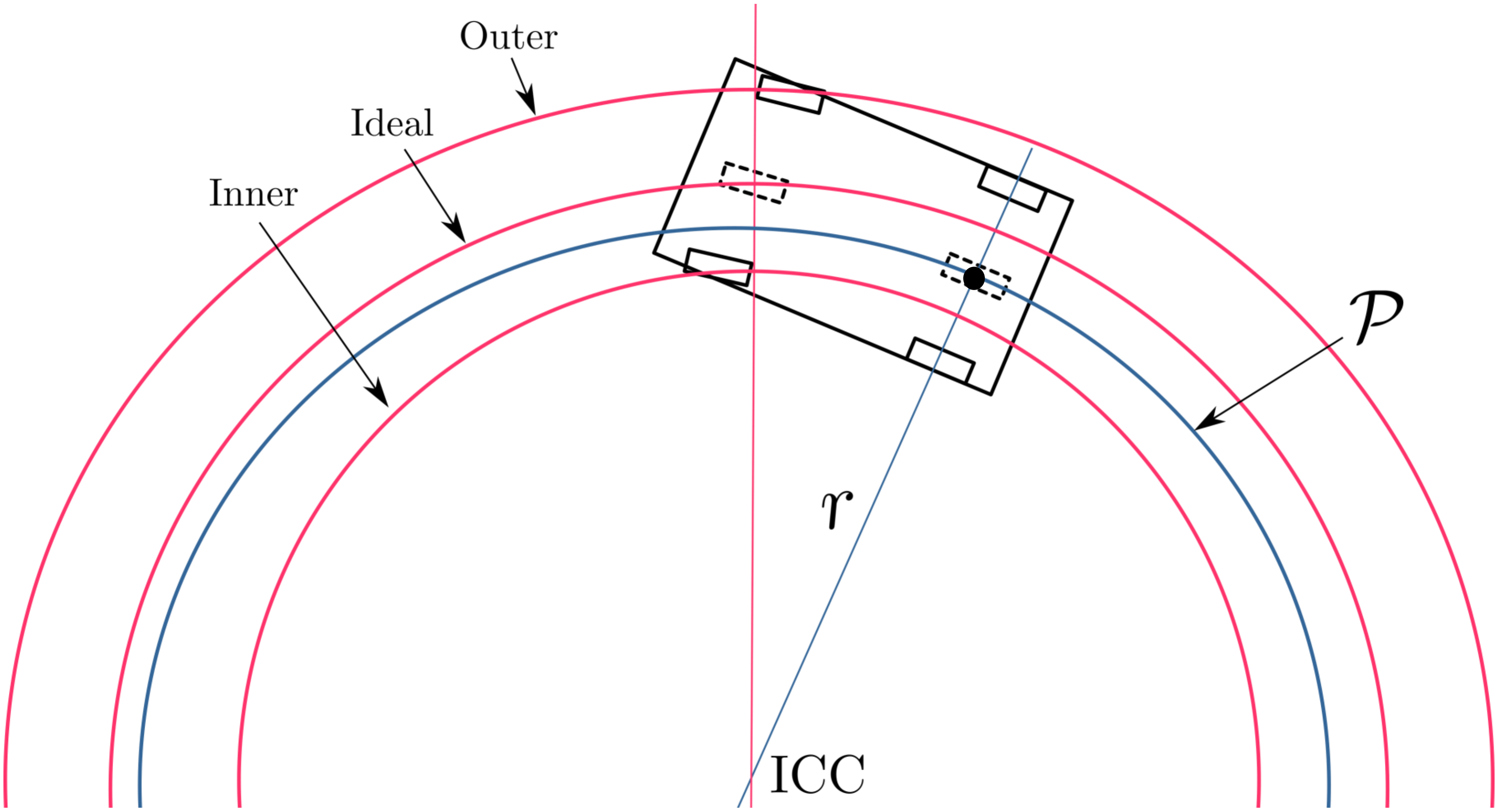
Ackermann Drive



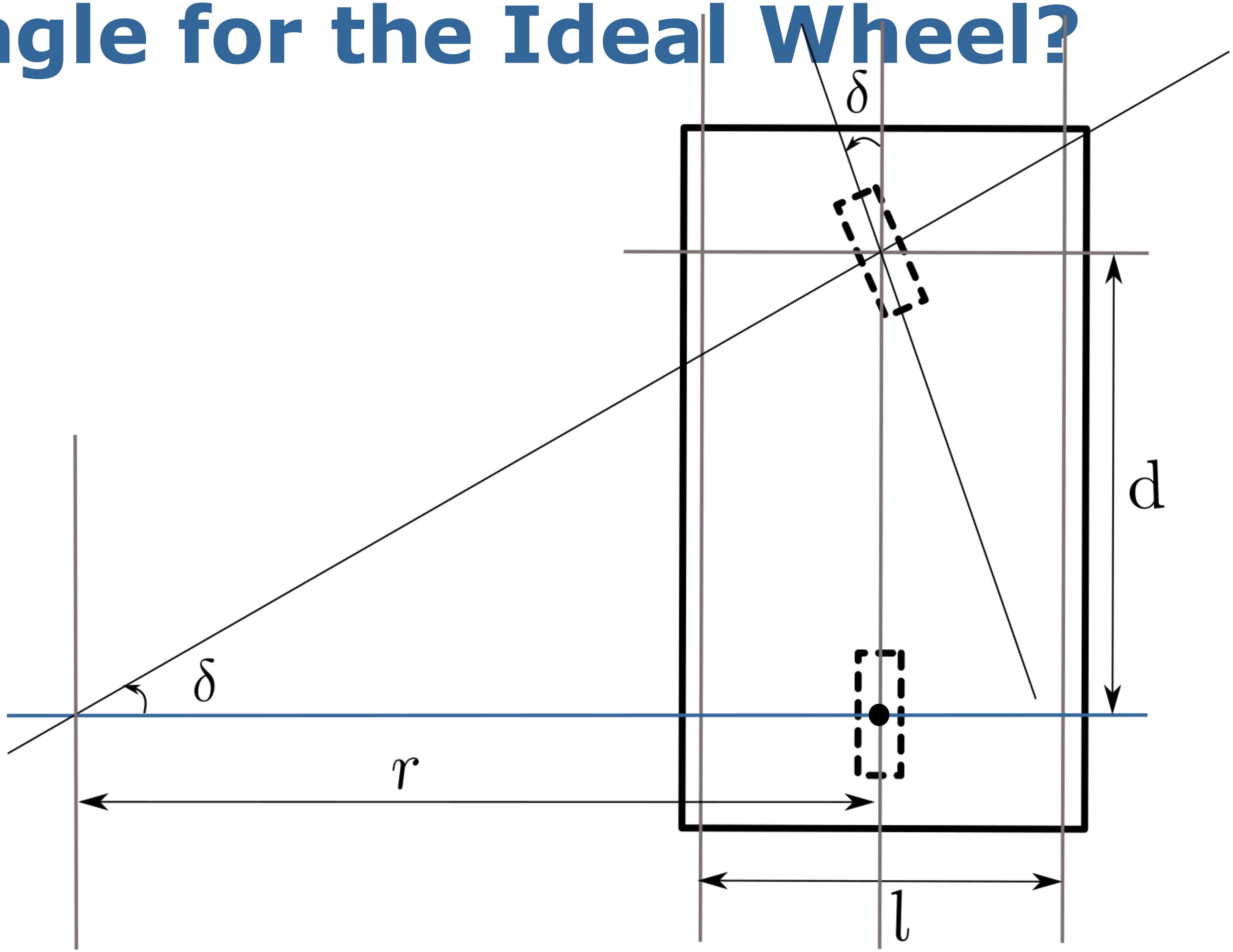
Slipping during turning



Ackermann Model

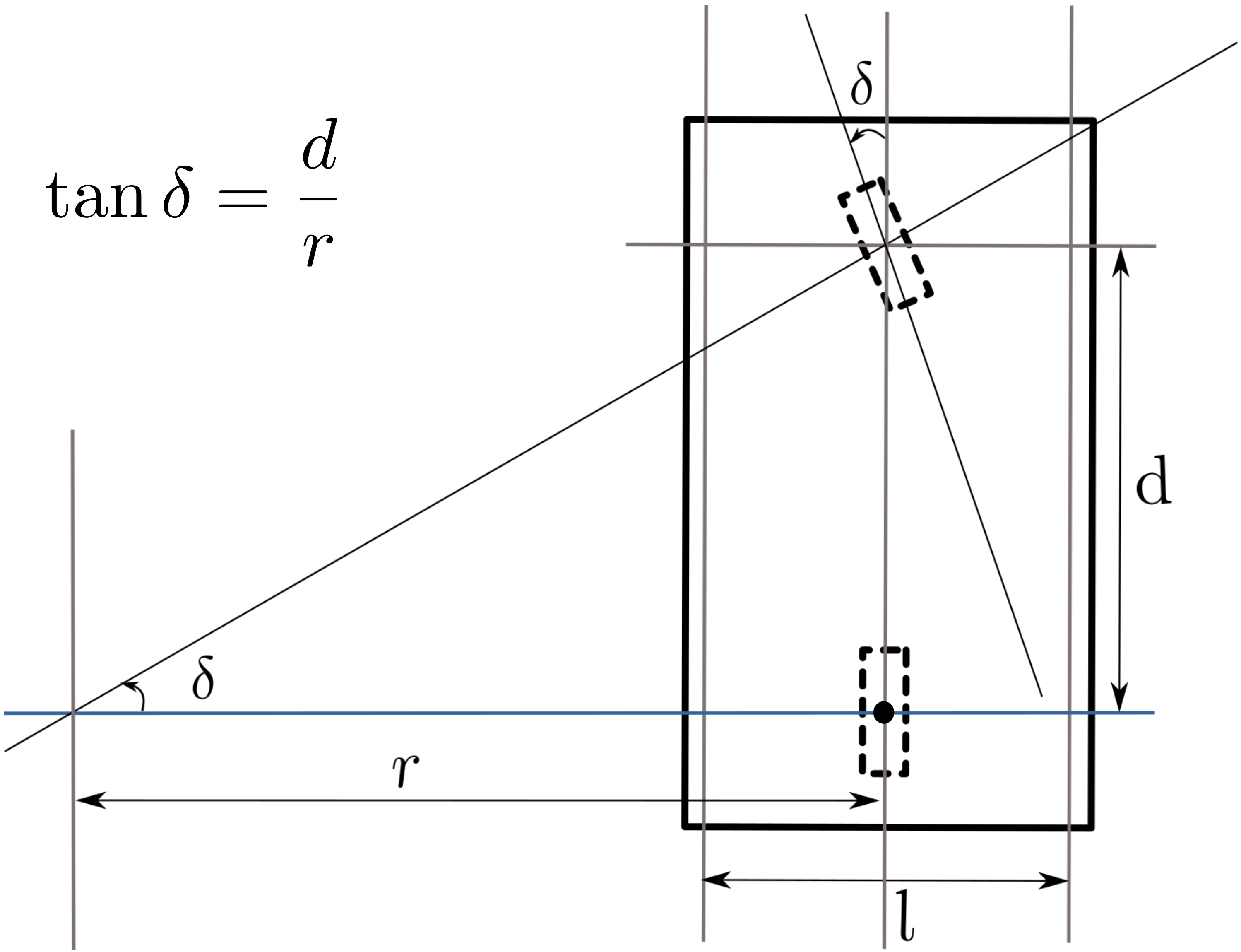


How to Determine the Steering Angle for the Ideal Wheel?

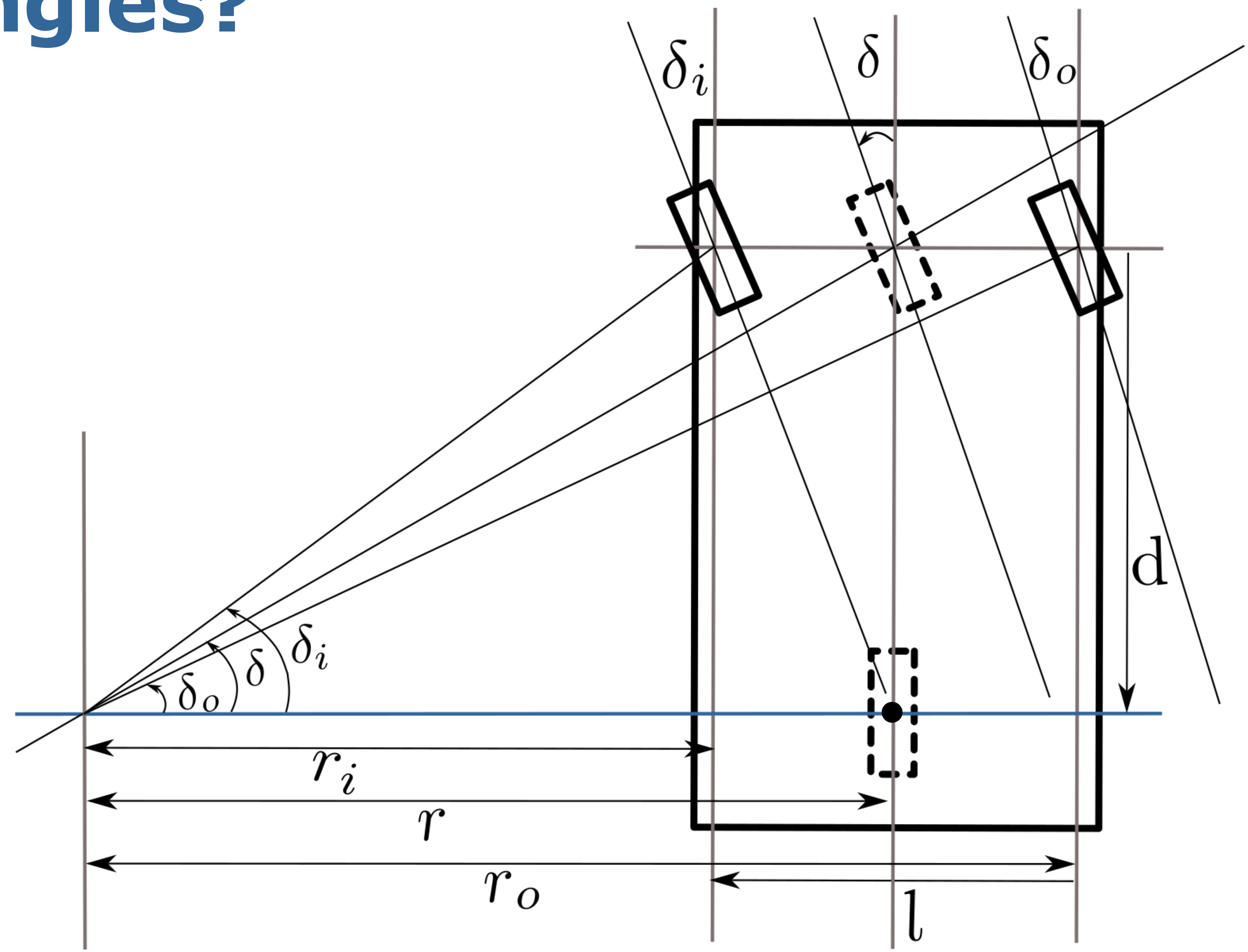


Ideal Wheel Steering Angle

$$\tan\delta = \frac{d}{r}$$



How to Set the Other Wheel Angles?

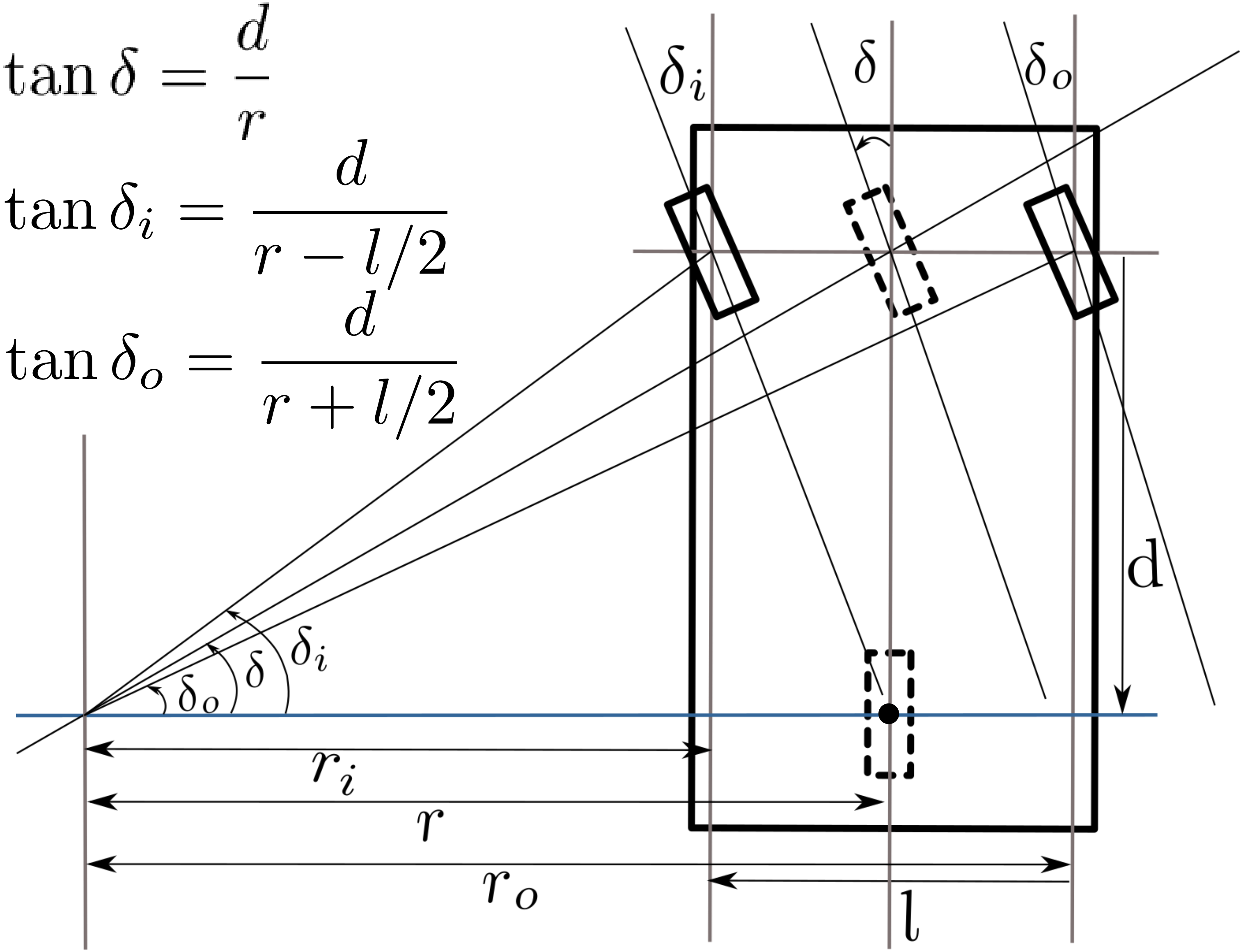


Ackermann Steering Angles

$$\tan \delta = \frac{d}{r}$$

$$\tan \delta_i = \frac{d}{r - l/2}$$

$$\tan \delta_o = \frac{d}{r + l/2}$$



Building an Ackermann Drive

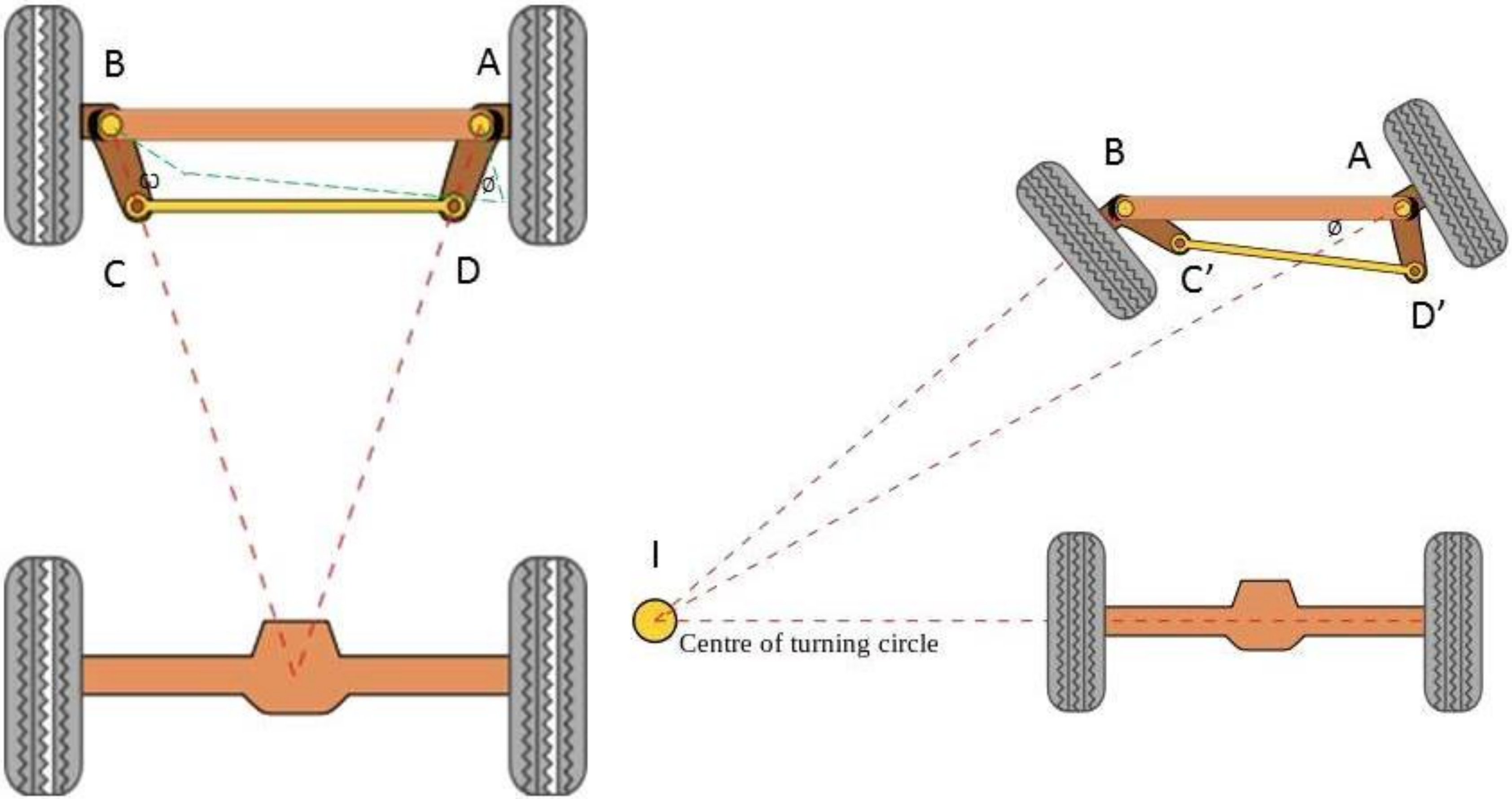
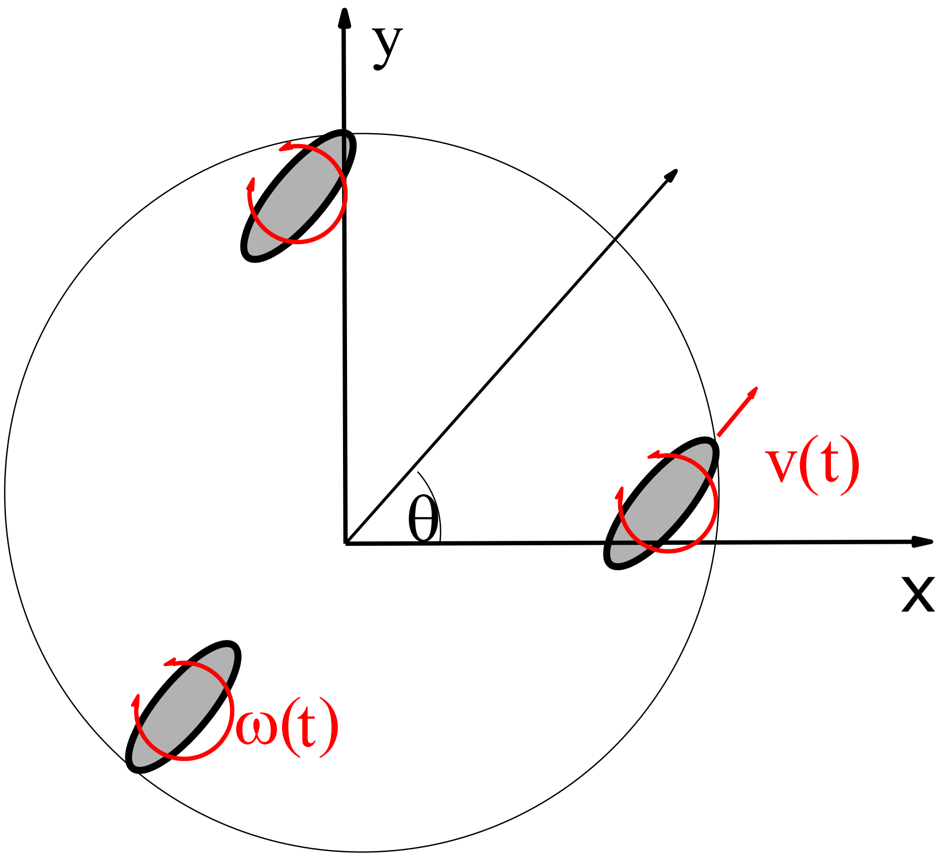


Image Courtesy: Wikipedia

Synchronous Drive

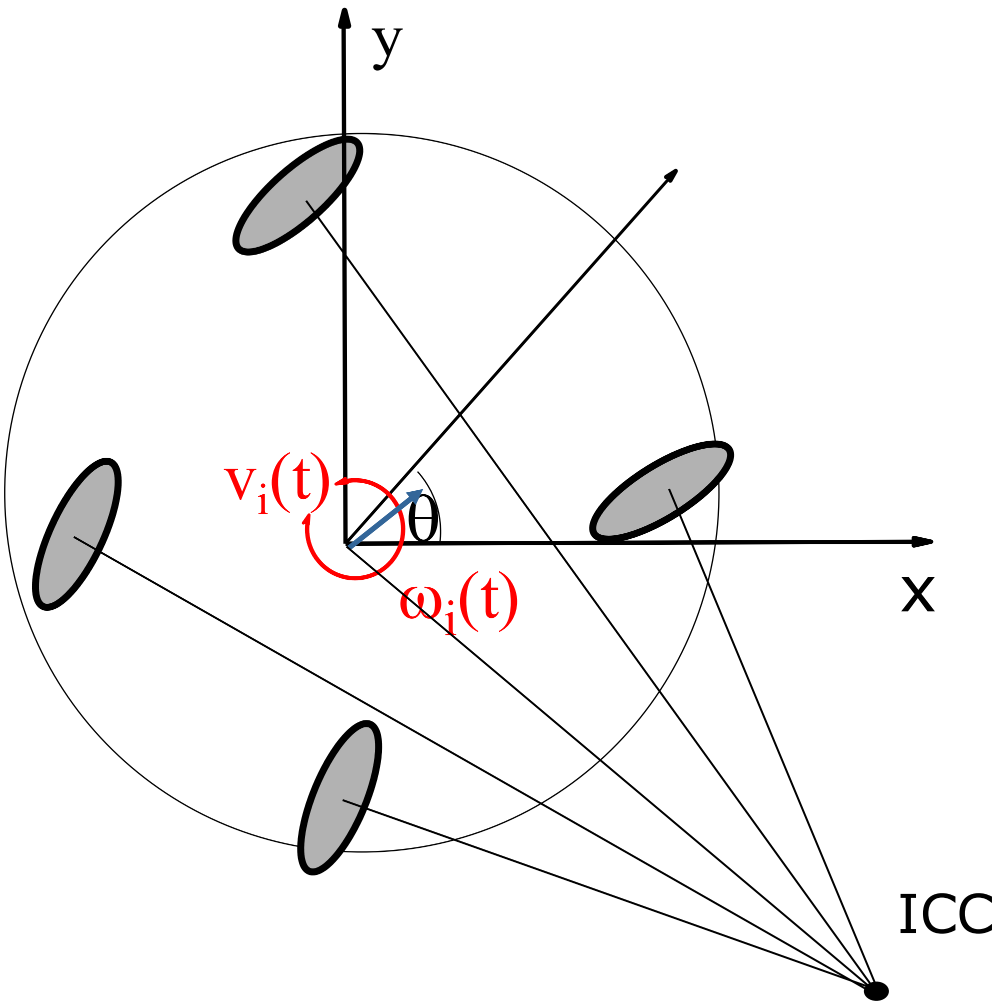


$$x_t = \int_0^t v(t') \cos \theta_{t'} \, dt'$$

$$y_t = \int_0^t v(t') \cos \theta_{t'} \, dt'$$

$$\theta_t = \int_0^t \omega(t') \, dt'$$

XR4000 Drive



$$x_t = \int_0^t v(t') \cos \theta_{t'} \, dt'$$

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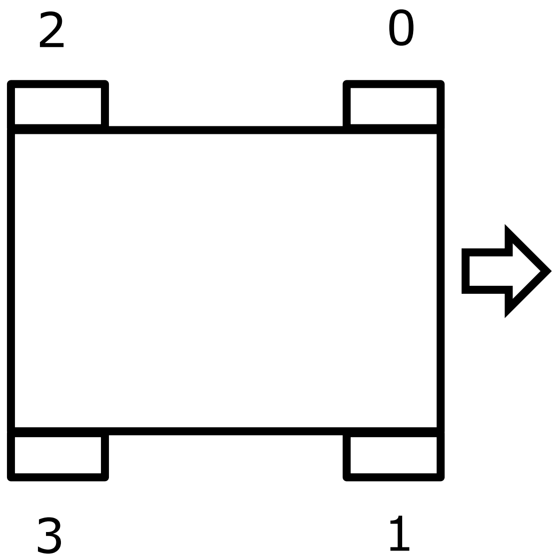
Mecanum Wheels



$$v_x = (v_0 + v_1 + v_2 + v_3)/4$$

$$v_y = (-v_0 + v_1 + v_2 - v_3)/4$$

$$v_\theta = (-v_0 + v_1 - v_2 + v_3)/4$$

$$v_{error} = (v_0 + v_1 - v_2 - v_3)/4 \stackrel{!}{=} 0$$

KUKA omniRob

Tracked Vehicles

Legged Robots: Humanoids

The Currently Probably Most Capable Humanoid Robot

Video Courtesy: Boston Dynamics

Non-Holonomic Constraints

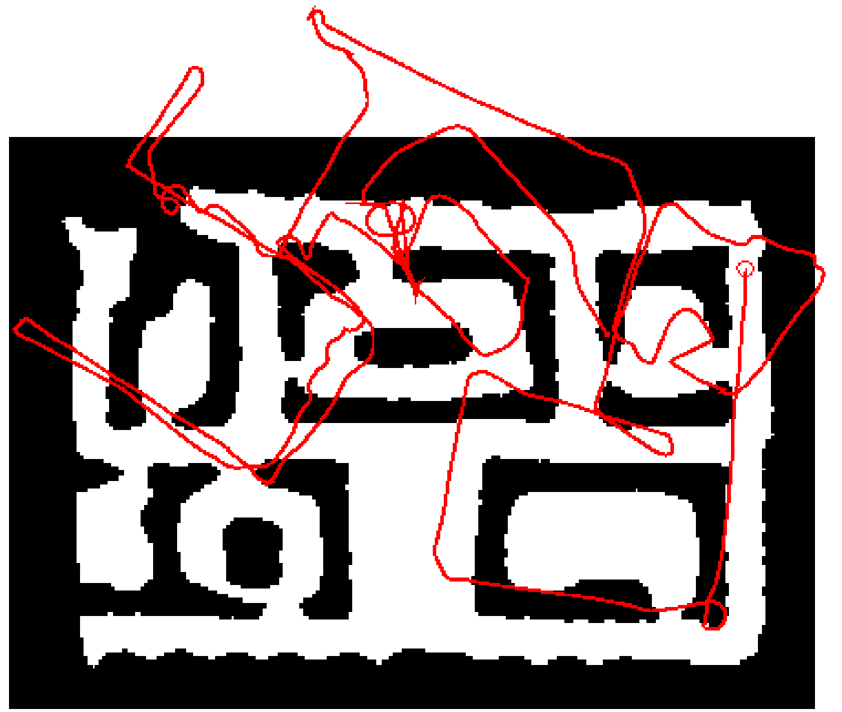
- Non-holonomic constraints limit the possible movements within the configuration space of the robot
- Differential & synchro-drive move on a circular trajectory and cannot move sideways
- Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints)

Holonomic vs. Non-Holonomic

- **Non-holonomic constraints** reduce the control space with respect to the current configuration
  - E.g., moving sideways is impossible.
- **Holonomic constraints** reduce the configuration space.
  - E.g., a train on tracks (not all positions and orientations are possible)

Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time



Summary

- Different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic/holonomic constraints
- Odometry and dead reckoning