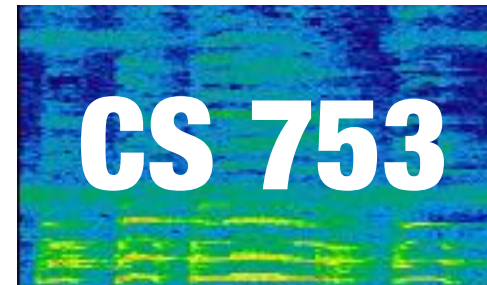


# Pre-midsem Revision

## Lecture 11

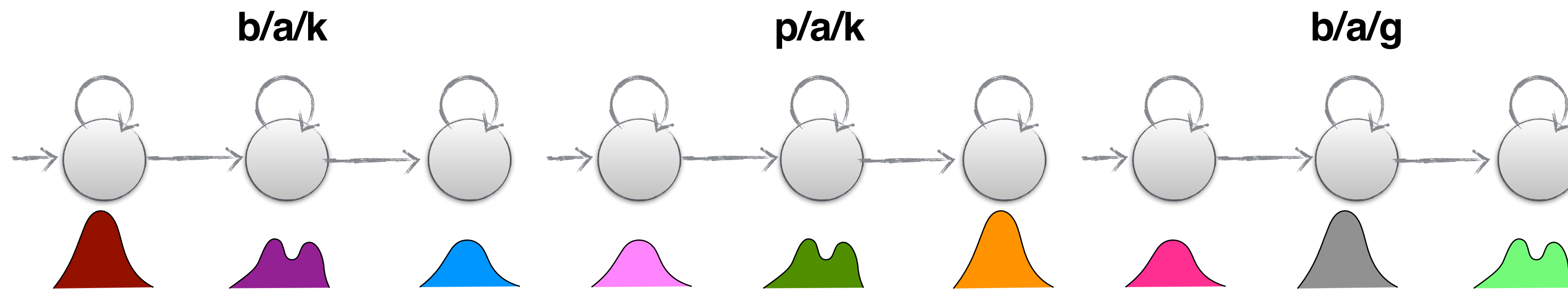


Instructor: Preethi Jyothi

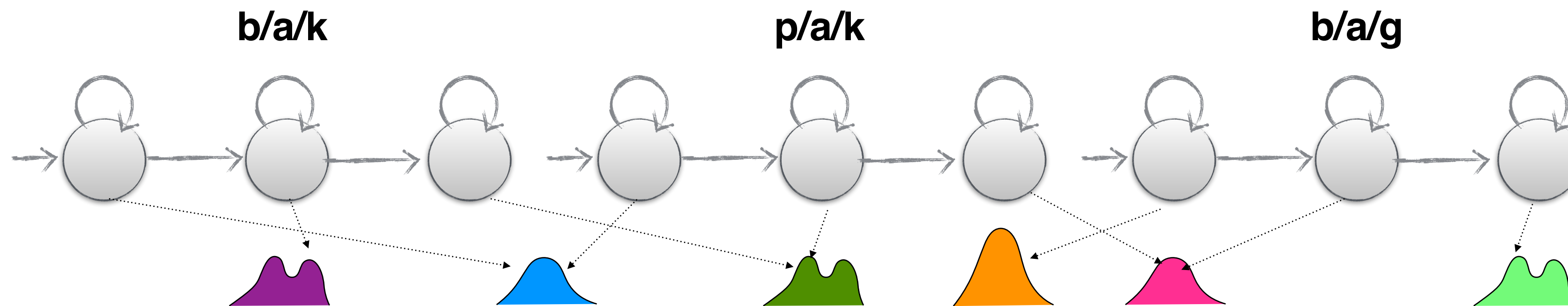
# **Tied-state Triphone Models**

# State Tying

- Observation probabilities are shared across triphone states which generate acoustically similar data



Triphone HMMs (No sharing)

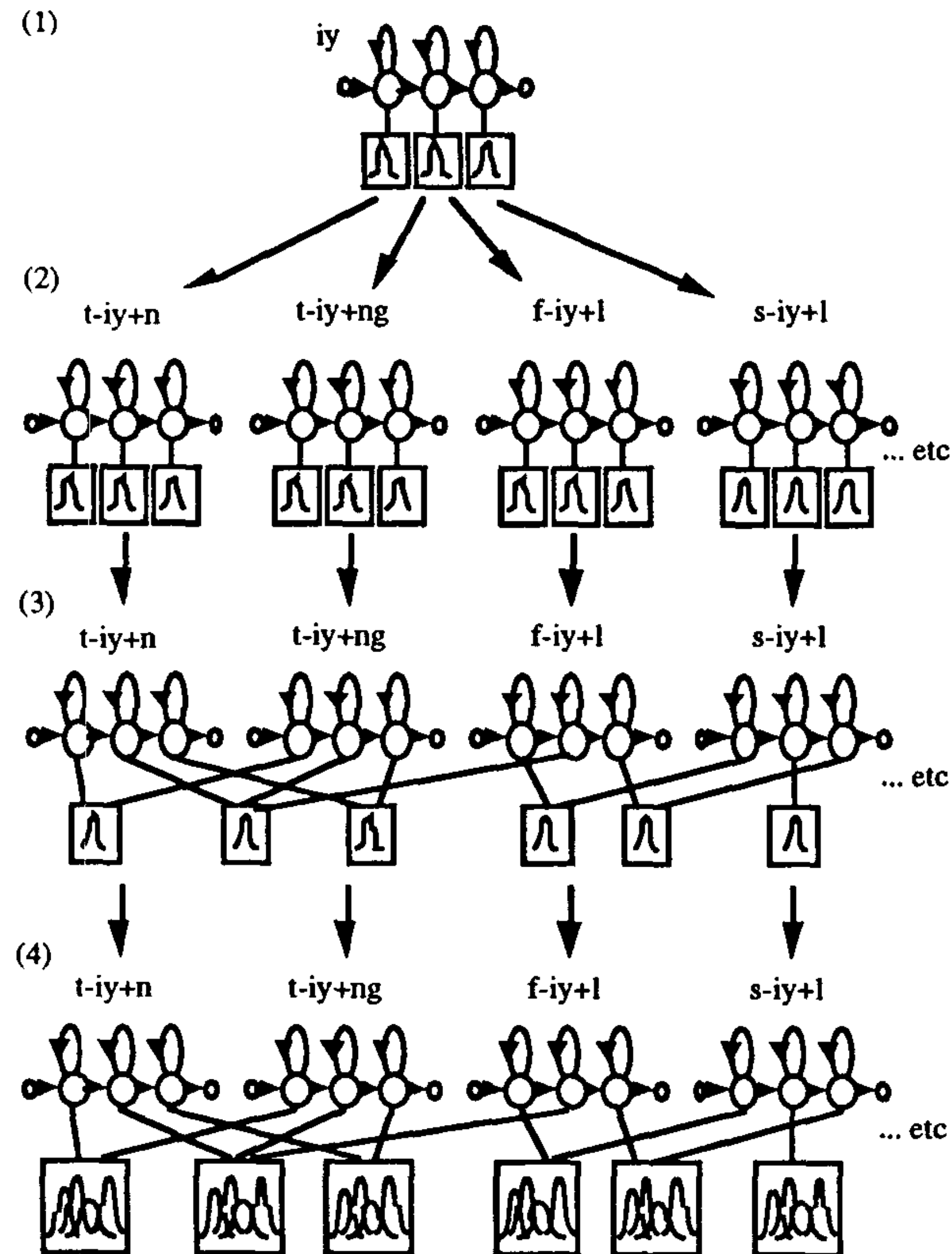


Triphone HMMs (State Tying)

# Tied state HMMs

Four main steps in building a tied state HMM system:

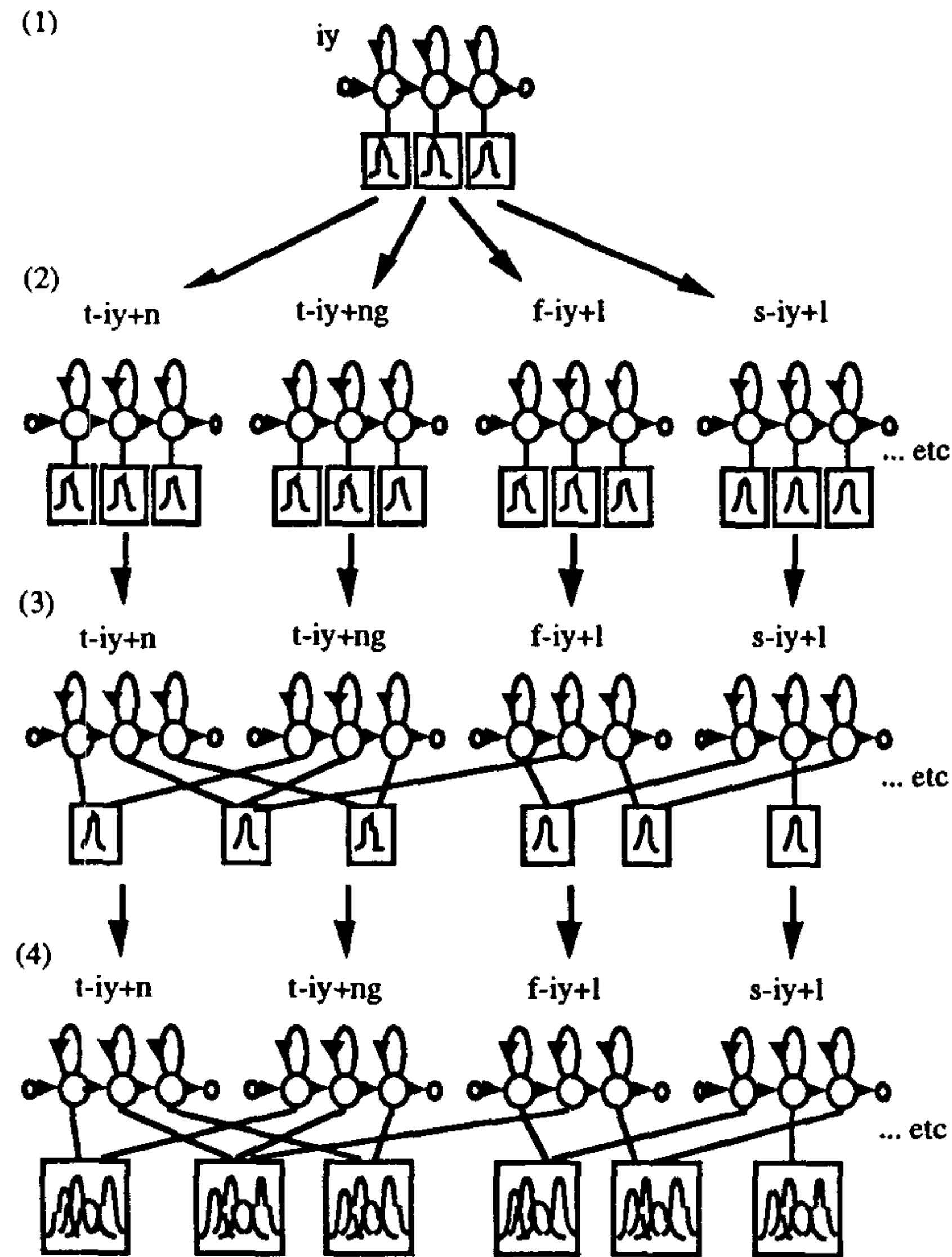
1. Create and train 3-state monophone HMMs with single Gaussian observation probability densities
2. Clone these monophone distributions to initialise a set of untied triphone models. Train them using Baum-Welch estimation. Transition matrix remains common across all triphones of each phone.
3. For all triphones derived from the same monophone, cluster states whose parameters should be tied together.
4. Number of mixture components in each tied state is increased and models re-estimated using BW





# Tied state HMMs

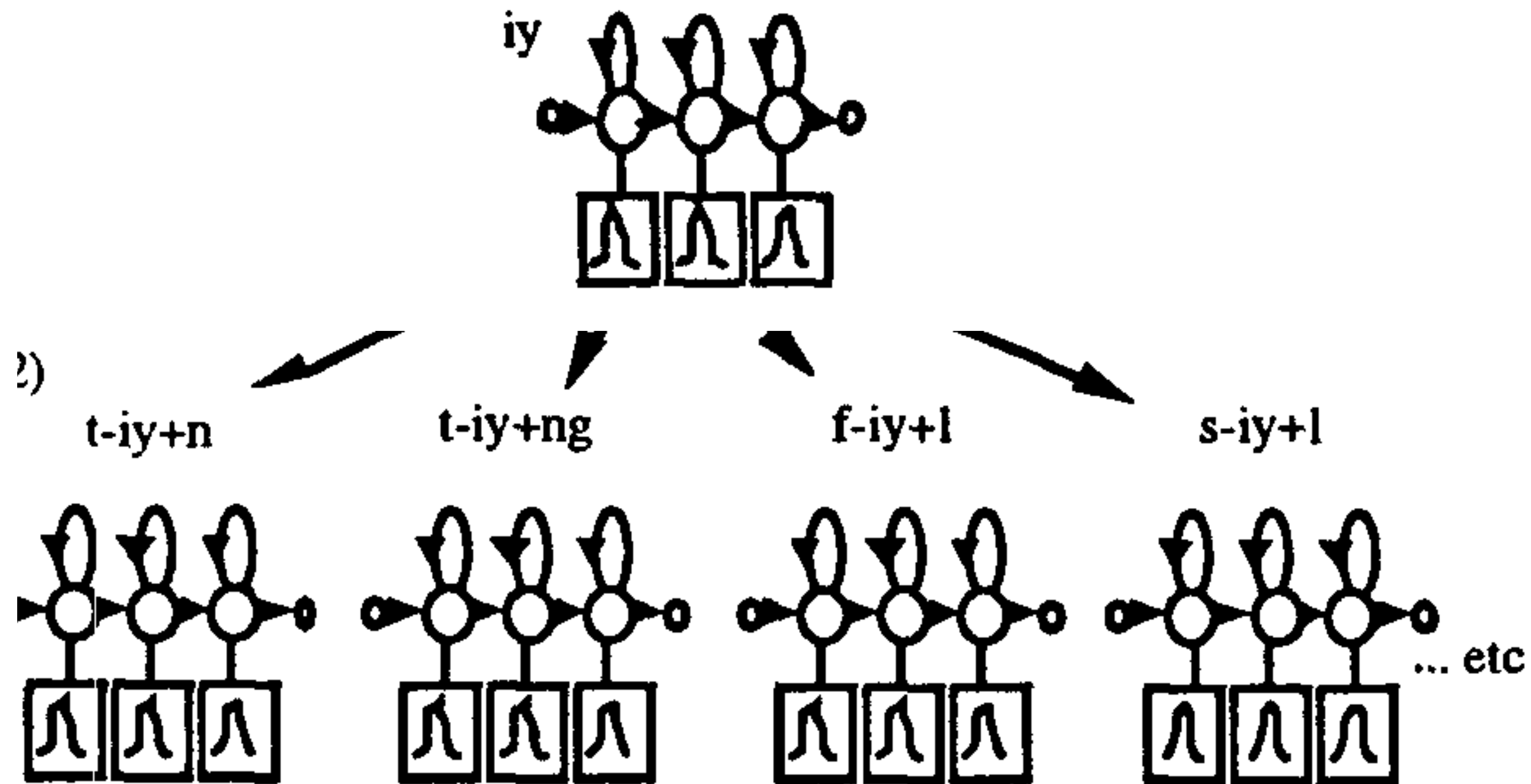
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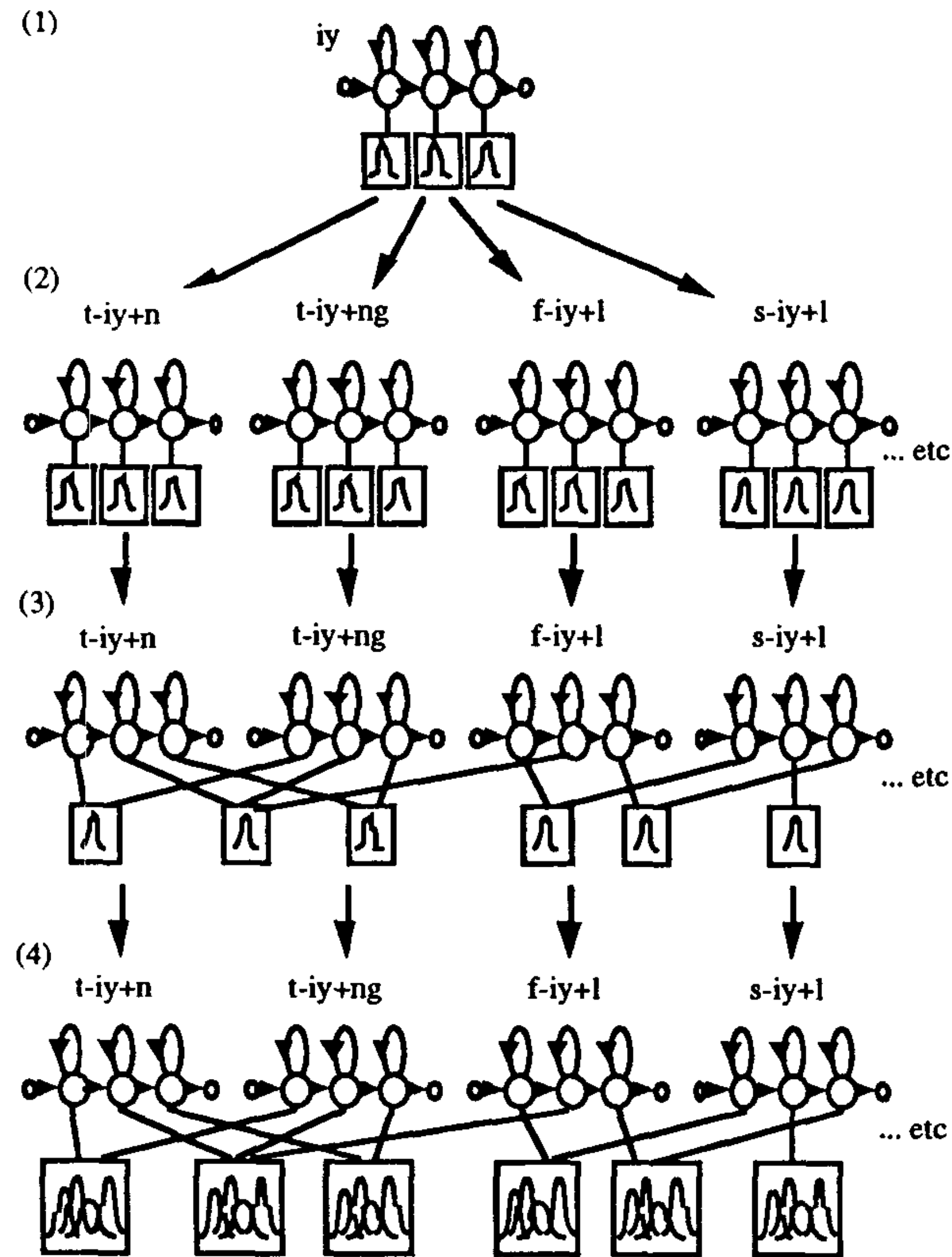
# Tied state HMMs: Step 2

Clone these monophone distributions to initialise a set of untied triphone models



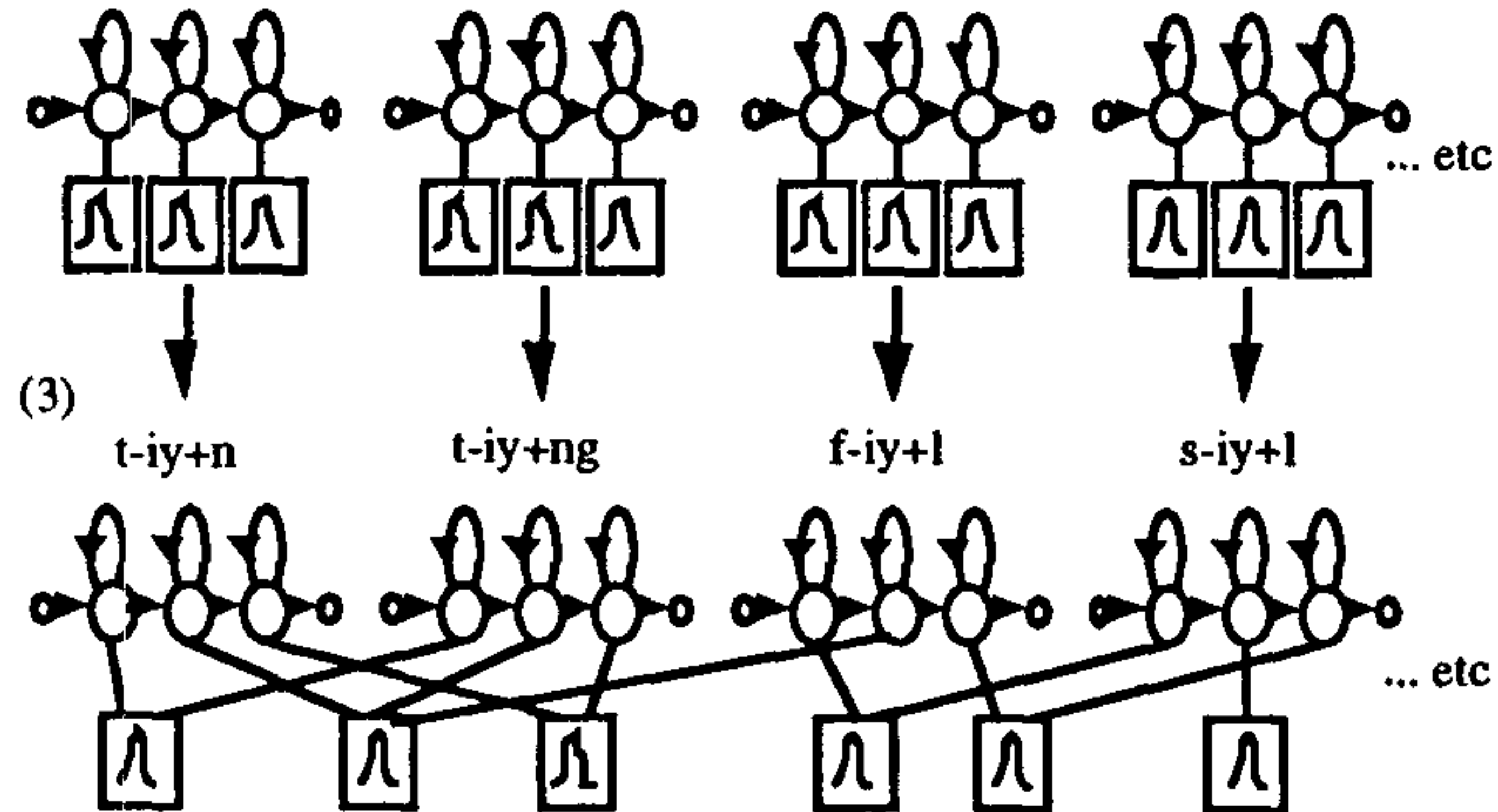
# Tied state HMMs

Four main steps in building a tied state HMM system:



1. Create and train 3-state monophone HMMs with single Gaussian observation probability densities
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# Tied state HMMs: Step 3



Use decision trees to determine which states should be tied together

# Example: Phonetic Decision Tree (DT)

One tree is constructed for each state of each monophone to cluster all the corresponding triphone states

Head node

*aa<sub>2</sub>/ow<sub>2</sub>/f<sub>2</sub>, aa<sub>2</sub>/ow<sub>2</sub>/s<sub>2</sub>,  
aa<sub>2</sub>/ow<sub>2</sub>/d<sub>2</sub>, h<sub>2</sub>/ow<sub>2</sub>/p<sub>2</sub>,  
aa<sub>2</sub>/ow<sub>2</sub>/n<sub>2</sub>, aa<sub>2</sub>/ow<sub>2</sub>/g<sub>2</sub>,  
...*

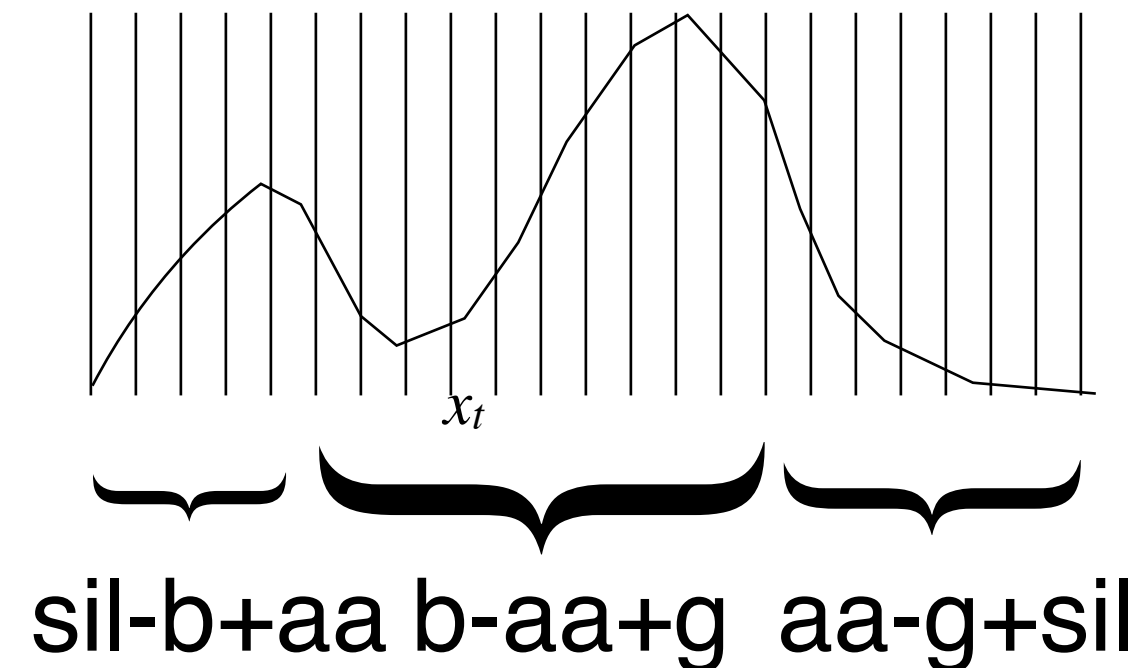


DT for center  
state of [ow]

Uses all training data  
tagged with \*-ow<sub>2</sub>+\*

# Training data for DT nodes

- Align training instance  $x = (x_1, \dots, x_T)$  where  $x_i \in \mathbb{R}^d$  with a set of triphone HMMs
- Use Viterbi algorithm to find the best HMM triphone state sequence corresponding to each  $x$
- Tag each  $x_t$  with ID of current phone along with left-context and right-context

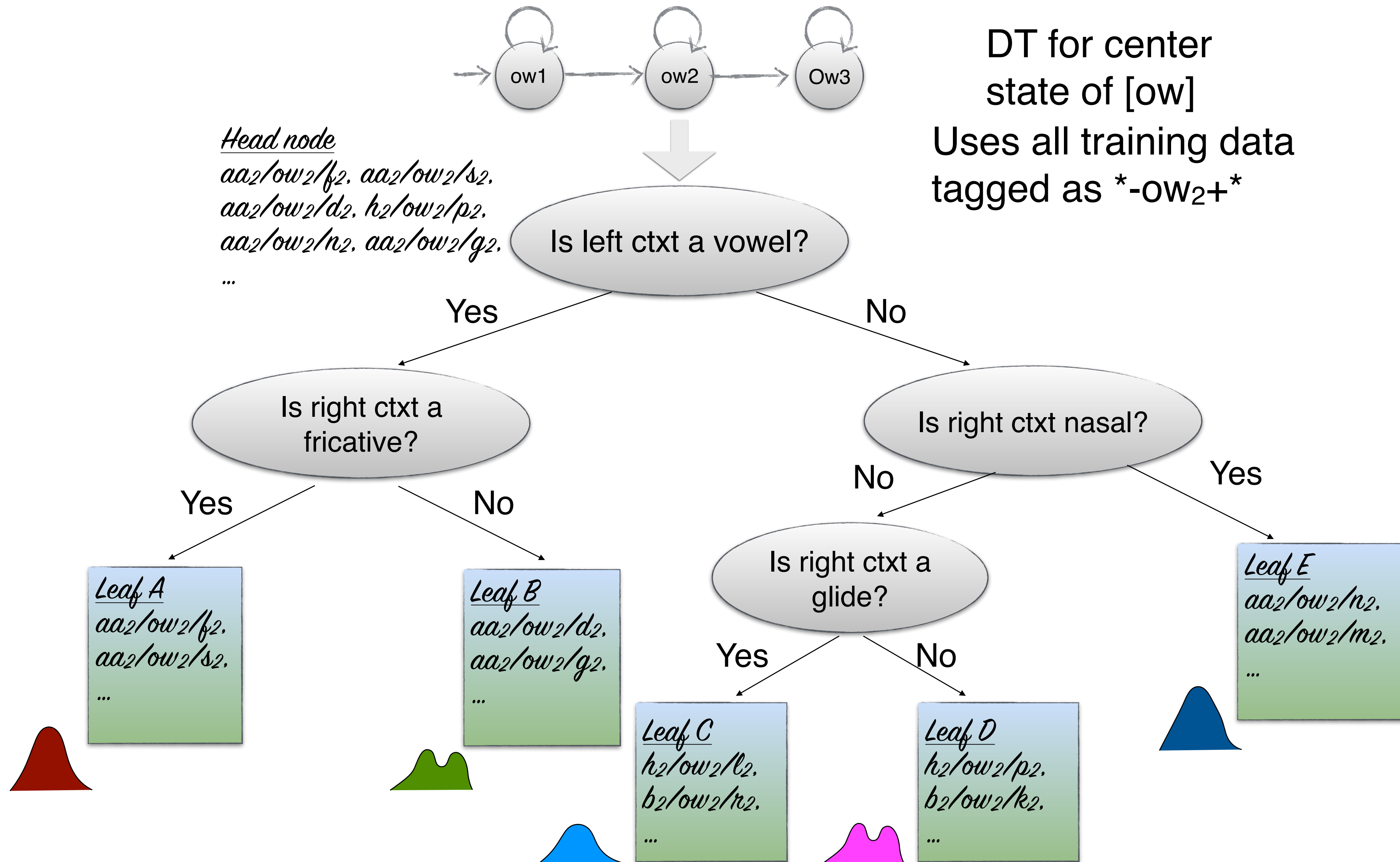


$x_t$  is tagged with ID  $b_2\text{-}aa_2\text{+}g_2$  i.e.  $x_t$  is aligned with the second state of the 3-state HMM corresponding to the triphone  $b\text{-}aa\text{+}g$

- Training data corresponding to state  $j$  in phone  $p$ : Gather all  $x_t$ 's that are tagged with ID  $^*\text{-}p_j\text{+}^*$

# Example: Phonetic Decision Tree (DT)

One tree is constructed for each state of each monophone to cluster all the corresponding triphone states



# How do we build these phone DTs?

## 1. What questions are used?

Linguistically-inspired binary questions: “Does the left or right phone come from a broad class of phones such as vowels, stops, etc.?” “Is the left or right phone [k] or [m]?”

## 2. What is the training data for each phone state, $p_j$ ? (root node of DT)

All speech frames that align with the  $j^{\text{th}}$  state of every triphone HMM that has  $p$  as the middle phone

## 3. What criterion is used at each node to find the best question to split the data on?

Find the question which partitions the states in the parent node so as to give the maximum increase in log likelihood



# Likelihood of a cluster of states

- If a cluster of HMM states,  $S = \{s_1, s_2, \dots, s_M\}$  consists of  $M$  states and a total of  $K$  acoustic observation vectors are associated with  $S$ ,  $\{x_1, x_2, \dots, x_K\}$ , then the log likelihood associated with  $S$  is:

$$\mathcal{L}(S) = \sum_{i=1}^K \sum_{s \in S} \log \Pr(x_i; \mu_S, \Sigma_S) \gamma_s(x_i)$$

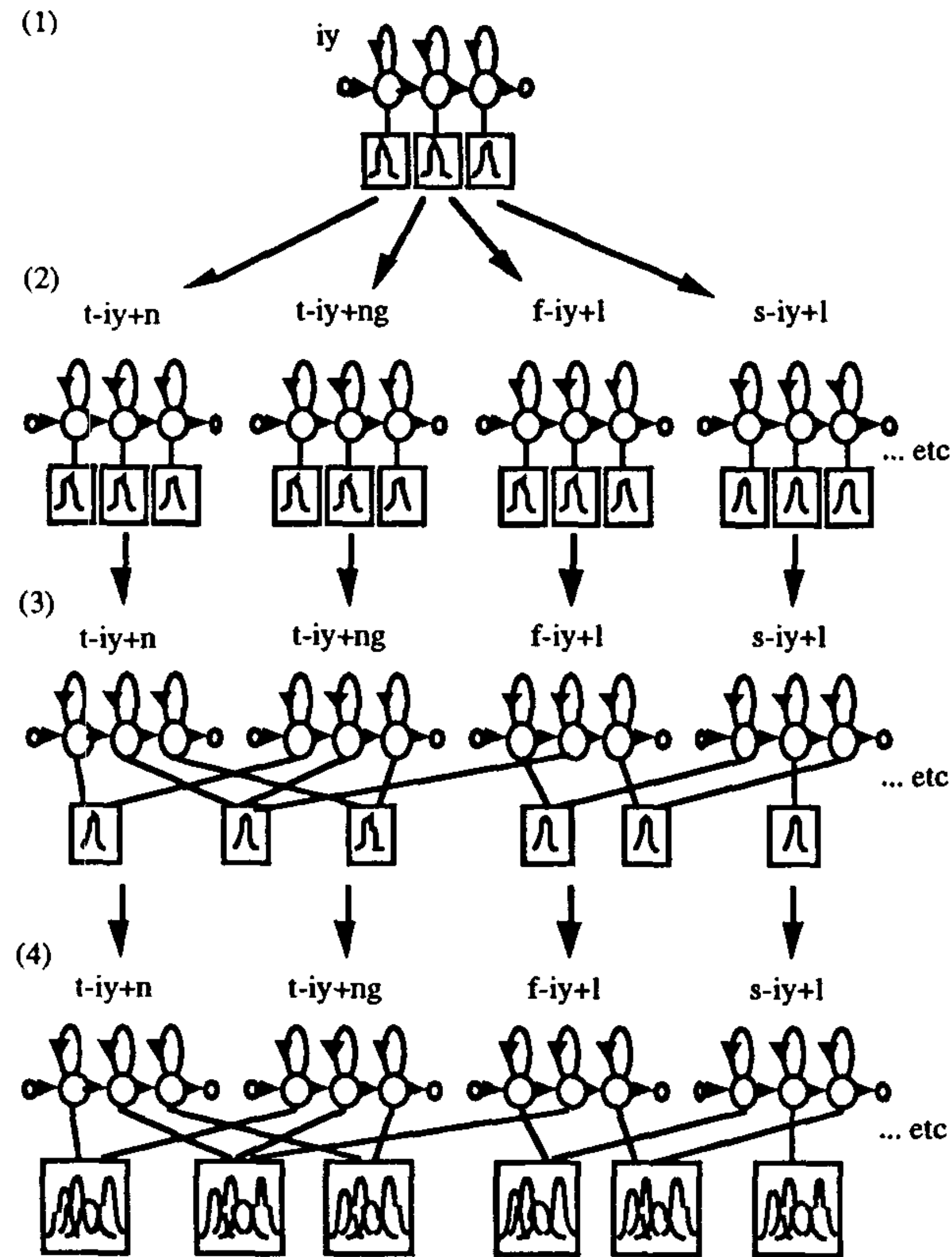
- For a question  $q$  that splits  $S$  into  $S_{\text{yes}}$  and  $S_{\text{no}}$ , compute the following quantity:

$$\Delta_q = \mathcal{L}(S_{\text{yes}}^q) + \mathcal{L}(S_{\text{no}}^q) - \mathcal{L}(S)$$

- Go through all questions, find  $\Delta_q$  for each question  $q$  and choose the question for which  $\Delta_q$  is the biggest
- Terminate when: Final  $\Delta_q$  is below a threshold or data associated with a split falls below a threshold

# Tied state HMMs

Four main steps in building a tied state HMM system:



1. Create and train 3-state monophone HMMs with single Gaussian observation probability densities
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# **WFSTs for ASR**

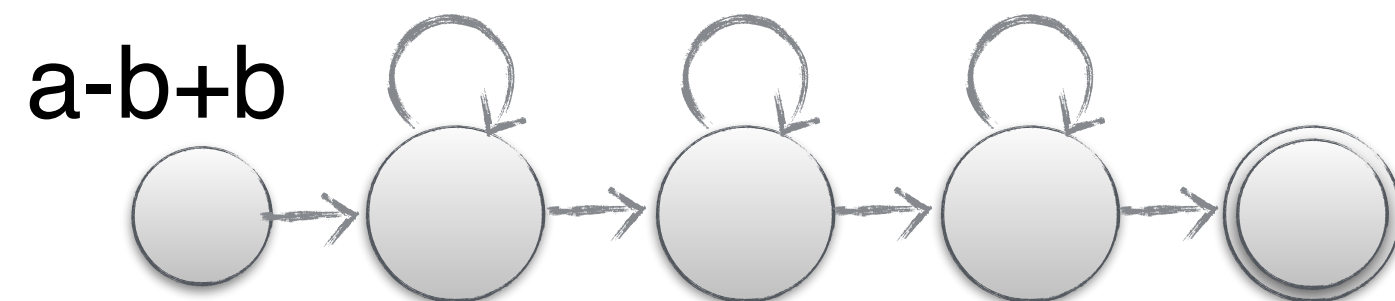
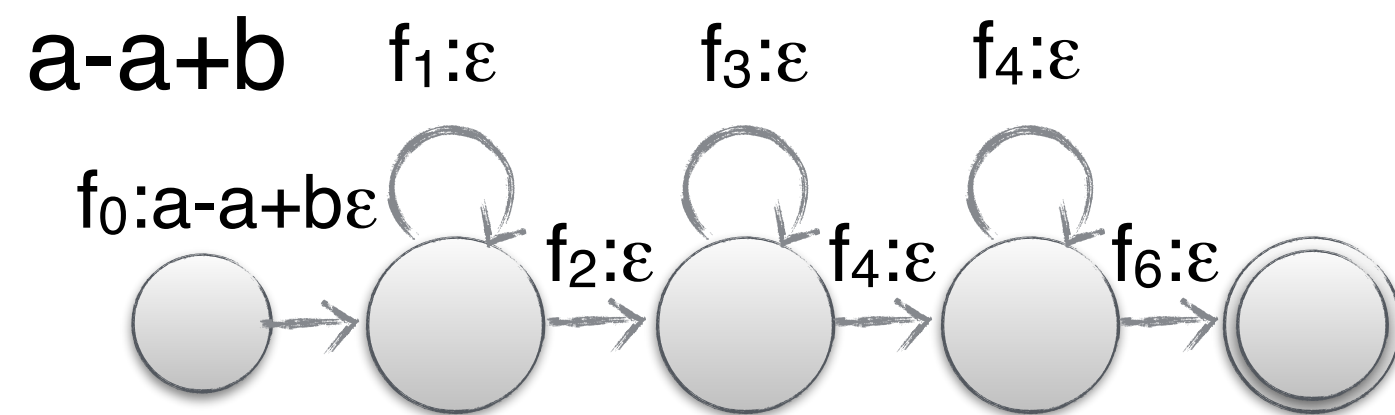
# WFST-based ASR System



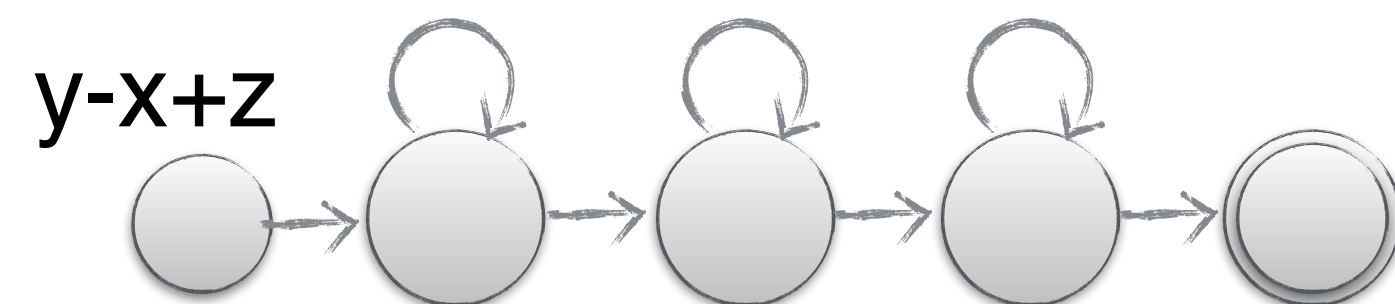
# WFST-based ASR System



H



⋮

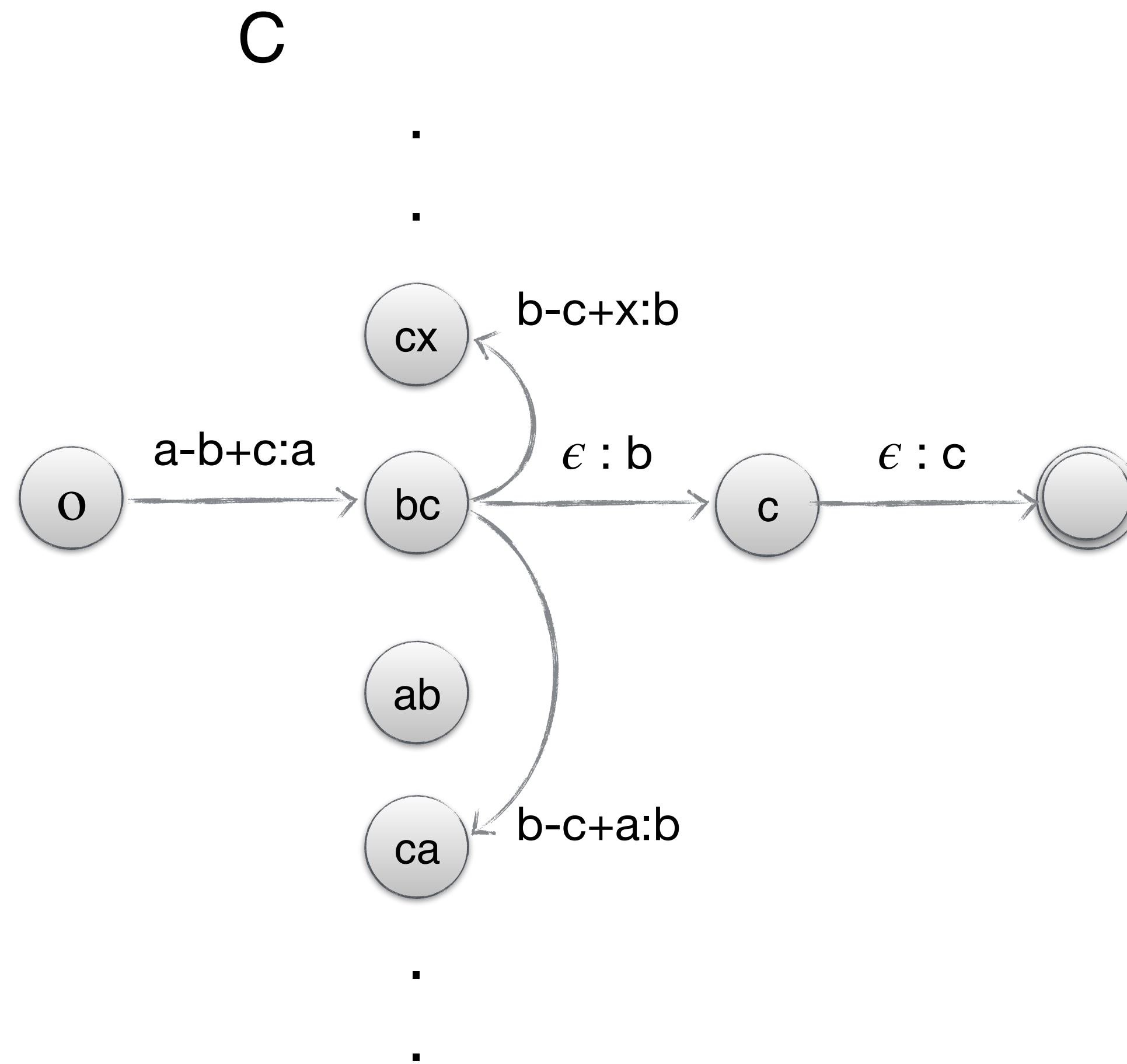
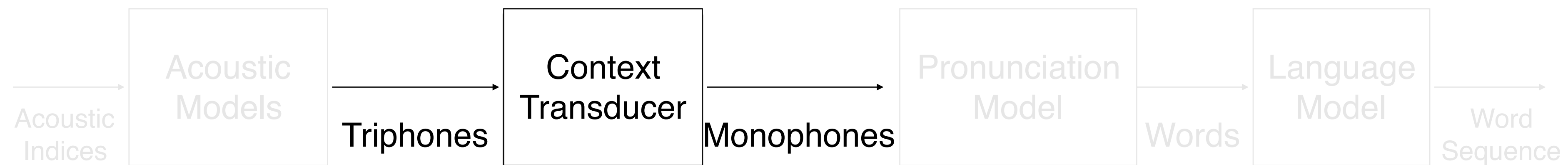


One 3-state  
HMM for  
each  
triphone

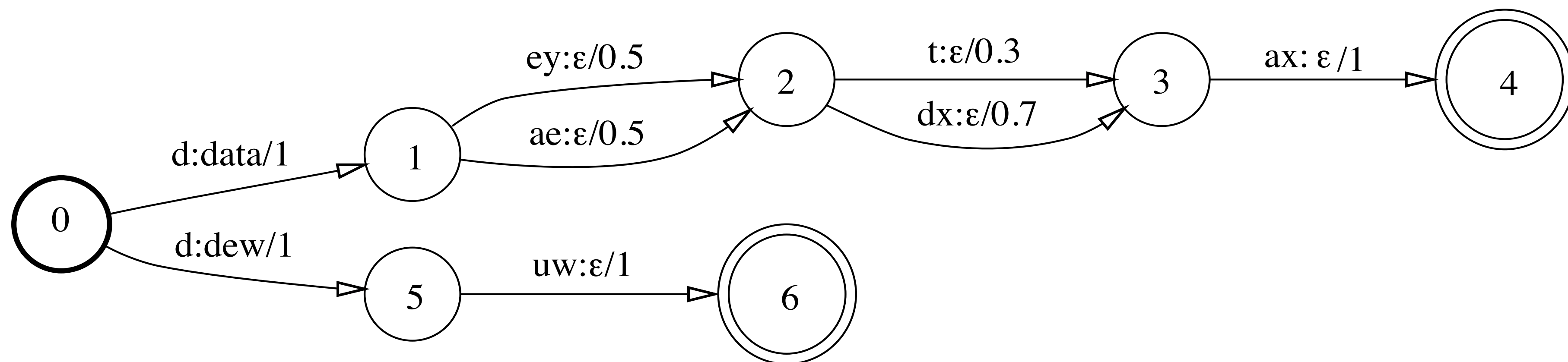
FST Union +  
Closure

Resulting  
FST  
H

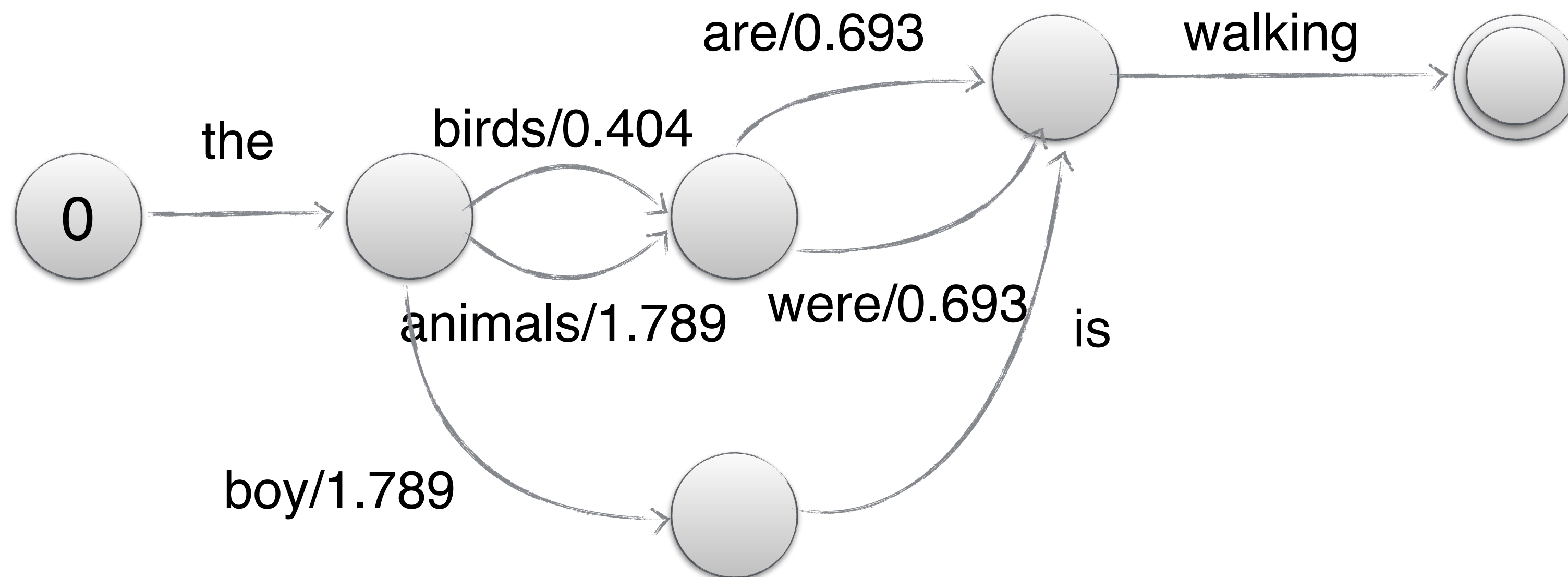
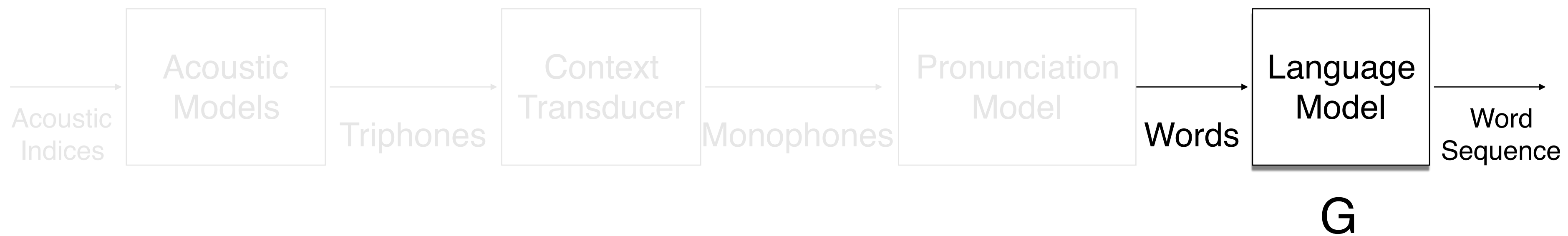
# WFST-based ASR System



# WFST-based ASR System

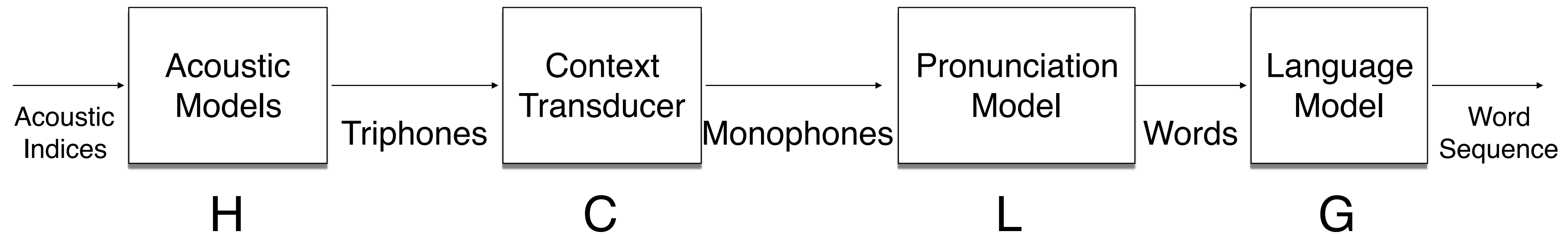


# WFST-based ASR System





# Decoding

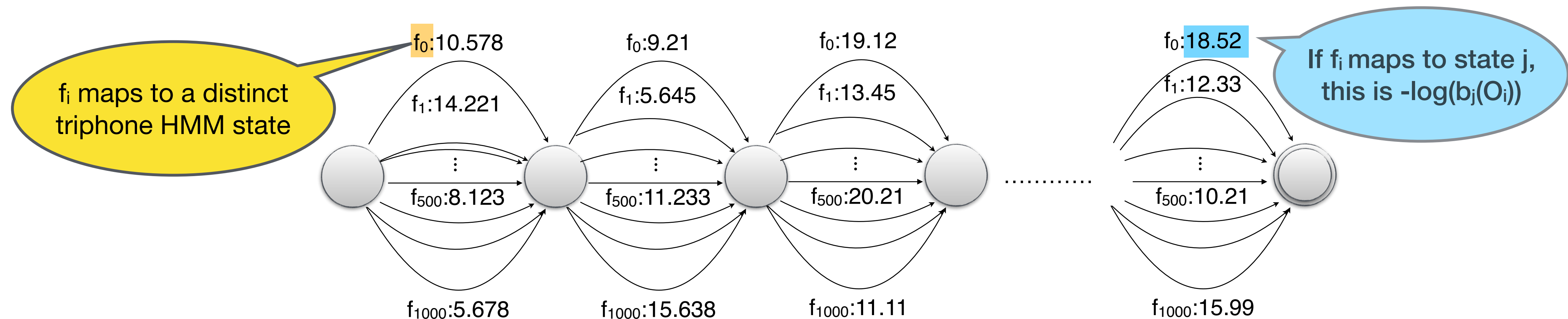


Carefully construct a decoding graph D using optimization algorithms:

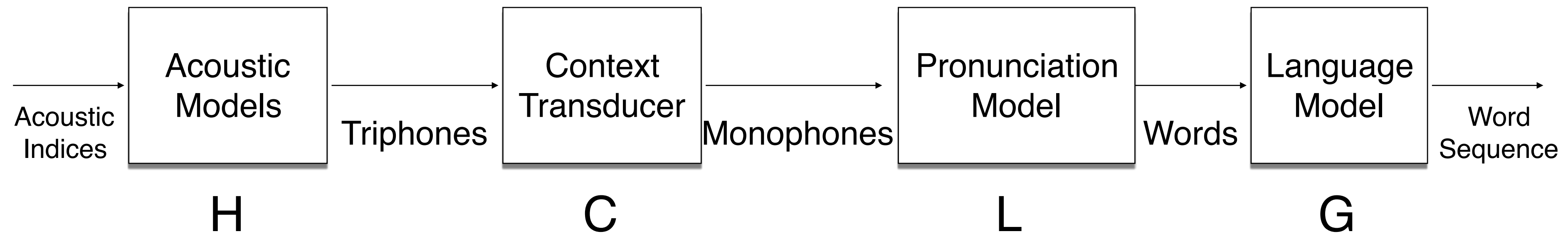
$$D = \min(\det(H \circ \det(C \circ \det(L \circ G))))$$

Given a test utterance O, how do I decode it?

Assuming ample compute, first construct the following machine X from O.



# Decoding

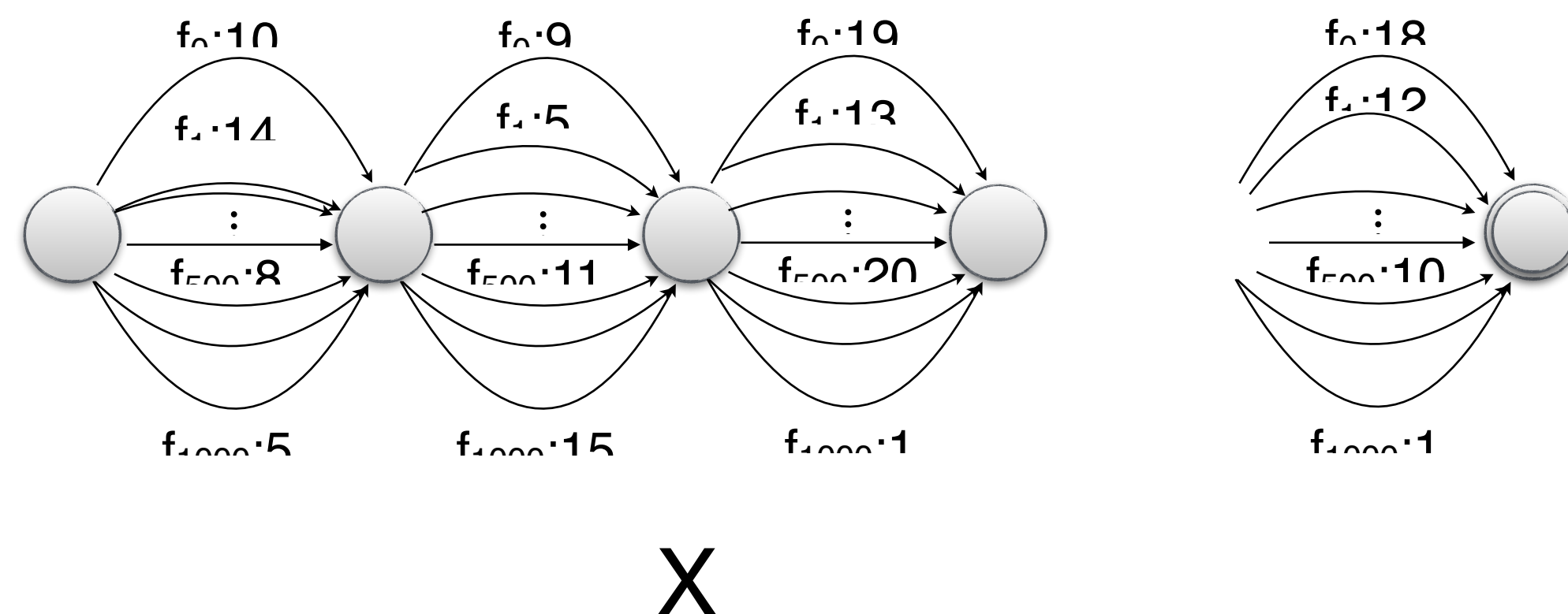


Carefully construct a decoding graph D using optimization algorithms:

$$D = \min(\det(H \circ \det(C \circ \det(L \circ G))))$$

Given a test utterance O, how do I decode it?

Assuming ample compute, first construct the following machine X from O.



$$W^* = \arg \min_{W=out[\pi]} X \circ D$$

where  $\pi$  is a path in the composed FST  
 $out[\pi]$  is the output label sequence of  $\pi$

X is never typically constructed;

D is traversed dynamically using approximate search algorithms  
 (discussed later in the semester)

# **Ngram LM Smoothing**

# Good-Turing Discounting

- Good-Turing discounting states that for any token that occurs  $r$  times, we should use an adjusted count  $r^* = \theta(r) = (r + 1)N_{r+1}/N_r$  where  $N_r$  is the number of tokens with  $r$  counts
- Good-Turing counts for unseen events:  $\theta(0) = N_1/N_0$
- For large  $r$ , many instances of  $N_{r+1} = 0$ .  
A solution: Smooth  $N_r$  using a best-fit power law  
once counts start getting small
- Good Turing discounting always used in conjunction with backoff or interpolation

# Katz Backoff Smoothing

- For a Katz bigram model, let us define:
- $\Psi(w_{i-1}) = \{w: \pi(w_{i-1}, w) > 0\}$
- A bigram model with Katz smoothing can be written in terms of a unigram model as follows:

$$P_{\text{Katz}}(w_i | w_{i-1}) = \begin{cases} \frac{\pi^*(w_{i-1}, w_i)}{\pi(w_{i-1})} & \text{if } w_i \in \Psi(w_{i-1}) \\ \alpha(w_{i-1}) P_{\text{Katz}}(w_i) & \text{if } w_i \notin \Psi(w_{i-1}) \end{cases}$$

$$\text{where } \alpha(w_{i-1}) = \frac{\left(1 - \sum_{w \in \Psi(w_{i-1})} \frac{\pi^*(w_{i-1}, w)}{\pi(w_{i-1})}\right)}{\sum_{w_i \notin \Psi(w_{i-1})} P_{\text{Katz}}(w_i)}$$

# Absolute Discounting Interpolation

- Absolute discounting motivated by Good-Turing estimation
- Just subtract a constant  $d$  from the non-zero counts to get the discounted count
- Also involves linear interpolation with lower-order models

$$\text{Pr}_{\text{abs}}(w_i|w_{i-1}) = \frac{\max\{\pi(w_{i-1}, w_i) - d, 0\}}{\pi(w_{i-1})} + \lambda(w_{i-1})\text{Pr}(w_i)$$

- However, interpolation with unigram probabilities has its limitations
- Cue in, Kneser-Ney smoothing that replaces unigram probabilities (how often does the word occur) with continuation probabilities (how often is the word a continuation)

# Kneser-Ney discounting

$$\text{Pr}_{\text{KN}}(w_i|w_{i-1}) = \frac{\max\{\pi(w_{i-1}, w_i) - d, 0\}}{\pi(w_{i-1})} + \lambda_{\text{KN}}(w_{i-1})\text{Pr}_{\text{cont}}(w_i)$$

Consider an example: “Today I cooked some yellow curry”

Suppose  $\pi(\text{yellow}, \text{curry}) = 0$ .  $\text{Pr}_{\text{abs}}[w \mid \text{yellow}] = \lambda(\text{yellow})\text{Pr}(w)$

Now, say  $\text{Pr}[\text{Francisco}] \gg \text{Pr}[\text{curry}]$ , as San Francisco is very common in our corpus.

But Francisco is not as common a “continuation” (follows only San) as curry is (red curry, chicken curry, potato curry, ...)

Moral: Should use probability of being a continuation!

c.f., absolute discounting

$$\text{Pr}_{\text{abs}}(w_i|w_{i-1}) = \frac{\max\{\pi(w_{i-1}, w_i) - d, 0\}}{\pi(w_{i-1})} + \lambda(w_{i-1})\text{Pr}(w_i)$$

# Kneser-Ney discounting

$$\Pr_{\text{KN}}(w_i|w_{i-1}) = \frac{\max\{\pi(w_{i-1}, w_i) - d, 0\}}{\pi(w_{i-1})} + \lambda_{\text{KN}}(w_{i-1})\Pr_{\text{cont}}(w_i)$$

$$\Pr_{\text{cont}}(w_i) = \frac{|\Phi(w_i)|}{|B|} \quad \text{and} \quad \lambda_{\text{KN}}(w_{i-1}) = \frac{d}{\pi(w_{i-1})} |\Psi(w_{i-1})|$$

where

$$\Phi(w_i) = \{w_{i-1} : \pi(w_{i-1}, w_i) > 0\}$$
$$B = \{(w_{i-1}, w_i) : \pi(w_{i-1}, w_i) > 0\}$$

$$\frac{d \cdot |\Psi(w_{i-1})| \cdot |\Phi(w_i)|}{\pi(w_{i-1}) \cdot |B|}$$

c.f., absolute discounting

$$\Pr_{\text{abs}}(w_i|w_{i-1}) = \frac{\max\{\pi(w_{i-1}, w_i) - d, 0\}}{\pi(w_{i-1})} + \lambda(w_{i-1})\Pr(w_i)$$



# Midsem Exam

- September 17th, 2019 (Tuesday)
- Time: 8.30 am to 10.30 am
- Venue: CC 101, 103 and 105
- Closed book exam. Will allow 1 A4 (two-sided) sheet of notes.
- Can bring calculators to the exam hall.

# Midsem Syllabus

- HMMs (Forward/Viterbi/Baum-Welch (EM) algorithms)
- Tied-state HMM models
- WFST algorithms
- WFSTs in ASR
- Feedforward NN-based acoustic models (Hybrid/Tandem/TDNNs)
- Language modeling (Ngram models + Smoothing techniques)
- There could be (no more than) one question on basic probability
  
- Topics covered in class that won't appear in the exam:
  - Basics of speech production
  - Role of epsilon filters in composition
  - RNN-based models

# Question 1: Phone recogniser

Suppose you are building a simple ASR system which recognizes only four words bowl, bore, pour, poll involving five phones p, b, ow, l, r (with obvious pronunciations for the words). We are given a phone recognizer which converts a spoken word into a sequence of phones, which is known to have the following behaviour:

Phone	p	ow	r	b	l
p	0.8	0	0	0.2	0
ow	0	1	0	0	0
r	0	0	0.6	0	0.4
b	0.2	0	0	0.8	0
l	0	0	0.4	0	0.6

The probability of recognizing a spoken phone  $x$  as a phone  $y$  is given in the row labeled by  $x$  and the column labeled by  $y$ . Let us assume a simple language model for our task:  $\Pr(\text{bowl}) = 0.1$ ,  $\Pr(\text{bore}) = 0.4$ ,  $\Pr(\text{pour}) = 0.3$  and  $\Pr(\text{poll}) = 0.2$ . Determine the most likely word (and the corresponding probability) given that the output from the phone recognizer is “p ow l”.

## Question 2: WFSTs for ASR

Recall the WFST-based framework for ASR that was described in class. Given a test utterance  $x$ , let  $D_x$  be a WFST over the tropical semiring (with weights specialized to the given utterance) such that decoding the utterance corresponds to finding the shortest path in  $D_x$ . Suppose we modify  $D_x$  by adding  $\gamma$  ( $> 0$ ) to each arc in  $D_x$  that emits a word. Let's call the resulting WFST  $D'_x$ .

A) Describe informally, what effect increasing  $\gamma$  would have on the word sequence obtained by decoding  $D'_x$ .

B) Recall that decoding  $D_x$  was used as an approximation for  $\arg\max \Pr(x|W) \Pr(W)$ . What would be the analogous expression for decoding from  $D'_x$ ?

## Question 3: FSTs in ASR

Words in a language can be composed of sub-word units called morphemes. For simplicity, in this problem, we consider there to be three sets of morphemes,  $V_{\text{pre}}$ ,  $V_{\text{stem}}$  and  $V_{\text{suf}}$  – corresponding to prefixes, stems and suffixes. Further, we will assume that every word consists of a single stem, and zero or more prefixes and suffixes. That is, a word is of the form  $w = p_1 \cdots p_k \sigma s_1 \cdots s_l$  where  $k, l \geq 0$ , and  $p_i \in V_{\text{pre}}$ ,  $s_i \in V_{\text{suf}}$  and  $\sigma \in V_{\text{stem}}$ . For example, a word like fair consists of a single morpheme (a stem), where as the word unfairness is composed of three morphemes, un + fair + ness, which are a prefix, a stem and a suffix, respectively.

- A) Suppose we want to build an ASR system for a language using morphemes instead of words as the basic units of language. Which WFST(s) in the  $H \circ C \circ L \circ G$  framework should be modified in order to utilize morphemes?
- B) Draw an FSA over morphemes ( $V_{\text{pre}} \cup V_{\text{stem}} \cup V_{\text{suf}}$ ) that accepts only words with at most four morphemes. Your FSA should not have more than 15 states. You may draw a single arc labeled with a set to indicate a collection of arcs, each labeled with an element in the set.

## Question 4: Probabilities in HMMs

	a	b	c
q <sub>1</sub>	0.5	0.3	0.2
q <sub>2</sub>	0.3	0.4	0.3
q <sub>3</sub>	0.2	0.1	0.7
q <sub>4</sub>	0.4	0.5	0.1
q <sub>5</sub>	0.3	0.3	0.4
q <sub>6</sub>	0.9	0	0.1

Consider the HMM shown in the figure. (The transition probabilities are shown in the finite-state machine and the observation probabilities corresponding to each state are shown on the left.) This model generates hidden state sequences and observation sequences of length 4. If  $S_1, S_2, S_3, S_4$  represent the hidden states and  $O_1, O_2, O_3, O_4$  represent the observations, then  $S_i \in \{q_1, \dots, q_6\}$  and  $O_i \in \{a, b, c\}$ .  $\Pr(S_1 = q_1) = 1$  i.e. the state sequence starts in  $q_1$ .

State whether the following three statements are true or false and justify your responses. If the statement is false, then state how the left expression is related to the right expression, using either  $=, <$  or  $>$  operators. (We use the following shorthand in the statements below:  $\Pr(O = abbc)$  denotes  $\Pr(O_1 = a, O_2 = b, O_3 = b, O_4 = c)$ )

A)  $\Pr(O = bbca, S_1 = q_1, S_4 = q_6) = \Pr(O = bbca \mid S_1 = q_1, S_4 = q_6)$

B)  $\Pr(O = acac, S_2 = q_2, S_3 = q_5) > \Pr(O = acac, S_2 = q_4, S_3 = q_3)$

C)  $\Pr(O = cbc b \mid S_2 = q_2, S_3 = q_5) = \Pr(O = baac, S_2 = q_4, S_3 = q_5)$

## Question 5: HMM training

Suppose we are given  $N$  observation sequences,  $X_i$ ,  $i = 1$  to  $N$  where each  $X_i$  is a sequence  $(x_i^1, \dots, x_i^{T_i})$  of length  $T_i$  where  $x_i^t$  is an acoustic vector  $\in \mathbb{R}^d$ . To estimate the parameters of an HMM with Gaussian output probabilities from this data, the Baum-Welch EM algorithm uses empirical estimates  $\xi_{i,t}(s, s')$  for the probability of being in state  $s$  at time  $t$  and  $s'$  at time  $t + 1$  given the observation sequence  $X_i$  and  $\gamma_{i,t}(s)$  for the probability of occupying state  $s$  at time  $t$  given  $X_i$ .

In a variant of EM known as Viterbi training, for each  $i$ , one computes the single most likely state sequence  $S_i^1, \dots, S_i^{T_i}$  for  $X_i$  by Viterbi decoding, and defines  $\xi_{i,t}$  and  $\gamma_{i,t}$  assuming that  $X_i$  was produced deterministically by this path. Give the expressions for  $\xi_{i,t}(s, s')$  and  $\gamma_{i,t}(s)$  in this case.