Graphical Models for Sequence Labeling in NLP

Anup Kulkarni

Indian Institute of Technology, Bombay

Sep 30, 2009

Under the guidance of Prof. Pushpak Bhattacharyya.

Outline

Graphical Models and Machine Learning

- Machine Learning like SVM, NB, K-NN have attributes as input and class label as output
- attributes influence class label but class label does not influence other class labels

- Graphical Model attributes as well as class labels influence class labels

- Text Processing, NLP
- Computer Vision eg. Scene Recognition, Object Labeling etc.

Goal of NLP

- Understanding Language

The heart operation was painful.

He plays violin in the play.

Goal of NLP

- Understanding Language

The heart operation was painful.

He plays violin in the play.

Goal of NLP

- Understanding Language

The heart operation was painful.

He plays violin in the play.

Goal of NLP

- Understanding Language
- Sequence Labeling

The heart operation was painful.

He plays violin in the play.

What Labels?

- Parts of Speech Tags

The_AT postman_NN collected_VBD letters_NNS and_CNJ left_VBD.
He_PN plays_VBZ violin_NN in_PP the_AT play_NN.

What Labels?

- Parts of Speech Tags: POS tags
- Chunk Tags

He$< B-NP >$ reckons$< B-VP >$ the$< B-NP >$ current$< I-NP >$ account$< I-NP >$ deficit$< I-NP >$ will$< B-VP >$ narrow$< I-VP >$.$< O >$

What Labels?

- Parts of Speech Tags: POS tags
- Chunk Tags
- Named Entity Recognition

I bought 100 shares of IBM Corp$< ORG >$.
IIT Bombay$< ORG >$ is located in Powai which is in north part of Bombay$< LOC >$.
Mr. Wesley$< PER >$ went to buy the book published by Addison Wesley$< ORG >$.

Imposing Graph on Sentence

- Words influence each other.
- Words can be represented using nodes.
- Relation between words can be represented using edges.

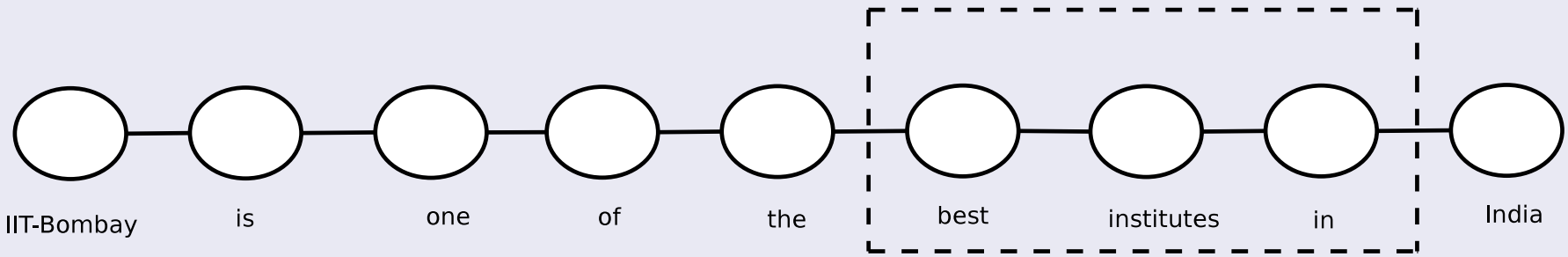


Figure: Graphical model induced on a sentence.

Imposing Graph on Sentence

- Words influence each other.
- Words can be represented using nodes.
- Relation between words can be represented using edges.

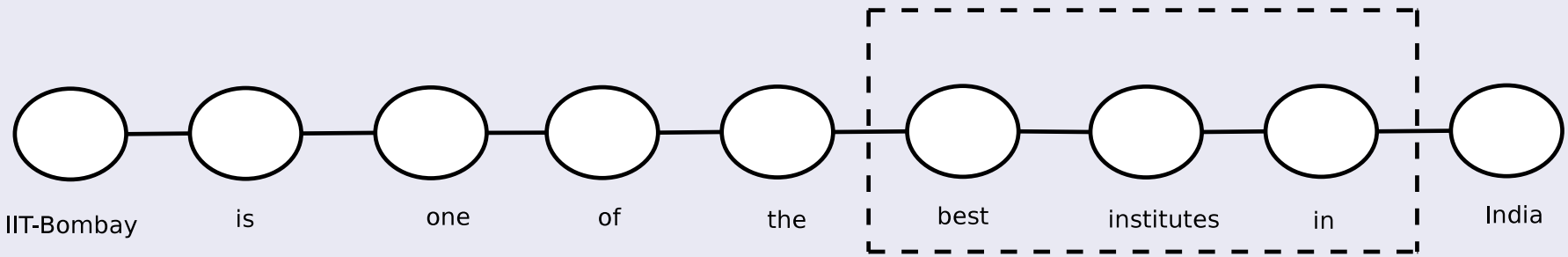


Figure: Graphical model induced on a sentence.

Markov Assumption Every node depends only on previous node.

Labeling Each Node

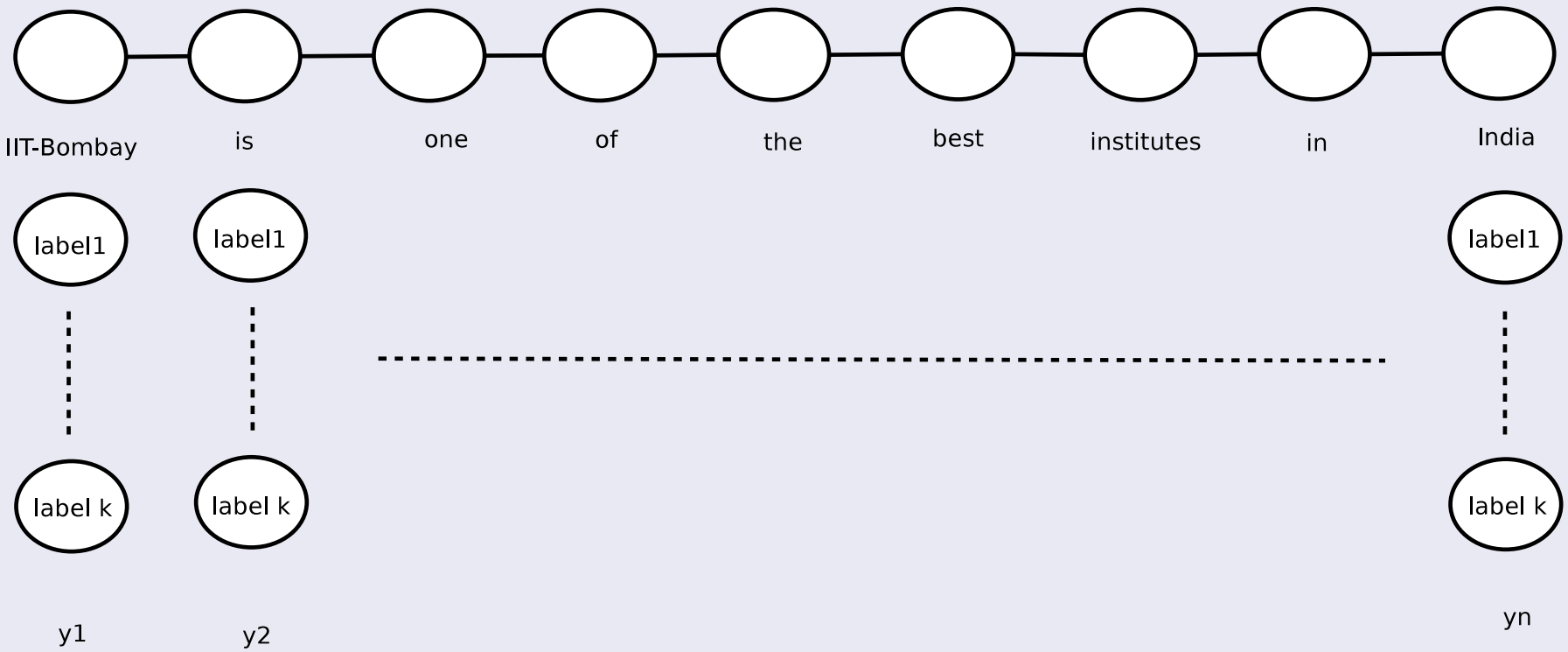


Figure: Graphical model induced on a sentence.

Random Field

- Each node can take all possible labels
- If we define y_i as random variable for node i taking some label, we will get random field for

$$Y = (y_1, y_2, ..., y_n)$$

- Probability of tag sequence is given by

$$\Pr(Y) = \Pr(y_1, y_2, ..., y_n)$$

- Our goal is to find best probable tag sequence, given sentence(or observation) $X = (x_1, x_2, ..., x_n)$

$$\Pr(Y|X) = \Pr(y_1, y_2, ..., y_n|X)$$

Hammersley-Clifford Theorem

Every Markov Random Field is equivalent to Gibbs Random Field

Hammersley-Clifford Theorem

Every Markov Random Field is equivalent to Gibbs Random Field

Markov Random Field is random field satisfying Markov Property

$$\Pr(y_i|y_{N-i}) = \Pr(y_i|Ne(i))$$

Gibbs Random Field is random field obeying Gibbs distribution

$$\Pr(y) = \frac{exp(-U(y))}{Z}$$

Where Z is defined as

$$Z = \sum_{y \in Y} exp(-U(y))$$

U(y) is called as energy function defined as

$$U(y) = \sum_{c \in C} V_c(y)$$

That is sum of all clique potentials.

Proof of HC Theorem $GRF \rightarrow MRF$

Let $D_i = Ne(i) \cup X_i$ be set of neighbors of X_i and X_i itself.
We have to prove that if Gibbs distribution is satisfied then markovian property is hold. That means given Gibbs distribution we have to arrive at

$$\Pr(X_i|X_{S-i}) = \Pr(X_i|X_{Ne(i)}) \quad (1)$$

Proof of HC Theorem *GRF* $\longrightarrow$ *MRF*

Starting with LHS

$$
\begin{aligned}
\Pr(X_i|X_{S-i}) &= \frac{\Pr(X_i, X_{S-i})}{\Pr(X_{S-i})} \\
&= \frac{\Pr(X_i, X_{S-i})}{\sum_{X_i} \Pr(X_i, X_{S-i})} \\
&= \frac{\frac{exp(\sum_{c \in C} V_c(X))}{Z(X)}}{\sum_{X_i} \frac{exp(\sum_{c \in C} V_c(X))}{Z(X)}} \\
&= \frac{exp(\sum_{c \in C} V_c(X))}{\sum_{X_i} exp(\sum_{c \in C} V_c(X))}
\end{aligned}
$$

Proof of HC Theorem *GRF* $\rightarrow$ *MRF*

Now cliques C in the graph is set of cliques A which contains node X_i and B which do not contain node X_i. Using this,

$$\Pr(X_i|X_{S-i}) = \frac{exp(\sum_{c\in C} V_c(X))}{\sum_{X_i} exp(\sum_{c\in C} V_c(X))}$$
$$= \frac{exp(\sum_{c\in A} V_c(X)) \times exp(\sum_{c\in B} V_c(X))}{\sum_{X_i} exp(\sum_{c\in A} V_c(X)) \times exp(\sum_{c\in B} V_c(X))}$$

since cliques in B do not contain node X_i, the term $exp(\sum_{c\in B} V_c(X))$ can be brought out of summation.

$$\Pr(X_i|X_{S-i}) = \frac{exp(\sum_{c\in A} V_c(X)) \times exp(\sum_{c\in B} V_c(X))}{\sum_{X_i} exp(\sum_{c\in A} V_c(X)) \times exp(\sum_{c\in B} V_c(X))}$$
$$= \frac{exp(\sum_{c\in A} V_c(X))}{\sum_{X_i} exp(\sum_{c\in A} V_c(X))}$$

The term in the numerator contains all the cliques which contains X_i.

Proof of HC Theorem *GRF* $\rightarrow$ *MRF*

We arrive at the same expression from RHS as

$$\begin{aligned}
\Pr(X_i|X_{Ne(i)}) &= \frac{\Pr(X_i, X_{Ne(i)})}{\Pr(X_{Ne(i)})} \\
&= \frac{\frac{exp(\sum_{c\in A} V_c(X))}{Z(X)}}{\sum_{X_i} \frac{exp(\sum_{c\in A} V_c(X))}{Z(X)}} \\
&= \frac{exp(\sum_{c\in A} V_c(X))}{\sum_{X_i} exp(\sum_{c\in A} V_c(X))}
\end{aligned}$$

Thus *GRF* $\rightarrow$ *MRF* is proved.

Proof of HC Theorem *MRF* $\rightarrow$ *GRF*

For this Mobius inversion principle is used,

$$F(A) = \sum_{B:B \subseteq A} G(B) \Leftrightarrow G(B) = \sum_{B:B \subseteq A} (-1)^{|B|-|C|} F(C)$$

This when written for energy function U(X) and potentials V(X),

$$U(x_v) = \sum_{B:B \subseteq V} V(x_b) \Longleftrightarrow V(x_b) = \sum_{C:C \subseteq B} (-1)^{|B|-|C|} U(x_c)$$

where V our graph and B is subgraph.

Proof of HC Theorem *MRF* $\longrightarrow$ *GRF*

Gibbs distribution requires energy to be defined over all cliques of the graph. So to prove that

$$V(x_b) = 0 \tag{2}$$

whenever B is not completely connected ie. B is not clique.
when B is not a clique, let Z_1 and Z_2 be two nodes such that they are not connected in B.
let S be separator : $Z_1 S Z2 = B$

Proof of HC Theorem *MRF* $\rightarrow$ *GRF*

$V(x_b) = \sum_{C:C\subseteq B}(-1)^{|B|-|C|}U(x_c)$
can be written as

$$V(x_b) = \sum_{C':C'\subseteq S}(-1)^{|B|-|C'|}U(x_{c'}) + \sum_{C':C'\subseteq S}(-1)^{|B|-|Z_1C'|}U(x_{z_1c'}) + \sum_{C':C'\subseteq S}(-1)^{|B|-|C'Z_2|}U(x_{c'z_2}) + \sum_{C':C'\subseteq S}(-1)^{|B|-|Z_1C'Z_2|}U(x_{z_1c'z_2})$$

adjusting exponents of (-1)

$$V(x_b) = \sum_{C':C'\subseteq S}(-1)^{|B|-|C'|}[U(x_{c'}) - U(x_{z_1c'}) - U(x_{c'z_2}) + U(x_{z_1c'z_2})] \quad (3)$$

Now we will prove that bracketed term [.] is 0. Taking exp of [.] term,

Proof of HC Theorem *MRF* $\longrightarrow$ *GRF*

$$
\begin{aligned}
exp[.] &= exp(U(x_{c'}) - U(x_{z_1 c'}) - U(x_{c' z_2}) + U(x_{z_1 c' z_2})) \\
&= \frac{exp(U(x_{c'})) \times exp(U(x_{z_1 c' z_2}))}{exp(U(x_{z_1 c'})) \times exp(U(x_{c' z_2}))} \\
&= \frac{\frac{exp(U(x_{c'}))}{Z} \times \frac{exp(U(x_{z_1 c' z_2}))}{Z}}{\frac{exp(U(x_{z_1 c'}))}{Z} \times \frac{exp(U(x_{c' z_2}))}{Z}} \\
&= \frac{\Pr(x_{c'})}{\Pr(x_{z_1 c'})} \frac{\Pr(x_{z_1 c' z_2})}{\Pr(x_{c' z_2})}
\end{aligned}
$$

Proof of HC Theorem *MRF* $\longrightarrow$ *GRF*

Using Baye's Therom,

$$exp[.] = \frac{\Pr(x_{z_1}|x_{c'z_2})}{\Pr(x_{z_1}|x_{c'})}$$

since X_{Z_1} and X_{Z_2} are not directly connected, using Markovian property,

$$\Pr(x_{z_1}|x_{c'z_2}) = \Pr(x_{z_1}|x_{c'})$$

$$exp[.] = 1$$

so [.] in 3 is 0. So by 2 Every MRF is GRF.

Graphical Models

1. Maximum Entropy Markov Model
2. Cyclic Dependency Network
3. Conditional Random Field

Maximum Entropy Principle

Entropy Entropy is a measure of the uncertainty associated with a random variable.

$$H(X) = -\sum_{i=1}^{n} \Pr(x_i) \log \Pr(x_i)$$

Maximum Entropy When incomplete information is available the only unbiased assumption which can be made is maximize uncertainty, ie. Maximize Entropy

Drawbacks of traditional HMM

- characteristics of text are not used
- learns joint distribution $\Pr(y, x)$
- assumes wrong model : word depends on tag

Drawbacks of traditional HMM

- characteristics of text are not used
- learns joint distribution $\Pr(y, x)$
- assumes wrong model : word depends on tag

$$\Pr(y, x) = \Pr(y) \Pr(y|x)$$

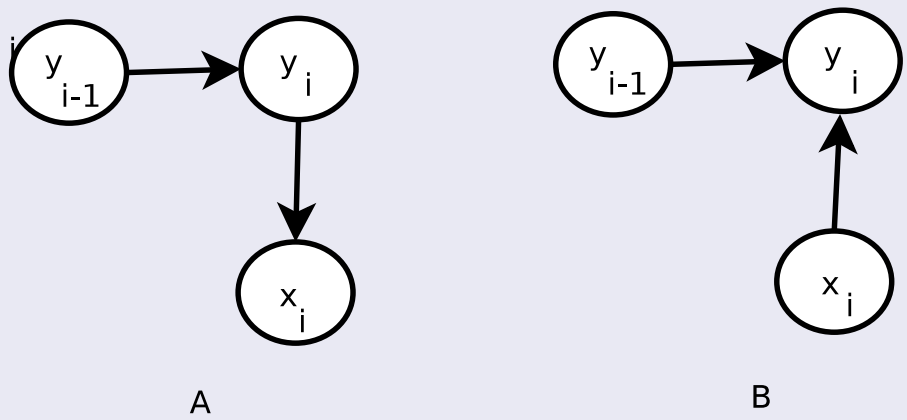


Figure: A.Dependency Graph for traditional HMM B.Dependency Graph for Maximum Entropy Markov Model(MEMM) Source: [McCallum et. al.,2000]

Maximum Entropy Principle

Entropy Entropy is a measure of the uncertainty associated with a random variable.

$$H(X) = -\sum_{i=1}^{n} \Pr(x_i) \log \Pr(x_i)$$

Maximum Entropy When incomplete information is available the only unbiased assumption which can be made is maximize uncertainty, ie. Maximize Entropy

Maximum Entropy Principle

Entropy Entropy is a measure of the uncertainty associated with a random variable.

$$H(X) = -\sum_{i=1}^{n} \Pr(x_i) \log \Pr(x_i)$$

Maximum Entropy When incomplete information is available the only unbiased assumption which can be made is maximize uncertainty, ie. Maximize Entropy

Incomplete Information $\sum_y \Pr(y|x) = 1$

Formulation of MEMM

- Given: pairs of (obs,label): $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$

Formulation of MEMM

- Given: pairs of (obs,label): $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$
- **Define** Empirical Distribution $\tilde{\Pr}(x, y)$

$$\tilde{\Pr}(x, y) = \frac{\text{No. of times (x,y) occurs in sample}}{N}$$

Formulation of MEMM

- Given: pairs of (obs,label): $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$
- **Define** Empirical Distribution $\tilde{\Pr}(x, y)$

$$\tilde{\Pr}(x, y) = \frac{\text{No. of times (x,y) occurs in sample}}{N}$$

Features Extra information and characteristics of the text are represented using features

$$f(x, y) = \left\{ \begin{array}{c} 1 \text{ if } x = play, y = verb \\ 0 \text{ otherwise} \end{array} \right\}$$

Constrain on MaxEnt model

constrain on MaxEnt model expectation of features in MaxEnt model should be equal to expectation of features in training data.

Expectation in Empirical Distribution [a]

$$\tilde{\Pr}(f) = \sum_{x,y} \tilde{\Pr}(x,y) f(x,y)$$

Expectation in MaxEnt Model

$$\Pr(f) = \sum_{x,y} \Pr(x,y) f(x,y)$$

[a] Using same notation as Berger et. al. 1996

Constrain on MaxEnt model

constrain on MaxEnt model expectation of features in MaxEnt model should be equal to expectation of features in training data.

Expectation in Empirical Distribution

$$Ex^{\sim}(f) = \sum_{x,y} \tilde{\Pr}(x,y) f(x,y)$$

Expectation in MaxEnt Model

$$Ex(f) = \sum_{x,y} \Pr(x,y) f(x,y)$$

$$\sum_{x,y} \tilde{\Pr}(x,y) f(x,y) = \sum_{x,y} \tilde{\Pr}(x) \Pr(y|x) f(x,y) \quad (4)$$

Using MaxEnt Principle

Maximum Entropy Model Model that will maximize the entropy

$$\overset{*}{\Pr}(y|x) = arg \max_{\Pr(y|x)} H(y|x)$$

where,

$$H(y|x) = -\sum_{(x,y)} \tilde{\Pr}(x) \Pr(y|x) log \Pr(y|x)$$

$$\sum_{x,y} \tilde{\Pr}(x,y) f(x,y) = \sum_{x,y} \tilde{\Pr}(x) \Pr(y|x) f(x,y)$$

$$\sum_{y} \Pr(y|x) = 1 \tag{5}$$

Formulating Dual Objective

Let $\lambda_{1...m}$ be Langrange Multiplier associated 4 for each feature $f_{1...m}$. Let λ_{m+1} be the Langrange Multiplier associated 5. Then MaxEnt dual objective is:

$$D(\Pr(y|x), \lambda) = H(y|x) + \sum_{i=1}^{m} \lambda_i (Ex(f_i) - Ex^{\sim}(f_i)) + \lambda_{m+1} (\sum_{y} \Pr(y|x) - 1)$$

Differentiating and equating to zero we get MaxEnt probability model as,

$$\Pr(y|x) = \frac{exp(\sum_{i=1}^{m} \lambda_i f_i(x, y))}{\sum_y exp(\sum_{i=1}^{m} \lambda_i f_i(x, y))} \quad (6)$$

Training

Notion of Weights Each λ_i can be viewed as weight associated with every feature f_i. The training problem is to find these weights.

Training

Notion of Weights Each λ_i can be viewed as weight associated with every feature f_i. The training problem is to find these weights.

Input Feature Functions $f_1, f_2, ..., f_m$;empirical distribution $\Pr^{\sim}(x, y)$
Output Optimal Parameters values Λ^*; optimal model $\Pr_{\Lambda^*}(y|x)$
repeat
  for $i = 1$ to m **do**
    δ_i be solution to
    $\delta_i = \frac{1}{T} log \frac{\sum_{x,y} \Pr^{\sim}(x,y) f_i(x,y)}{\sum_x \Pr^{\sim}(x) \sum_y \Pr_{\Lambda}(y|x) f_i(x,y)}$
    update values as $\lambda_i^{n+1} \leftarrow \lambda_i^n + \delta_i$
until Λ is converged

Inferencing

- Viterbi Algorithm Used

alpha probability [a]

$$\alpha_{t+1}(y) = \sum_{y' \in Y} \Pr_{y'}(y|x_{t+1})\alpha_t(y')$$

beta probability

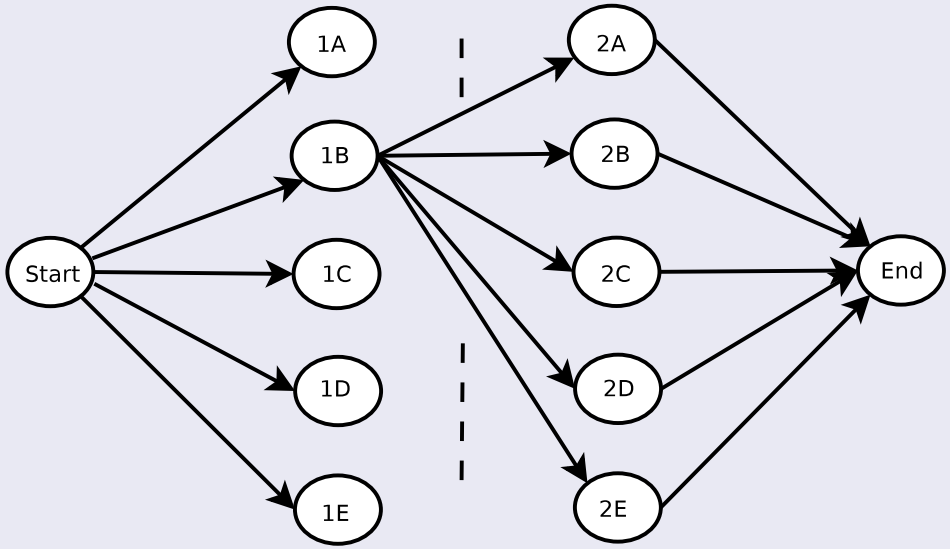
$$\beta_t(y') = \sum_{y \in Y} \Pr_{y'}(y|x_t)\beta_{t+1}(y)$$

delta probability

$$\delta_{t+1}(y) = \max_{y' \in Y} \Pr_{y'}(y|x_{t+1})\delta_t(y')$$

[a] Using same notation as McCallum et.al., 2000

Inferencing



Application Specific Features

Condition	Features
x_i is not rare	$x_i = W, y_i = K$
x_i is rare	W is prefix of x_i , $\|W\| \leq 4, y_i = K$ W is suffix of x_i , $\|W\| \leq 4, y_i = K$ x_i contains number , $y_i = K$ x_i contains upper case character , $y_i = K$ x_i contains hyphen , $y_i = K$ x_{i-1} is "have", "to", "be"
$\forall x_i$	$y_{i-1} = T, y_i = K$ $y_{i-2}y_{i-1} = T_2T_1, y_i = K$ $x_{i-2} = W, y_i = K$ $x_{i-1} = W, y_i = K$

Table: Table summarizing features used by [Ratnaparakhi,96]

Graphical Models

1. Maximum Entropy Markov Model
2. Cyclic Dependency Network
3. Conditional Random Field

Label Bias Problem

- Particular label is biased towards some next state label

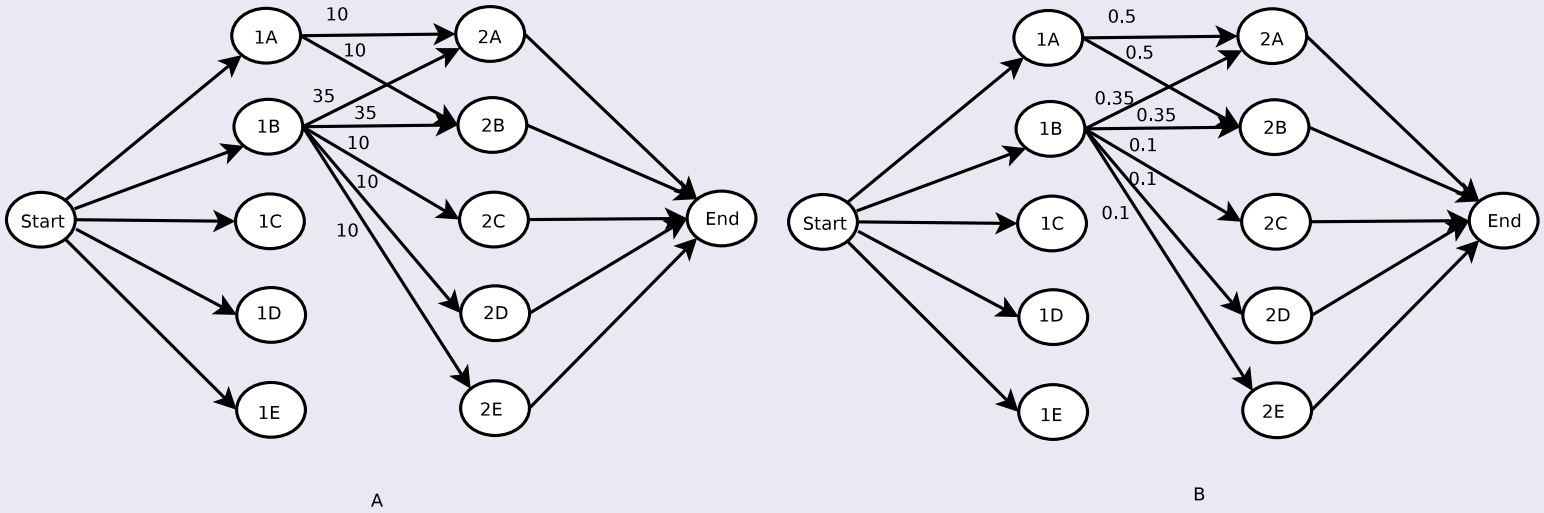


Figure: A.Transitions with un-normalized scores B.Transitions with probability values

Label Bias Problem

- Particular label is biased towards some next state label
- do not normalize scores per state

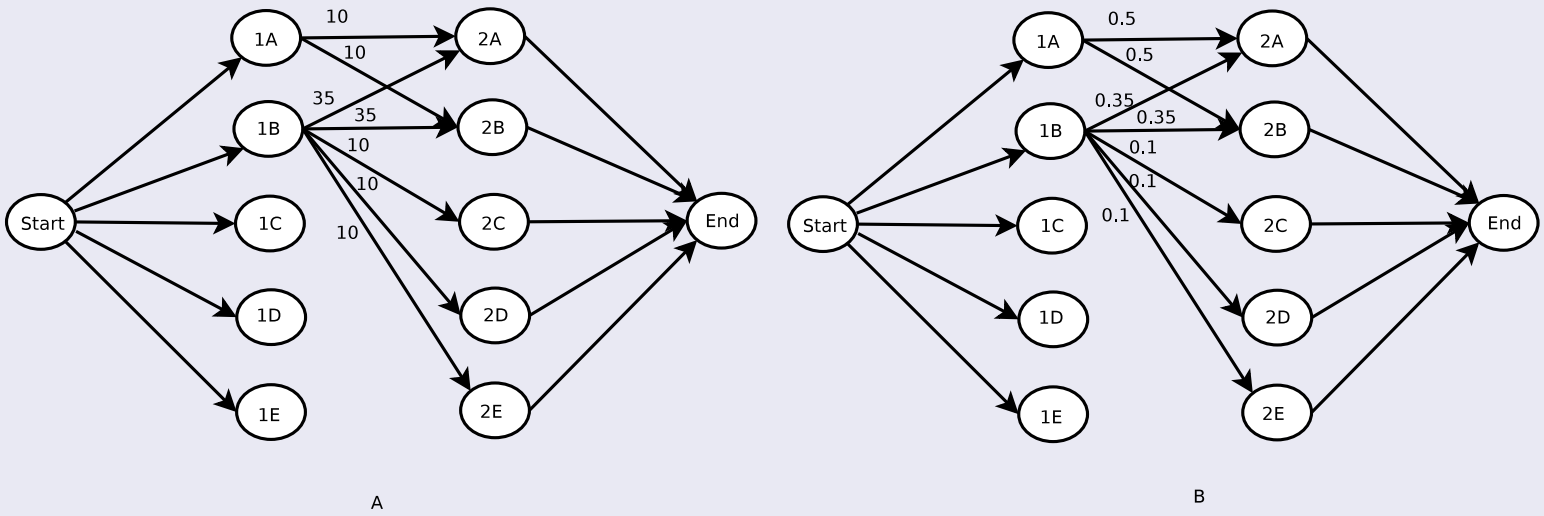


Figure: A.Transitions with un-normalized scores B.Transitions with probability values

CRF formulation

Conditional Random Field is defined as Markov Random Field globally conditioned on X

CRF formulation

Conditional Random Field is defined as Markov Random Field globally conditioned on X

$$
\begin{aligned}
\Pr(Y|X) &= \frac{\Pr(X,Y)}{\Pr(X)} \\
&= \frac{\Pr(X,Y)}{\sum_Y \Pr(X,Y)} \\
&= \frac{exp(\sum_{c \in C} V_c(X_c, Y_c))}{\sum_Y exp(\sum_{c \in C} V_c(X_c, Y_c))} \\
&= \frac{exp(\sum_{j=1}^{n} \sum_{i=1}^{M} \lambda_i f_i(y_{j-1}, y_j, X, j))}{Z(X)}
\end{aligned}
$$

Parameter Estimation

Voted Perceptron

$$\Delta^t = F(Y, X) - F(Y^*, X)$$

where Y^* is best scoring labels obtained using model.

$$\Lambda^{t+1} = \Lambda^t + \frac{1}{k} \sum_i \delta_i^t$$

Inferencing

1. Initialization: The values for all steps from the start state $\perp$ to all possible first states s are set to the corresponding factor value.

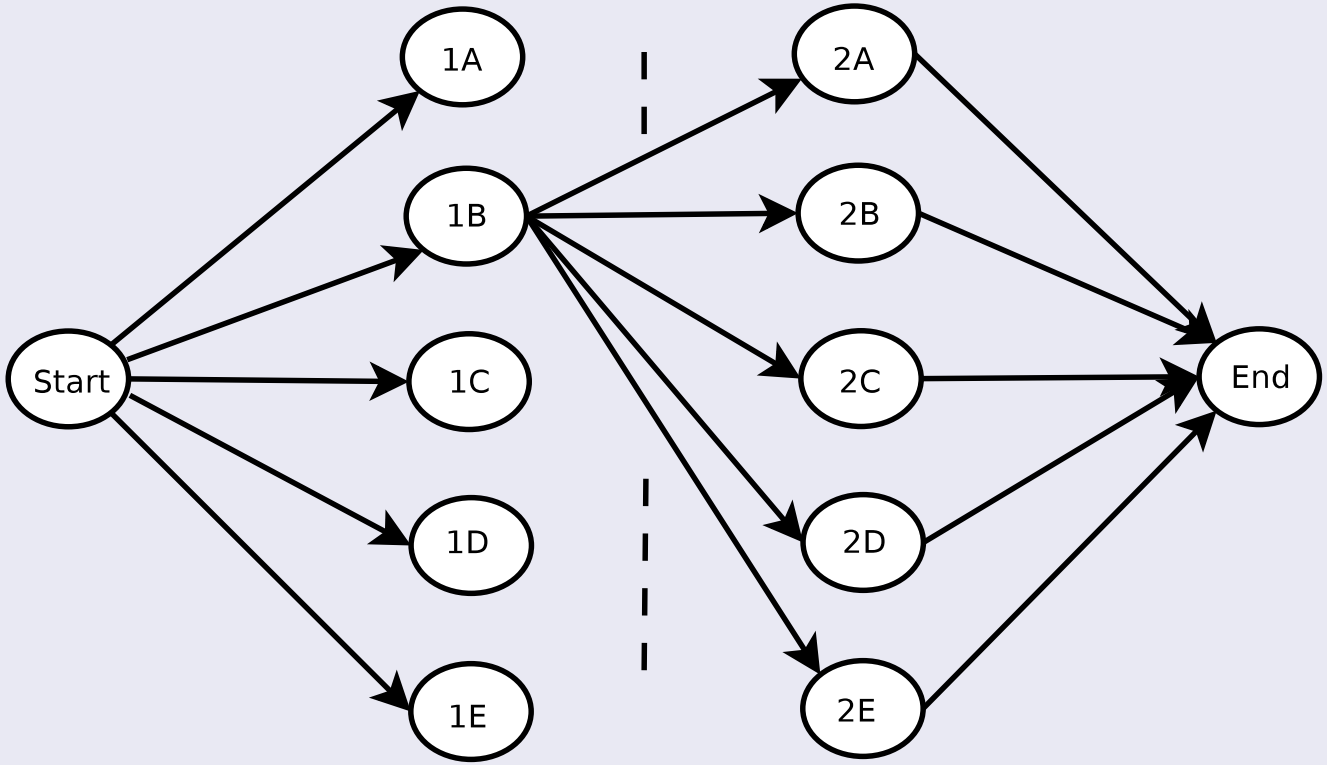
$$\begin{aligned} \forall s \in S : \delta_1(s) &= V_1(X, \perp, s) \\ &= \psi_1(s) = \perp \end{aligned}$$

2. Recursion: The values for the next steps are computed from the current value and the maximum values regarding all possible succeeding states s'

$$\forall s \in S : 1 \leq j \leq n : \delta_j(s) = \max_{s' \in S} \delta_{j-1}(s') V_j(X, s', s)$$

$$\psi_j(s) = \arg\max_{s' \in S} \delta_{j-1}(s') V_j(X, s', s)$$

Inferencing



Inferencing

1. Termination:

$$p^* = \max_{s' \in S} \delta_n(s')$$

$$y_n^* = \arg\max_{s' \in S} \delta_n(s')$$

2. Path Backtracking: Recomputing the optimal path from the lattice using the track keeping values ψ_t

$$y_t^* = \psi_{t+1}(y_{t+1}^*)$$

where $t = n-1, n-2, \ldots, 1$