Basic Observation Models

Nived Chebrolu

Slides adapted from Cyrill Stachniss at Uni. Bonn

State Estimation

- Estimate the state x of a system given observations z and controls u
- **Goal:**

$$p(x \mid z, u)$$

Motion and Observation Model

- Prediction step

$$\overline{bel}(x_t) = \int \underbrace{p(x_t \mid u_t, x_{t-1})}_{} \, bel(x_{t-1}) \, dx_{t-1}$$

motion model

- Correction step

$$bel(x_t) = \eta \, \underbrace{p(z_t \mid x_t)}_{} \, \overline{bel}(x_t)$$

observation model
(also: measurement or sensor model)

Recursive Bayes Filter

$$
\begin{aligned}
bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
&= \eta \, \underline{p(z_t \mid x_t)} \int p(x_t \mid x_{t-1}, u_t) \, bel(x_{t-1}) \, dx_{t-1}
\end{aligned}
$$

observation model

Observation Model

$$bel(x_t) = \eta \boxed{p(z_t \mid x_t)} \overline{bel}(x_{t-1})$$

Range Sensors

Model for Laser Scanners

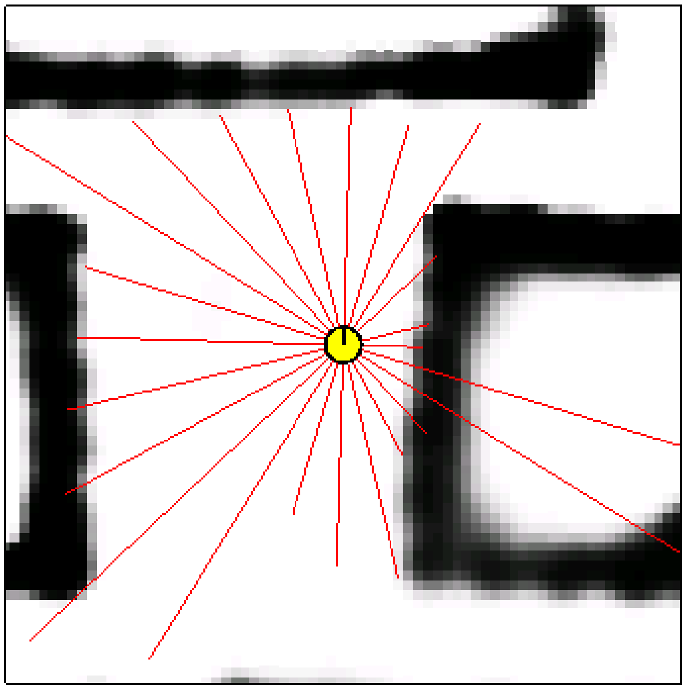
- Scan *z* consists of *K* measurements.

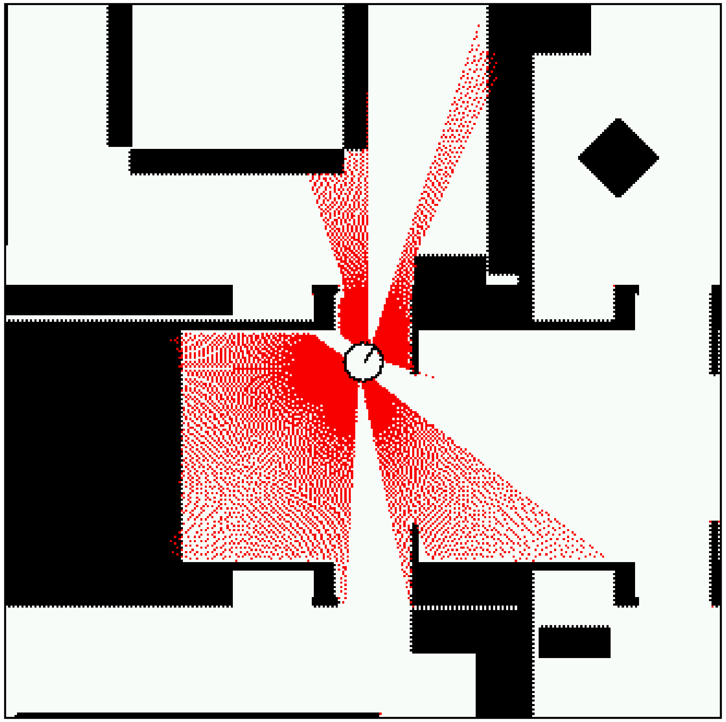
$$z_t = \{z_t^1, \ldots, z_t^k\}$$

- Individual measurements are independent given the sensor position

$$p(z_t \mid x_t, m) = \prod_{i=1}^{k} p(z_t^i \mid x_t, m)$$

Beam-based Sensor Model

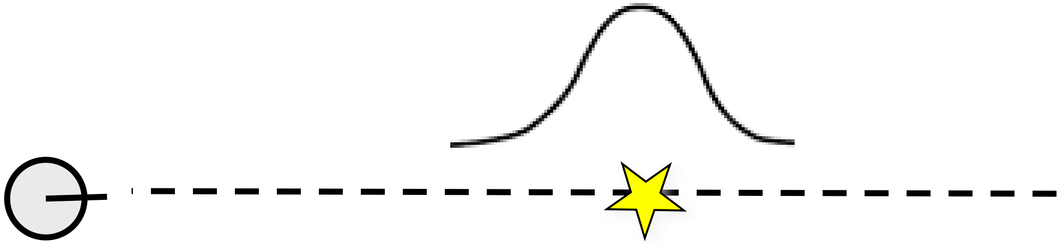




$$p(z_t \mid x_t, m) = \prod_{i=1}^{k} p(z_t^i \mid x_t, m)$$

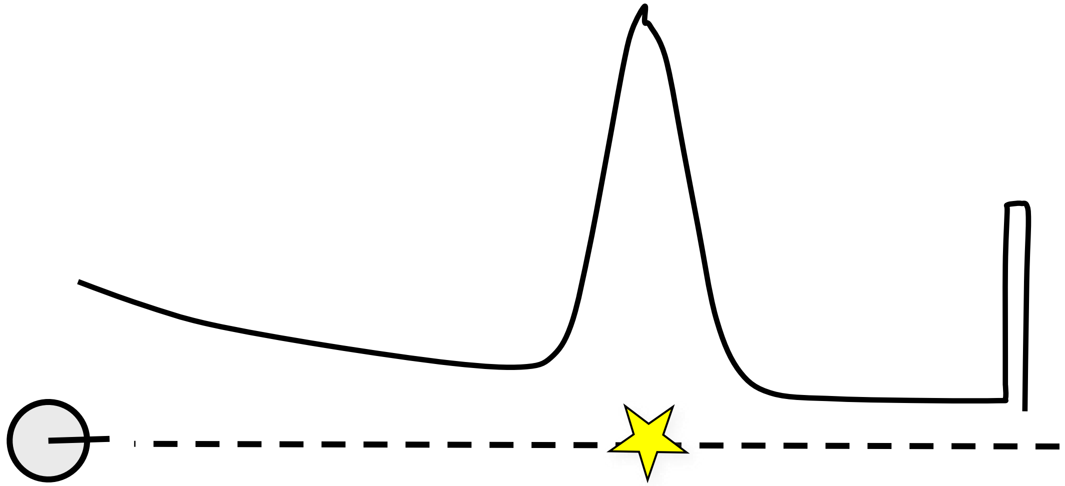
Simplest Ray-Cast Model

- Ray-cast models consider the first obstacle along the line of sight
- Gaussian noise in the distance



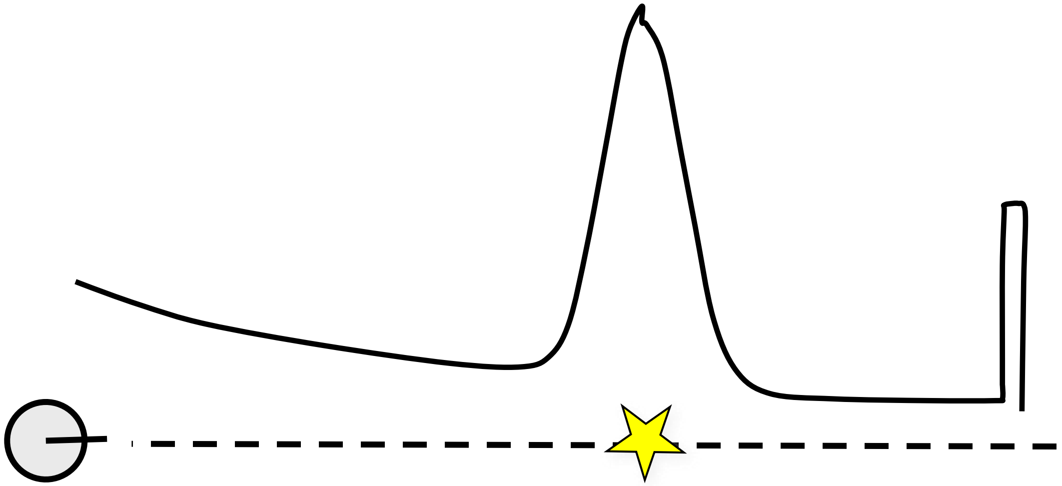
More Advanced Ray-Cast Model

- Ray-cast models consider the first obstacle along the line of sight
- A more advanced model may look like that. Why?



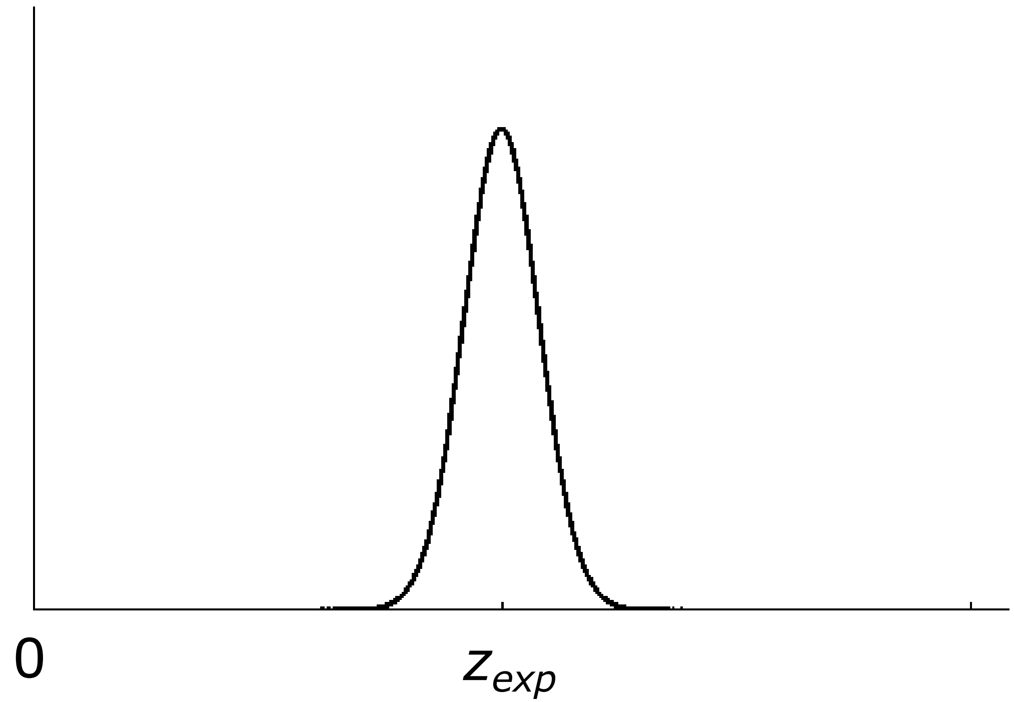
More Advanced Ray-Cast Model

- Ray-cast models consider the first obstacle along the line of sight
- Mixture of four models: considers different effects (dynamic objects, random, max-range, noise)



Beam-Based Proximity Model

measurement noise

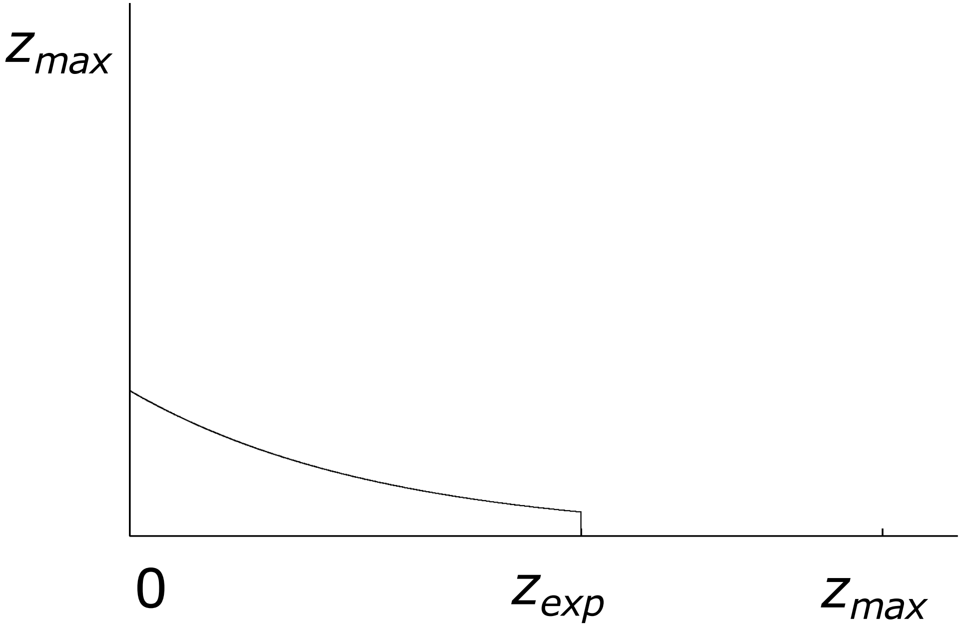


$$P_{hit}(z \mid x, m) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{(z - z_{exp})^2}{\sigma^2}\right)$$

Image courtesy: Burgard

Beam-Based Proximity Model

unexpected obstacles



$$P_{unexp}(z \mid x, m) = \begin{cases} \lambda \exp\left(-\lambda z\right) & z < z_{exp} \\ 0 & \text{otherwise} \end{cases}$$

Image courtesy: Burgard

Unexpected Obstacles

- Why an exponentially decaying function to model dynamics?
- Assume a uniform distribution of objects in the space
- Only the closest obstacle matters
- Analogy to bit-strings

Unexpected Obstacles

- "0" = free space
- "1" = obstacle
- How many bit strings exists with:

1***

01**

001*

0001

Unexpected Obstacles

- "0" = free space
- "1" = obstacle
- How many bit strings exists with:

1* -> 1*2*2*2 = 2^3=8**

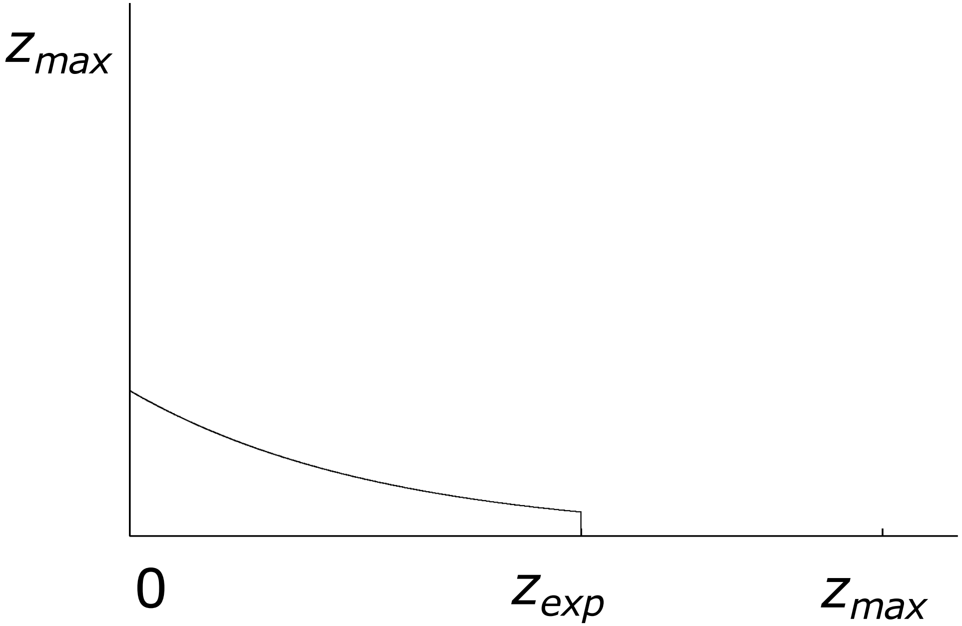
01 -> 1*1*2*2 = 2^2=4**

001* -> 1*1*1*2 = 2^1=2

0001 -> 1*1*1*1 = 2^0=1

Beam-Based Proximity Model

unexpected obstacles

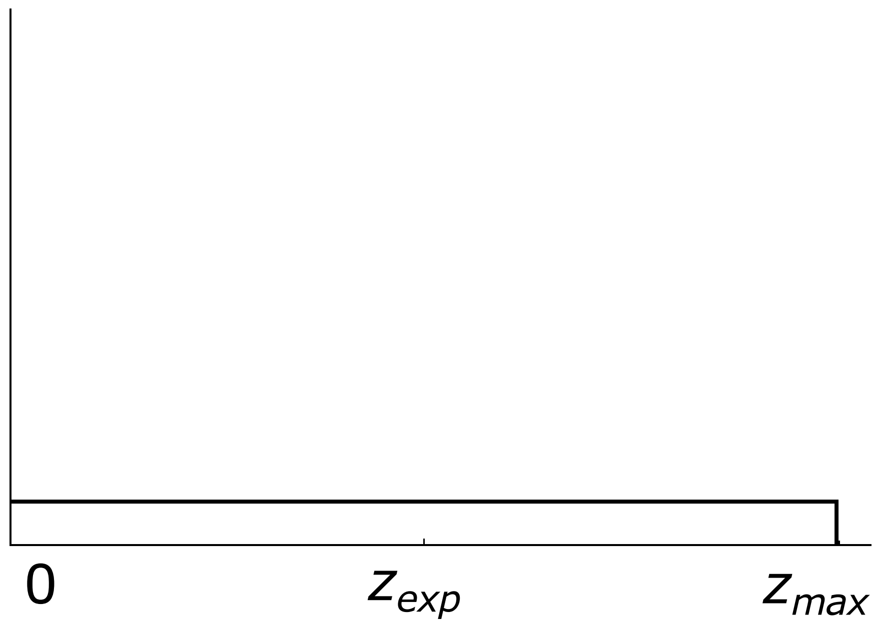


$$P_{unexp}(z \mid x, m) = \begin{cases} \lambda \exp(-\lambda z) & z < z_{exp} \\ 0 & \text{otherwise} \end{cases}$$

Image courtesy: Burgard

Beam-Based Proximity Model

random measurement

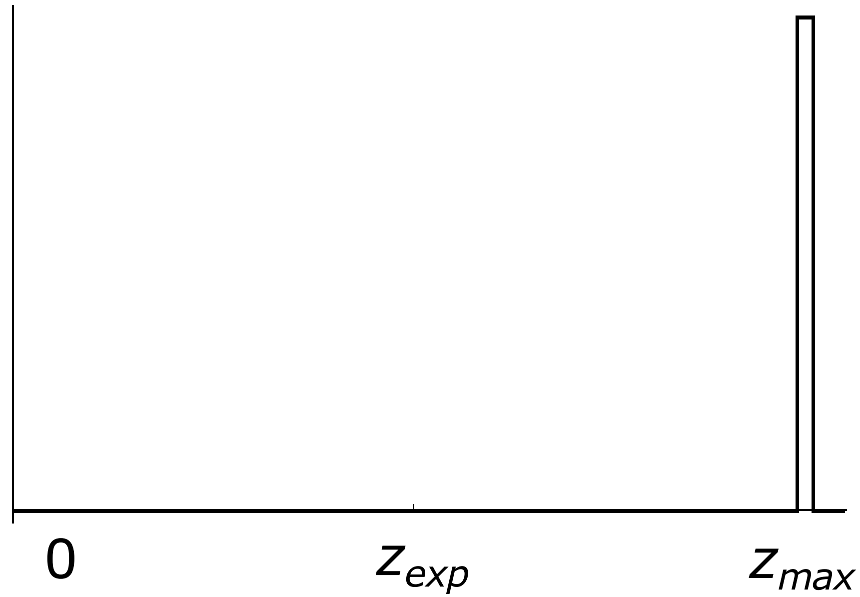


$$P_{rand}(z \mid x, m) = \text{const.}$$

Image courtesy: Burgard

Beam-Based Proximity Model

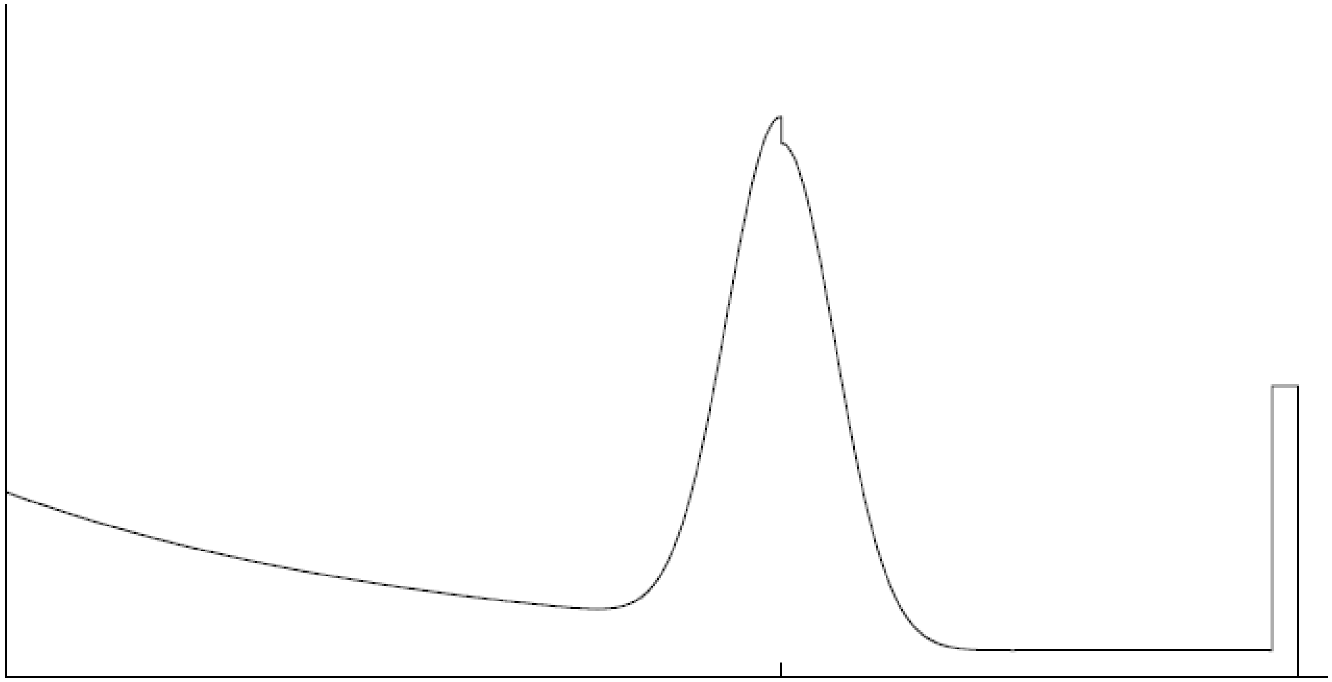
max range/no return



$$P_{max}(z \mid x, m) = \begin{cases} \text{const.} & z = z_{max} \\ 0 & \text{otherwise} \end{cases}$$

Image courtesy: Burgard

Resulting Mixture Density

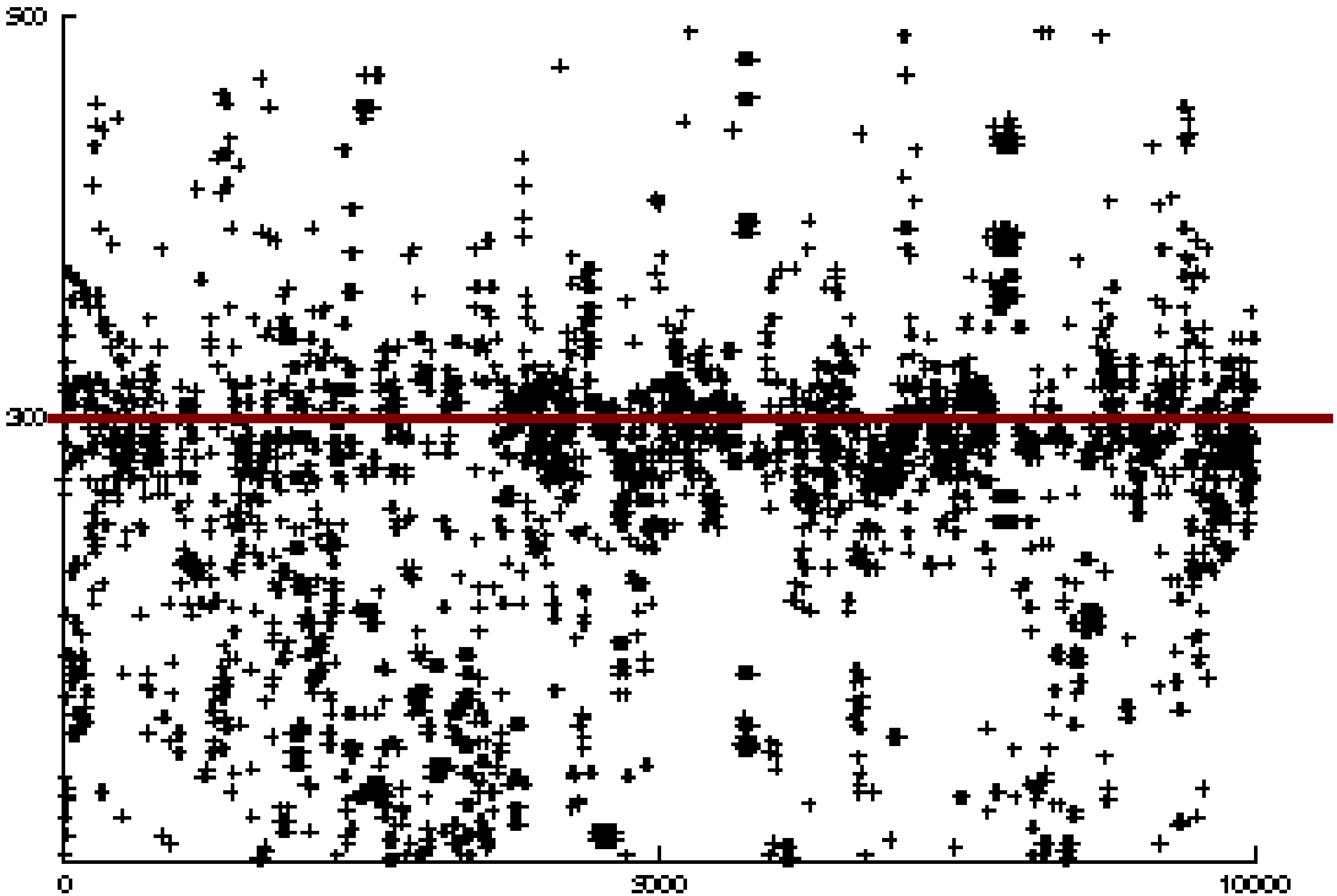


How can we determine the parameters?

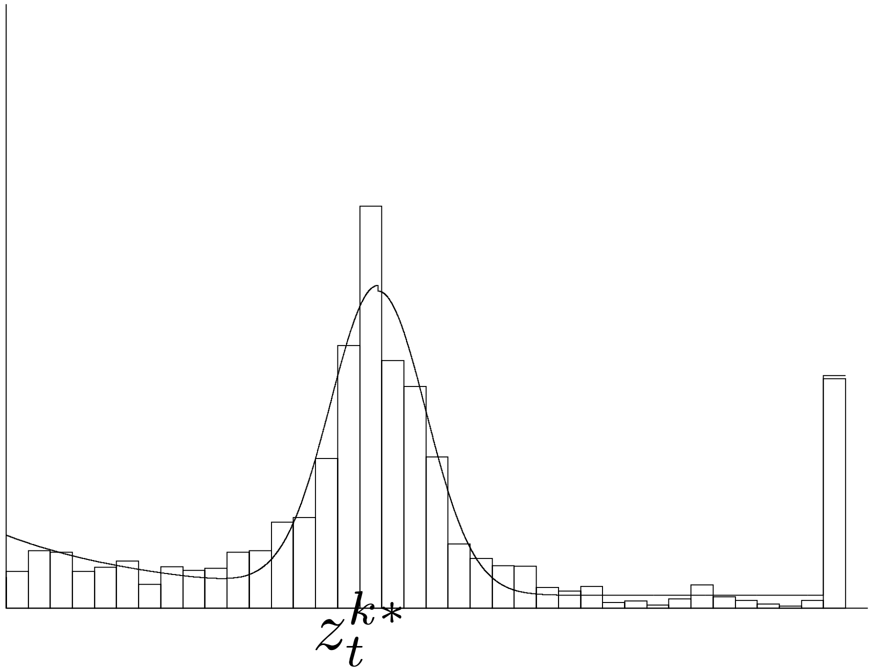
Image courtesy: Burgard

Raw Sensor Data

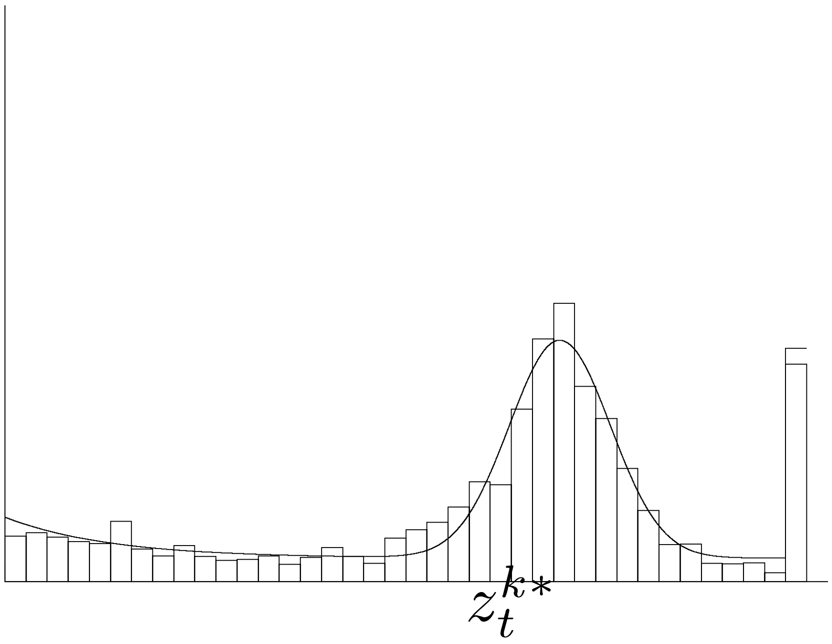
Measured distances for expected distance of 3m.



Results



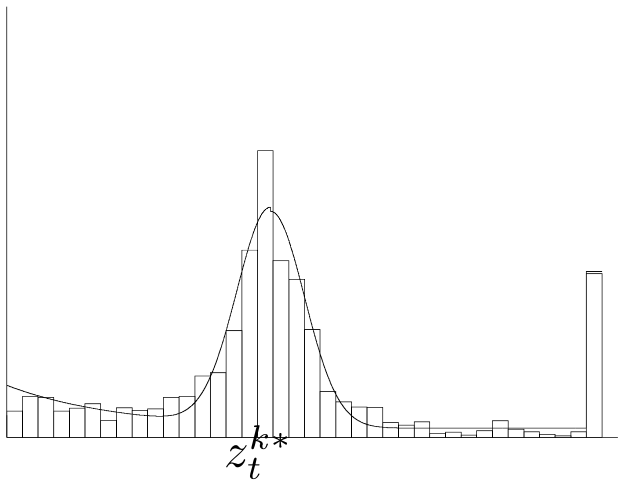
300cm

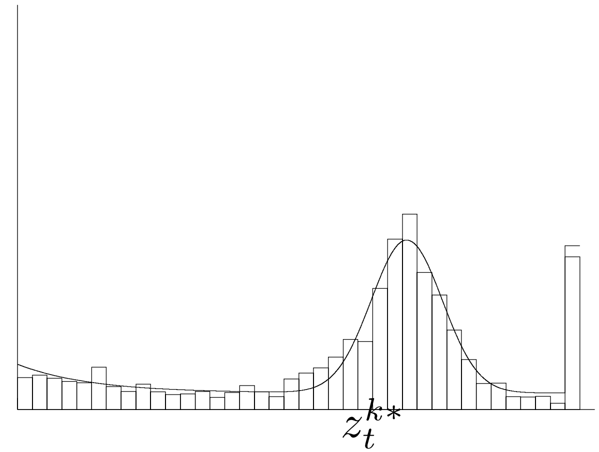


400cm

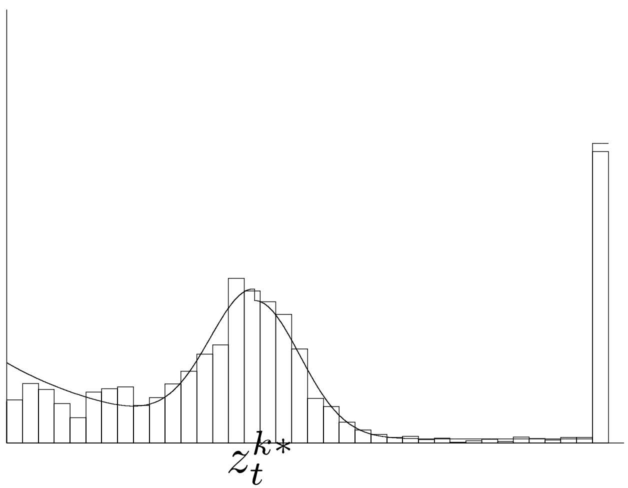
Image courtesy: Burgard

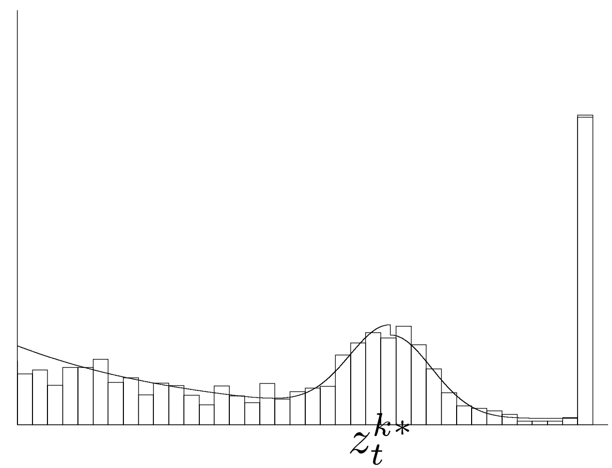
Similar Results for Sonars





Laser





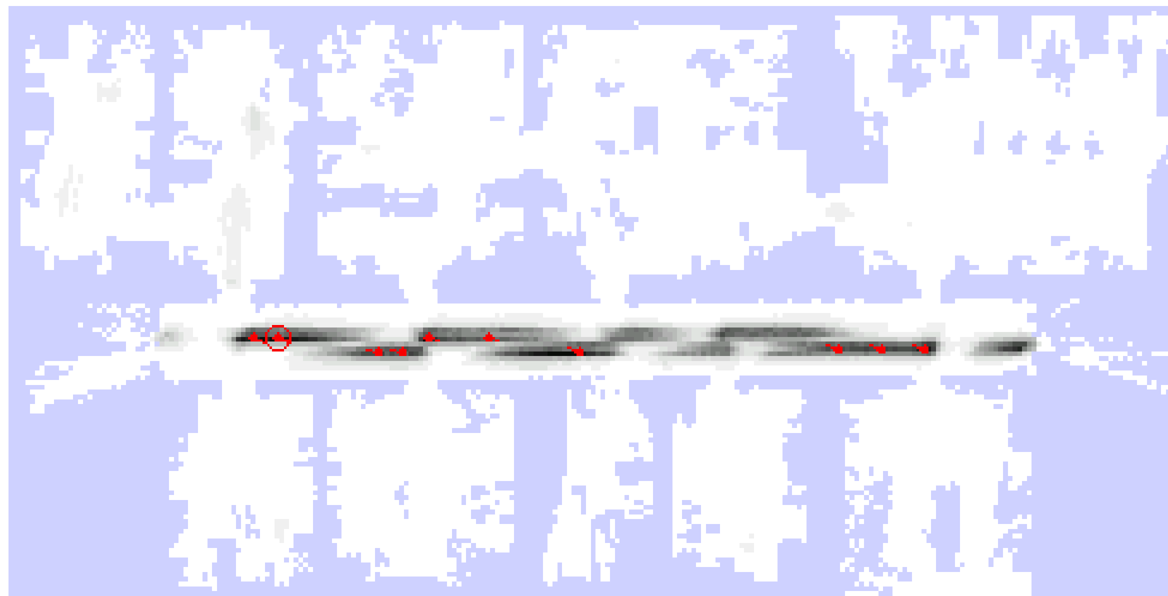
Sonar

300cm

400cm

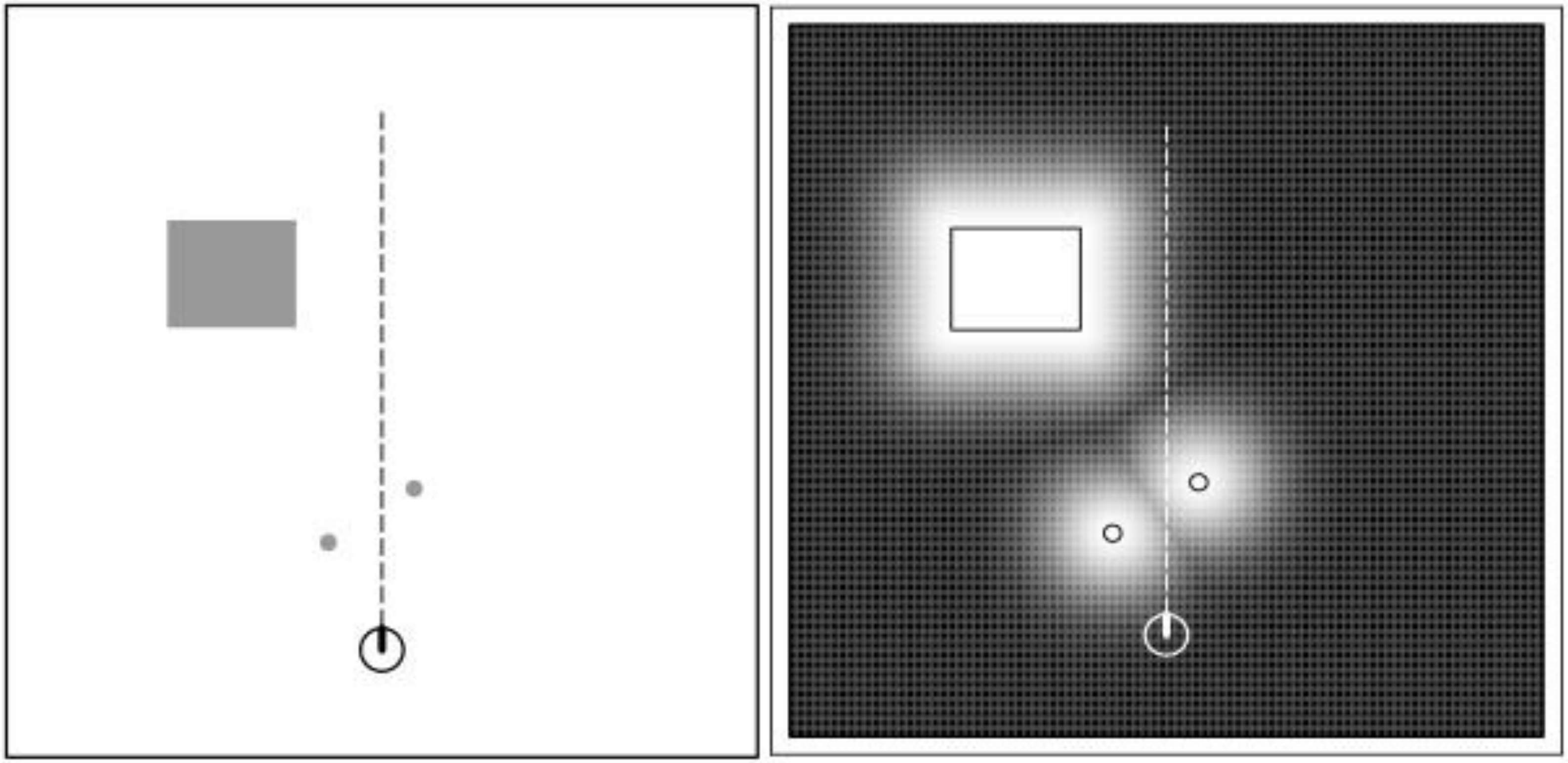
Example

z



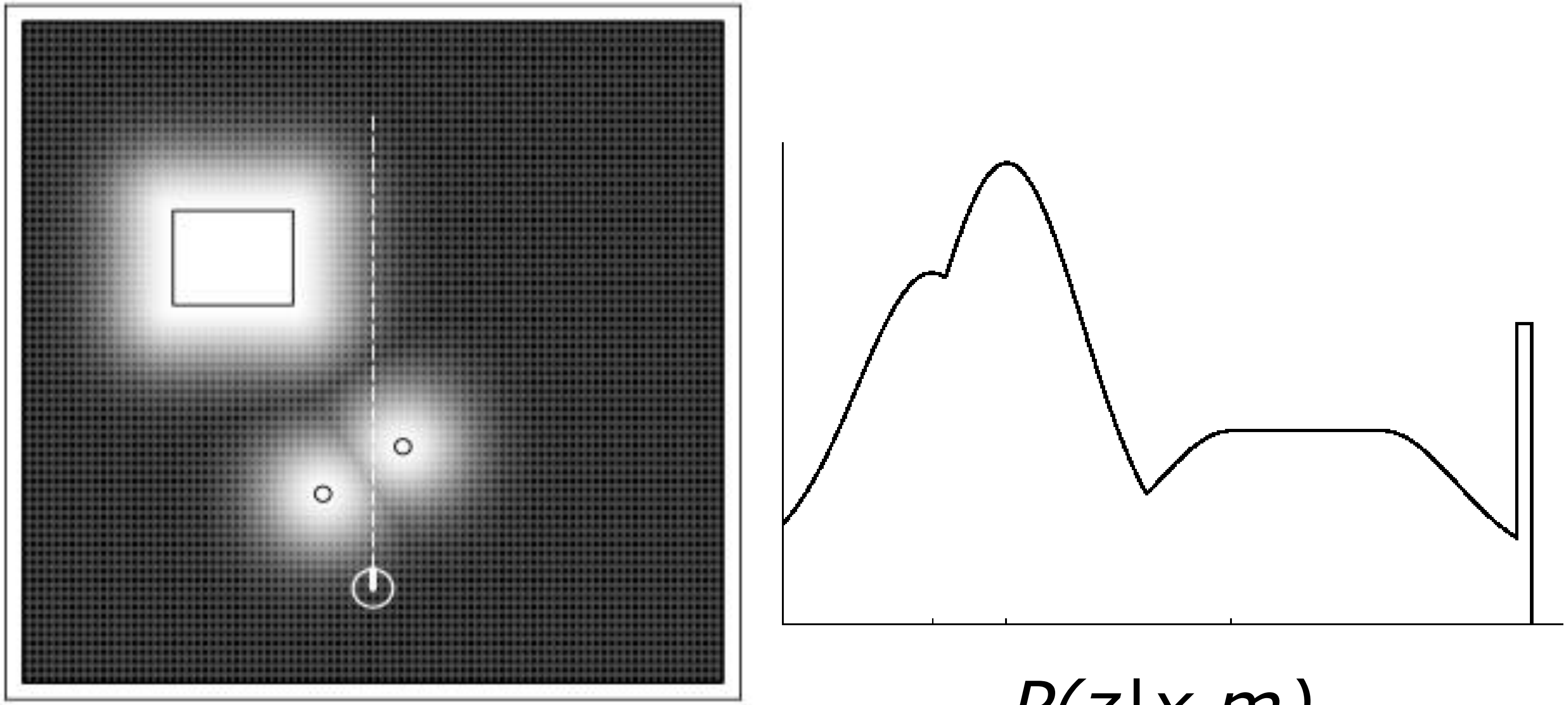
$P(z|x,m)$

Beam-endpoint Model



Courtesy: Thrun, Burgard, Fox

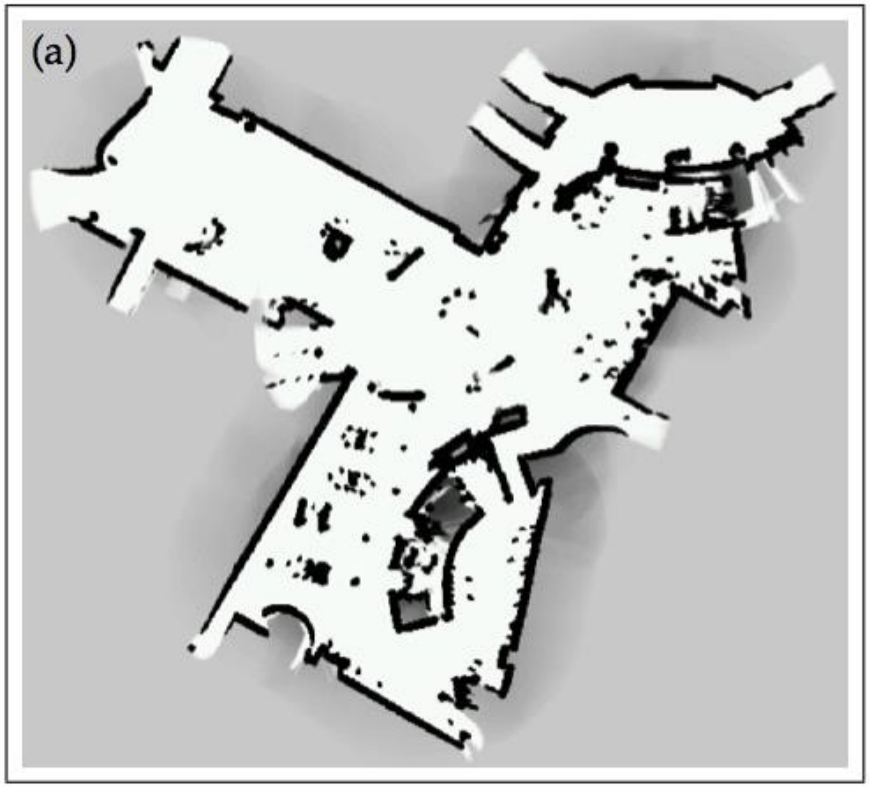
Beam-endpoint Model



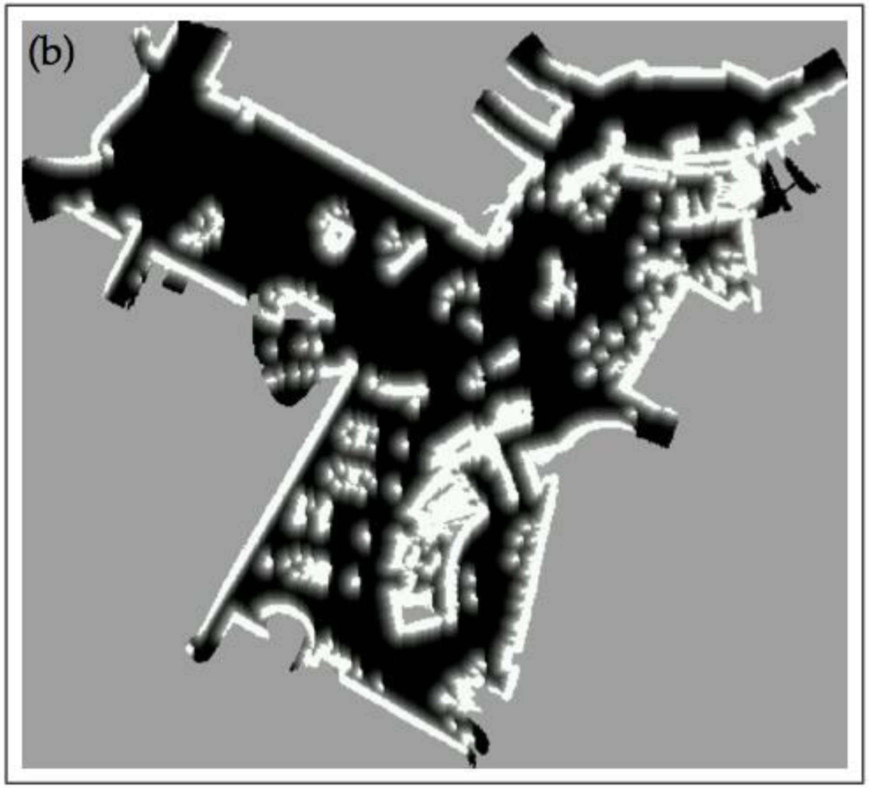
$P(z|x,m)$

Courtesy: Thrun, Burgard, Fox

Beam-endpoint Model



map



likelihood field

Courtesy: N. Roy

Models for Landmarks

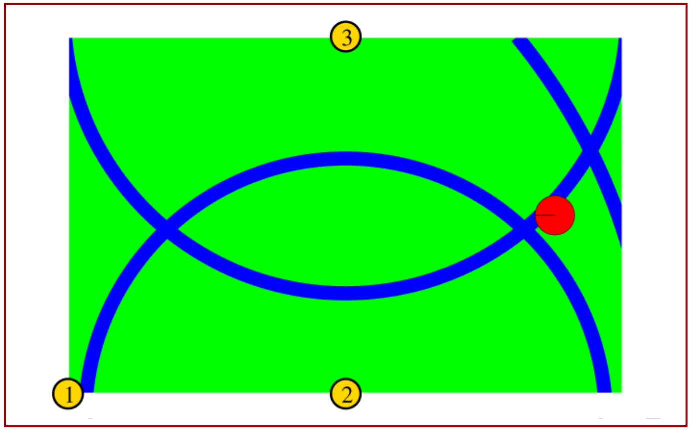
Model for Perceiving Landmarks with Range-Bearing Sensors

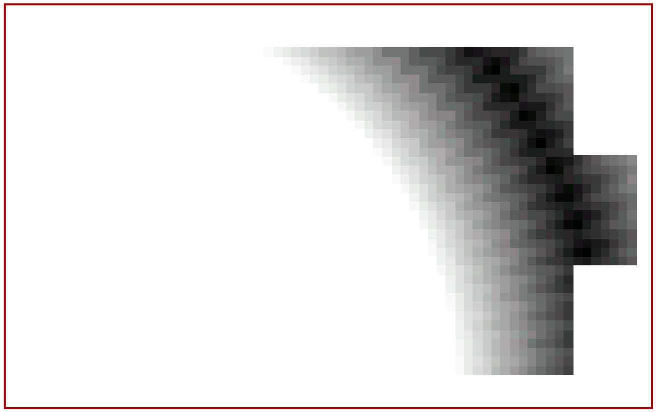
- Range-bearing $z_t^i = (r_t^i, \phi_t^i)^T$
- Pose $(x, y, \theta)^T$
- Expected observation of feature j at location $(m_{j,x}, m_{j,y})^T$

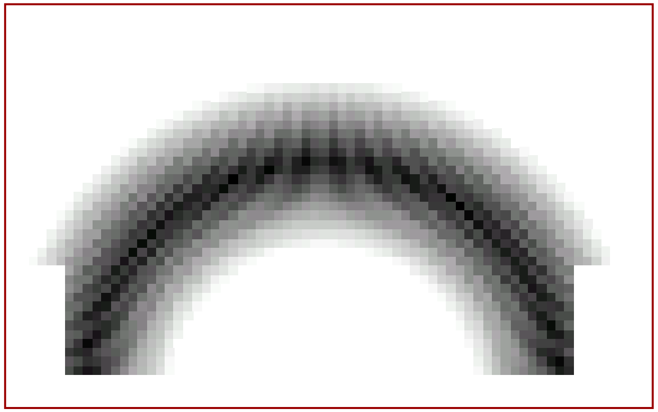
$$\begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix}$$

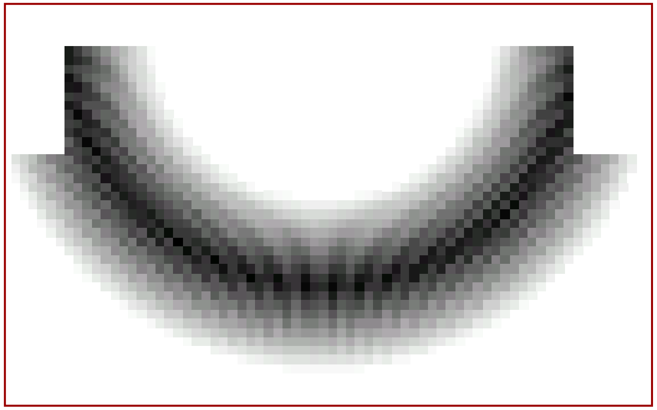
- Gaussian noise with covariance Q_t in range and bearing measurement

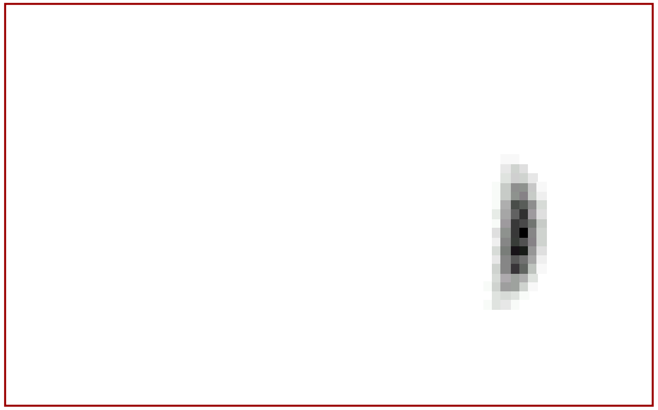
Distributions











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Model for Perceiving Landmarks with Bearing Sensors

- Bearing $z_t^i = \phi_t^i$
- Pose $(x, y, \theta)^T$
- Expected observation of feature j at location $(m_{j,x}, m_{j,y})^T$

$$\phi_t^i = \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta$$

- Gaussian noise with covariance Q_t in the bearing measurement

Summary

- Bayes filter is a framework for state estimation
- Recursive estimation: “from t to t+1”
- There are different realizations of the Bayes filter (e.g., EKF, particle filter)
- Motion and observation model are central models in the Bayes filter
- Examples of observation models

Summary

- Observation (and motion) models are central models in the Bayes filter
- Examples of observation models
- Raycast models for range sensors
- Beam-endpoint models
- Models for features observations

Literature

On observation models

- Thrun et al. “Probabilistic Robotics”, Chapter 6