Assignment 2

- Due Date: **24 Aug (Fri), 2012**
- DO NOT CHEAT! Copy cases will be handled severely (FR).
- Solve each question on a separate sheet of paper.
- Using outside sources is discouraged. However, if used, do mention them.

1. Let $X = \{1, 2, \ldots, n\}$ and consider the partial order $(\mathcal{P}(X) \subseteq)$.
   Prove the following: (3+2)
   (a) A chain is maximal if no other element can be added to it. There are at most $n!$ maximal chains.
   (b) Let $\mathcal{M}$ be any anti-chain. Let $\mathcal{P} = \{(C, L) \mid C \text{ is a maximal chain}, \ L \in C \cap \mathcal{M}\}$. Using (a), prove that $|\mathcal{P}| \leq n!$
2. Prove equivalence of WOP, induction and strong induction. (5+5)
3. **Schröder-Bernstein** (10)
   Recall that in class we defined the following: Let A, B be two sets. Let there be an injective map g from A to B and another injective map h from B to A. Let $B_1 = \{b \in B \mid \forall a \in A : g(a) \neq b\}$. An element $b \in B$ is called h-good if $\exists \beta \in B_1, \exists n \in \mathbb{N}$ s.t. $b = (g \odot h)^n(\beta)$, where $(g \odot h)(\beta) = g(h(\beta))$ and if $b = (g \odot h)^n(\beta)$ then $(g \odot h)^{n+1}(\beta) = g(h(b))$.
   Using the notion of h-good, we defined the following function:
   $$f(a) = \begin{cases} h^{-1}(a) & \text{if } g(a) \text{ is } h\text{-good} \\ g(a) & \text{otherwise} \end{cases}$$
   Prove that $f : A \to B$ is surjective. (Recall, in class we proved that f is injective.)
4. Induction (5)
   Let $x_1, \ldots, x_n$ be positive integers such that $x_1 \leq x_2 \ldots \leq x_n$. Prove that
   $$\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n} = 1 \Rightarrow x_n < 2^{n!}$$