

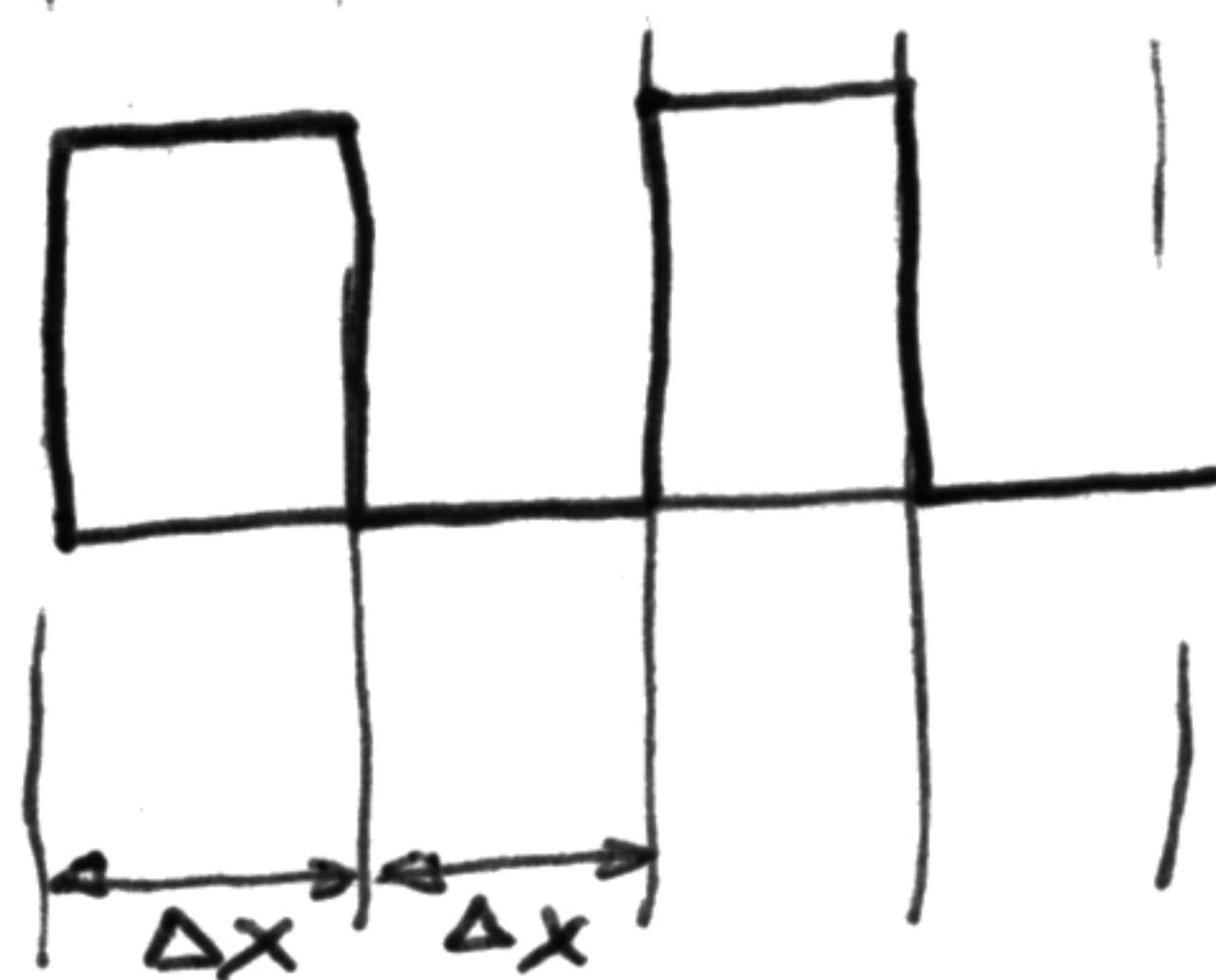
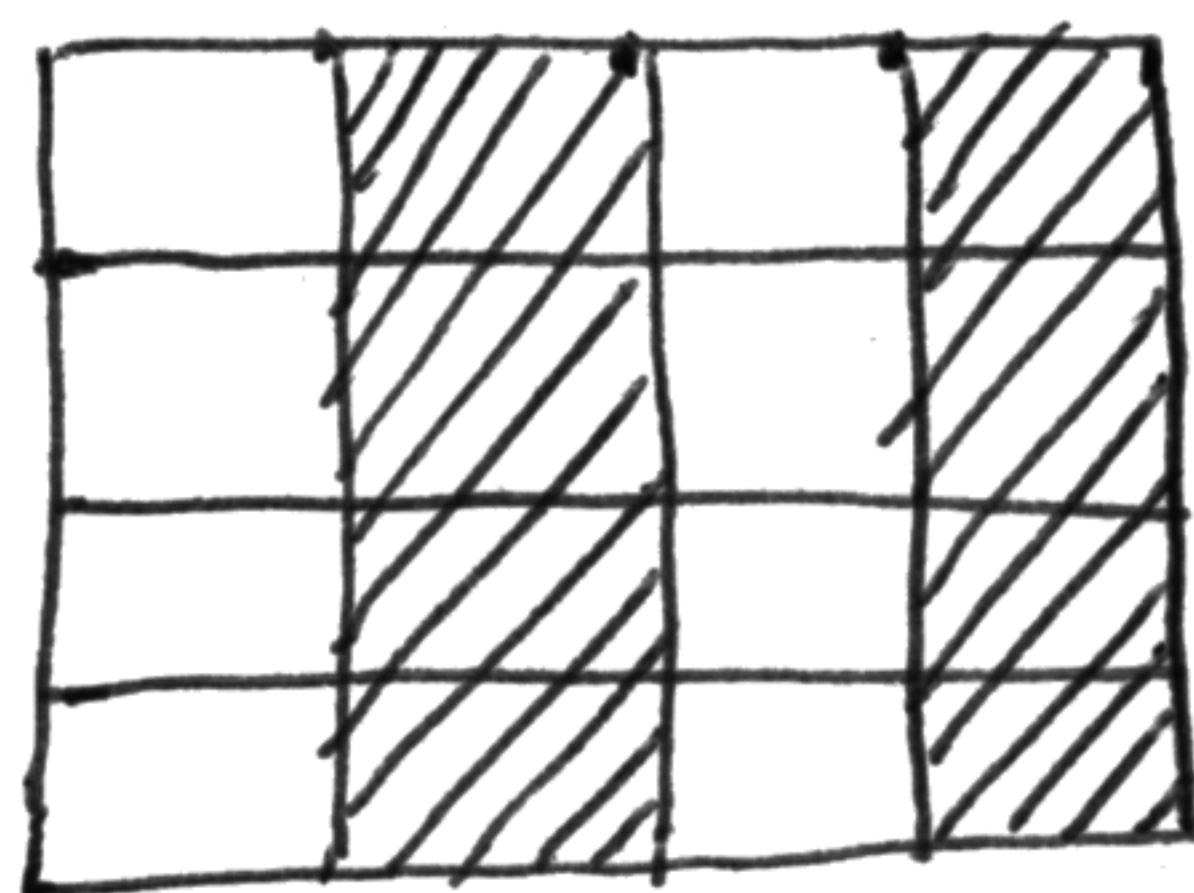
# PIXELS, SAMPLING AND THE NYQUIST LIMIT

- a) If the size of a pixel in an image is  $\Delta x$  units then the highest frequency signal that this image can contain without any aliasing is  $\frac{1}{2\Delta x}$

To see this consider the pixel grid made up of square pixels of side  $\Delta x$ .

On this grid, the pattern of highest

(spatial) frequency that can be formed without aliasing is a pattern formed of alternating white and black lines each being 1 pixel thick.



This represents a signal that completes 1 cycle every 2 pixels, so the frequency of the signal is  $\frac{1}{2\Delta x}$ .

- b)  $\Rightarrow$  Given any sampling with the sampling interval as  $\Delta d$  then the Nyquist limit of the sampling represents the highest frequency signal that this sampling can reconstruct without any aliasing and this is given by  $\frac{1}{2\Delta d}$

From a) and b) we can conclude that the Nyquist limit of a pixel of side  $\Delta x$  is  $\frac{1}{2\Delta x}$ .

c) What is  $\Delta x$ ?

Let us consider a digital camera with a CCD sensor of size  $W \times H$  ~~mm~~ (with  $W$  and  $H$  in mm). Let the image of highest resolution that this camera can take be of the size  $L \times B$  (with  $L$  and  $B$  in pixels). Let's put some numbers in place of the symbols.

Sensor size : 6.4 mm  $\times$  4.8 mm (W  $\times$  H)

Max. image resolution : 1024  $\times$  768 (L  $\times$  B)

$\Rightarrow$  that there are 160 pixels per mm of sensor dimension

Size of a pixel =  $\frac{1}{160}$  mm =  $\Delta x$

Nyquist limit =  $\frac{1}{2\Delta x} = \frac{1}{2 \times \frac{1}{160}} = 80$  pixels per mm

This means that when an image is formed on the CCD sensor, on that image any frequency that is more than 80 cycles per mm will cause aliasing.

+ Similarly for displaying images on the screen, or printing on a printer or scanning on a scanner, the sampling rate depends on how many pixels cover how much "physical" area.

To know more about this try reading about "DPI" or "dots per inch"