Lecture 19 - Ray Tracing 2
Ray Tracing

• Transforming normals

We also need the normal at the point of intersection. How does the normal transform when the object undergoes an affine transformation?
Ray Tracing

- Transforming normals

Incorrect

Correct
Ray Tracing

- Transforming normals – think about the tangents instead

- If $V_T$ is a vector in the tangent plane then after the transformation it becomes $V'_T = M \cdot V_T$

- The correct transformation to be applied to the normal should keep it perpendicular to the tangent vector.
Ray Tracing

- Transforming normals – think about the tangents instead

\[
N^T V_T = 0 \\
\Rightarrow N^T (M^{-1} M) V_T = 0 \\
\Rightarrow (N^T M^{-1}) (M V_T) = 0 \\
\Rightarrow (N^T M^{-1}) V'_T = 0 \\
\Rightarrow N'^T V'_T = 0 \\
\Rightarrow N' = (N^T M^{-1})^T = (M^{-1})^T N
\]
Ray Tracing

- Ray Casting – Single Step Ray Tracing
  - For every pixel in the image
    - Shoot a ray
    - Find closest intersection with object
    - Find normal at the point of intersection
    - Compute illumination at point of intersection
    - Assign pixel color
Ray Tracing

- Recursive Ray Tracing
Ray Tracing

• Recursive Ray Tracing

• Primary Rays - from the Eye
• Secondary Rays – Reflection, Refraction
• Shadow Rays
Ray Tracing

- Reflected Ray
  
  - Angle of Incidence = Angle of Reflection

\[
\vec{R} = \vec{I} - 2(\vec{I} \cdot \vec{N})\vec{N}\
\]

\(\vec{I}\) Incident Ray
\(\vec{R}\) Reflected Ray
\(\vec{N}\) Surface Normal
Ray Tracing

- Transmitted Ray

Snell-Descartes Law
\[ \frac{\eta_I \sin \theta_I}{\eta_T} = \frac{\sin \theta_T}{\sin \theta_I} = \eta_R \]

\[ \vec{I} = M \sin \theta_I - N \cos \theta_I \quad - \text{Incident Ray} \]

\[ \vec{M} = (I + N \cos \theta_I)/\sin \theta_I \]

So the transmitted ray is given by:
\[ \vec{T} = -N \cos \theta_T + M \sin \theta_T \]
\[ = -N \cos \theta_T + (I + N \cos \theta_I) \frac{\sin \theta_T}{\sin \theta_I} \]
\[ = \eta_R \vec{I} + (\eta_R \cos \theta_I - \cos \theta_T) \vec{N} \]
\[ = \eta_R \vec{I} + (\eta_R \cos \theta_I - \sqrt{1 - \eta_R^2 (1 - \cos^2 \theta_I)}) \vec{N} \]
\[ = \eta_R \vec{I} + (\eta_R (-\vec{N} \cdot \vec{I}) - \sqrt{1 - \eta_R^2 (1 - (-\vec{N} \cdot \vec{I})^2)}) \vec{N} \]

Normalize the result!
Ray Tracing

- Transmitted Ray

Snell-Descartes Law

\[ \eta_I \sin \theta_I = \eta_T \sin \theta_T \]
\[ \frac{\eta_I}{\eta_T} = \frac{\sin \theta_T}{\sin \theta_I} = \eta_R \]

\[ \vec{T} = M \sin \theta_I - N \cos \theta_I \]
- Incident Ray

⇒ \( \vec{M} = \left( I + N \cos \theta_I \right) / \sin \theta_I \)

So the transmitted ray is given by:

\[ \vec{T} = -N \cos \theta_T + M \sin \theta_T \]
\[ = -N \cos \theta_T + (I + N \cos \theta_I) \frac{\sin \theta_T}{\sin \theta_I} \]
\[ = \eta_R \vec{I} + \left( \eta_R \cos \theta_I - \cos \theta_T \right) \vec{N} \]
\[ = \eta_R \vec{I} + \left( \eta_R \cos \theta_I - \sqrt{1 - \eta_R^2 \left( 1 - \cos^2 \theta_I \right)} \right) \vec{N} \]
\[ = \eta_R \vec{I} + \left( \eta_R \left( -\vec{N} \cdot \vec{I} \right) - \sqrt{1 - \eta_R^2 \left( 1 - (\vec{N} \cdot \vec{I})^2 \right)} \right) \vec{N} \]

What happens when the square root is imaginary?
Ray Tracing

- Transmitted Ray

\[ \overrightarrow{I} \]
\[ n_I = 1 \]
\[ n_T = n_m \]
\[ \overrightarrow{T} \]

\[ \overrightarrow{N} \]

Entering and Leaving transmissive material is different – check dot product with normal.
Ray Tracing

• Total Internal Reflection
Ray Tracing

Ray Tracing

Ray Tracing

- Illumination: The Phong Model
  - For a single light source total illumination at any point is given by:
    \[ I = k_a I_a + k_d I_d + k_s I_s \]

    where
    - \( k_a I_a \) is the contribution due to ambient reflection
    - \( k_d I_d \) is the contribution due to diffuse reflection
    - \( k_s I_s \) is the contribution due to specular reflection
Ray Tracing

- Components of the Phong Model

- Ambient Illumination: $I_a$
  - Represents the reflection of all indirect illumination.
  - Has the same value everywhere.
  - Is an approximation to computing Global Illumination.

Ray Tracing

• Components of the Phong Model

• Diffuse Illumination:  \( I_d = I_L \cos \theta_L \)
  
  - Assumes Ideal Diffuse Surface – that reflects light equally in all direction.
  
  - Surface is very rough at microscopic level. For e.g., Chalk and Clay.
Ray Tracing

- Components of the Phong Model
- Diffuse Illumination: \( I_d = I_L \cos \theta_L \)
  - Reflects light according to *Lambert’s Cosine Law*

\[
I_d = I_L \cos \theta_L = I_L (\vec{L} \cdot \vec{N})
\]

\( \vec{L} \): vector to the light source
\( I_L \): intensity of the light source
\( \vec{N} \): surface normal
Ray Tracing

- Components of the Phong Model

- Specular Illumination: $I_s = I_L \cos^n \alpha_v = I_L (\vec{R} \cdot \vec{V})^n$
  - Ideal specular surface reflects only along one direction.
  - Reflected intensity is view dependent – Mostly it is along the reflected ray but as we move away some of the reflection is slightly offset from the reflected ray due to microscopic surface irregularities.
Ray Tracing

- Components of the Phong Model
- Specular Illumination: \( I_s = I_L \cos^n \alpha_v = I_L (\mathbf{R} \cdot \mathbf{V})^n \)
  - \( n \) is called the coefficient of shininess and \( I_L = I_t/r^2 \)
Ray Tracing

- The Phong Illumination Model

\[ I = k_a I_a + k_d I_d + k_s I_s \]

- \( k_a, k_d, k_s \) are material constants defining the amount of light that is reflected as ambient, diffuse and specular. They may be defined in as three values with R, G, B components.

http://en.wikipedia.org/wiki/Phong_shading
Ray Tracing

• Local Illumination Model

\[ I_{local} = k_a I_a + \sum_{1 \leq i \leq m} (k_d I_{di} + k_s I_{si}) \]

Global Illumination Model

\[ I_{global} = I_{local} + k_r I_{reflected} + k_t I_{transmitted} \]

• Reflected and transmitted components may also be attenuated based on distance the ray travels.
Ray Tracing

- Surface Material Properties
- Colour – For each object there can be a
  - Diffuse colour, Specular colour, Reflected colour and Transmitted colour
  - Remember differently coloured light is at different wavelength so:

\[
I_\lambda = k_a C_d \lambda I_a + \sum_{1 \leq i \leq m} (k_d C_d \lambda I_{di} + k_s C_s \lambda I_{si}) + k_r C_r \lambda I_r + k_t C_t \lambda I_t
\]

- Accounting for shadows:

\[
I_\lambda = k_a C_d \lambda I_a + \sum_{1 \leq i \leq m} S_i (k_d C_d \lambda I_{di} + k_s C_s \lambda I_{si}) + k_r C_r \lambda I_r + k_t C_t \lambda I_t
\]
Ray Tracing

- Recursive Ray Tracing

Eye

Eye

A

D

B

C

E

F

D

L_1

L_2
Ray Tracing

- Recursive Ray Tracing
Ray Tracing

• Recursive Ray Tracing

• Stop
  - When a ray leaves the scene
  - Contributed intensity is too less
Ray Tracing

• Recursive Ray Tracing

• Complexity?
Ray Tracing

- Recursive Ray Tracing

- Complexity = $h \times w \times N_{\text{objects}} \times \text{intersection cost} \times \text{depth of recursion} \times N_{\text{shadow\_rays}} \times ...$
Ray Tracing

- Aliasing – Discrete samples of a continuous world
Ray Tracing

- Anti-aliasing – Shoot more rays per pixel - super sample!
Ray Tracing

- Sampling strategies for anti-aliasing

  - Regular super sampling
  - Adaptive super sampling
  - Stochastic or jittered super sampling

- Aliasing vs. Noise

- Aggregating the samples.
Ray Tracing
Ray Tracing
Distributed Ray Tracing

http://web.cs.wpi.edu/~matt/courses/cs563/talks/dist_ray/dist.html

http://www.cs.utexas.edu/~fussell/
The Rendering Equation

\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t) (-\omega \cdot n) \, d\omega' \]

- \( L_o(x, \omega, \lambda, t) \) is the total amount of light of wavelength \( \lambda \), directed outward along direction \( \omega \) at time \( t \), from a particular position \( x \).
- \( L_e(x, \omega, \lambda, t) \) is the emitted light.
- \( L_i(x, \omega', \lambda, t) \) is the light of wavelength \( \lambda \), coming inward toward \( x \) from direction \( \omega' \) at time \( t \).
- \( f_r(x, \omega', \omega, \lambda, t) \) is the bidirectional reflectance distribution function (BRDF), i.e., the proportion of light reflected from \( \omega' \) to \( \omega \) at position \( x \), time \( t \), and at wavelength \( \lambda \).
- \((-\omega \cdot n)\) is the attenuation of incident light due to incident angle.
- \( \int_{\Omega} \ldots d\omega' \) is the integral over a sphere of inward directions.
The Rendering Equation

\[ L_o(x, \omega, \lambda, t) = L_e(x, \omega, \lambda, t) + \int_{\Omega} f_r(x, \omega', \omega, \lambda, t) L_i(x, \omega', \lambda, t) (-\omega \cdot n) \, d\omega' \]

- Is this enough?
- BTDF - Refraction, BSDF – Sub surface scattering
- Phosphoresence
- Diffraction
- Atmospheric Scattering