

Tutorial 3

- Solve all questions. Discuss solutions with TAs during TA meeting hours.

1. Partial order

Let (S, R) denote a partially ordered set S under the order R . An antichain is the subset $S' \subseteq S$ of S such that $\forall x, y \in S', x \neq y, x \not R y$.

- Give an example of an antichain.
- What is the length of a maximal antichain in $(\mathbb{N}, \leq)$, $(\mathcal{P}_A, \subseteq)$. Here $A = \{1, 2, \dots, 10\}$ and $\mathcal{P}_A$ denotes the power set of A .
- Show that a chain and an antichain intersect in at most 1 point.

2. Relational data of BTech 2

Consider the database of BTech2 consisting of three parameters: roll numbers, JEE rank, and CPI. Let x and y be two students. We define two relation $\succ, \succ>$ as follows: $x \succ y$ iff $CPI(x) - CPI(y) > 1.5$ and $x \succ> y$ iff $JEE(x) \leq JEE(y)$ and $CPI(x) \geq CPI(y)$.

- Which among the following properties are satisfied by $\succ$ and $\succ>$: reflexive, transitive, symmetric, antisymmetric.
 - A relation is called an equivalence relation if it is reflexive, transitive, and symmetric. Define a relation on this database which is an equivalence relation.
 - Let $\sim$ be an equivalence relation. Let $C_x = \{y \mid x \sim y\}$. The set C_x is called an equivalence class of x . How many equivalence classes are there in the relation that you defined in the previous subpart?
- How many functions exist from $\{1, 2, \dots, m\}$ to $\{1, 2, \dots, n\}$? How many injective functions exist from $\{1, 2, \dots, m\}$ to $\{1, 2, \dots, n\}$?
 - Each bead on a necklace with three beads is colored either black or white. Necklaces N_1, N_2 are said to be related if N_2 is N_1 or can be obtained from N_1 by flipping around the center of N_1 . Is it an equivalence relation? If not, then which property does it not satisfy? If it is, then what are the equivalence classes?