Aliasing

- Discrete samples of continuous information

Aliasing

- A signal processing view: Aliasing is caused by inadequate sampling of continuous information.

Aliasing

- The effect of sampling interval

Aliasing

- The samples seem to represent a signal at a lower frequency which are known as aliases (i.e., lost higher frequency information reappears as impersonating lower frequencies) - hence the name “Aliasing.”
Aliasing

- Nyquist-Shannon Sampling Theorem:
  - A continuous bandlimited function of a single variable can be completely represented by a set of samples made at equally spaced intervals.
  - The intervals between such samples must be less than half the period (or greater than twice the frequency) of the highest frequency component in the function.

\[ f_{\text{max}} = \frac{1}{2 \Delta x} \text{ or } f_s > 2f_{\text{max}} \]

Fourier Theory

- The Fourier Transform – Any signal, \( f(\cdot) \) can be considered to be made up of a weighted sum of sine and cosine waves.

\[ F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i ux} \, dx \]

- The Fourier Transform is reversible

\[ f(x) = \int_{-\infty}^{\infty} F(u) e^{2\pi i ux} \, du \]

- A Fourier transform converts the function from a spatial/temporal domain representation to a spectral/frequency domain representation.

Fourier Transform in 1D

<table>
<thead>
<tr>
<th>Spacial domain</th>
<th>Frequency domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>impulse or &quot;delta&quot; function</td>
<td>1/0</td>
</tr>
<tr>
<td>ramp or ramp function</td>
<td>( \sin ) or ( \cos ) function</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>( F(u) )</td>
</tr>
</tbody>
</table>

Convolution Operator

\[ f(x) \ast g(x) = \int_{-\infty}^{\infty} f(\tau) g(x-\tau) \, d\tau \]

The Convolution Theorem

\[ F(f(x) \ast g(x)) = F(f)G(u) \]
\[ F(f(x) g(x)) = F(f) \ast G(u) \]

Aliasing

And this reconstruction of the original signal from samples works only when

\[ f_{\text{max}} = \frac{1}{2 \Delta x} \text{ or } f_s > 2f_{\text{max}} \]
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  - The intervals between such samples must be less than half the period (or greater than twice the frequency) of the highest frequency component in the function.

\[ f_{\text{max}} \leq \frac{1}{2 x} \quad \text{or} \quad f_s \geq 2 f_{\text{max}} \]

Aliasing in images

- Images can be treated as 2D signals.
- Aliasing in images also happens when information at a higher frequency is sampled at less than the Nyquist limit.

A pixel of side \( \Delta x \) has a Nyquist limit of \( 1/2 \Delta x \).