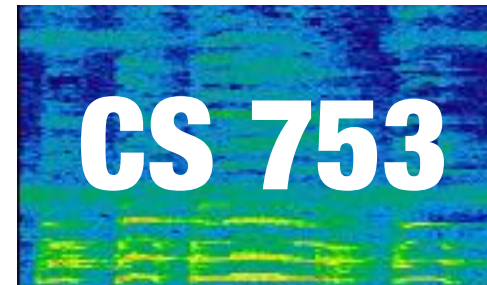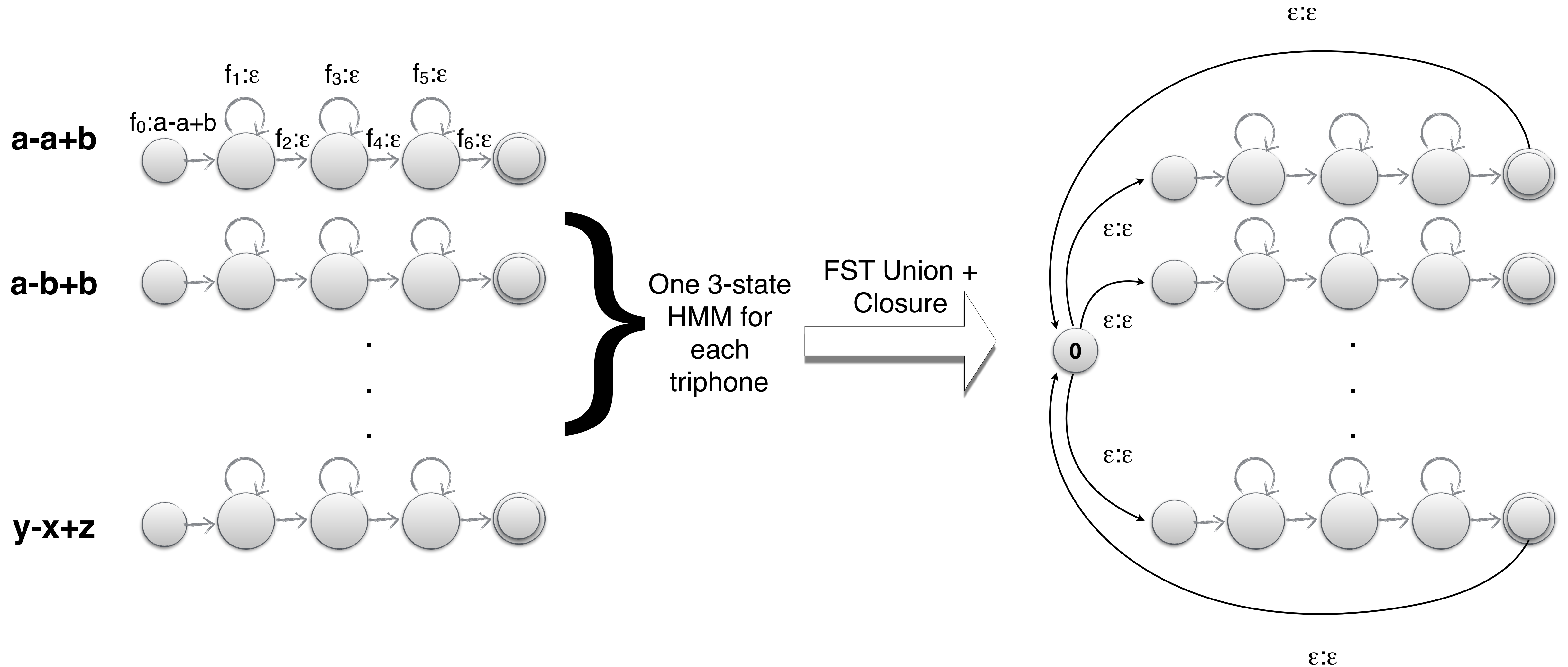# WFSTs continued

Lecture 5
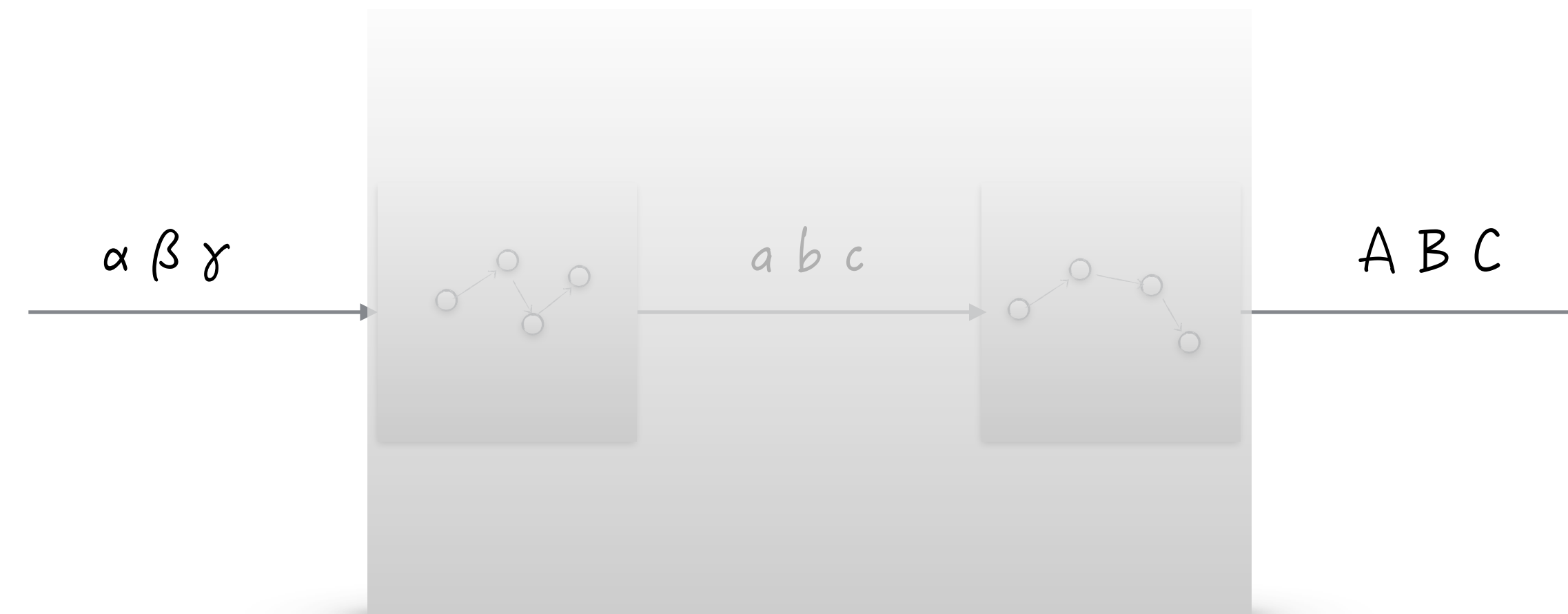

Instructor: Preethi Jyothi

Recap: Construct acoustic Model WFST using simple rational operations

Recap: Composition

α β γ   a b c   A B C
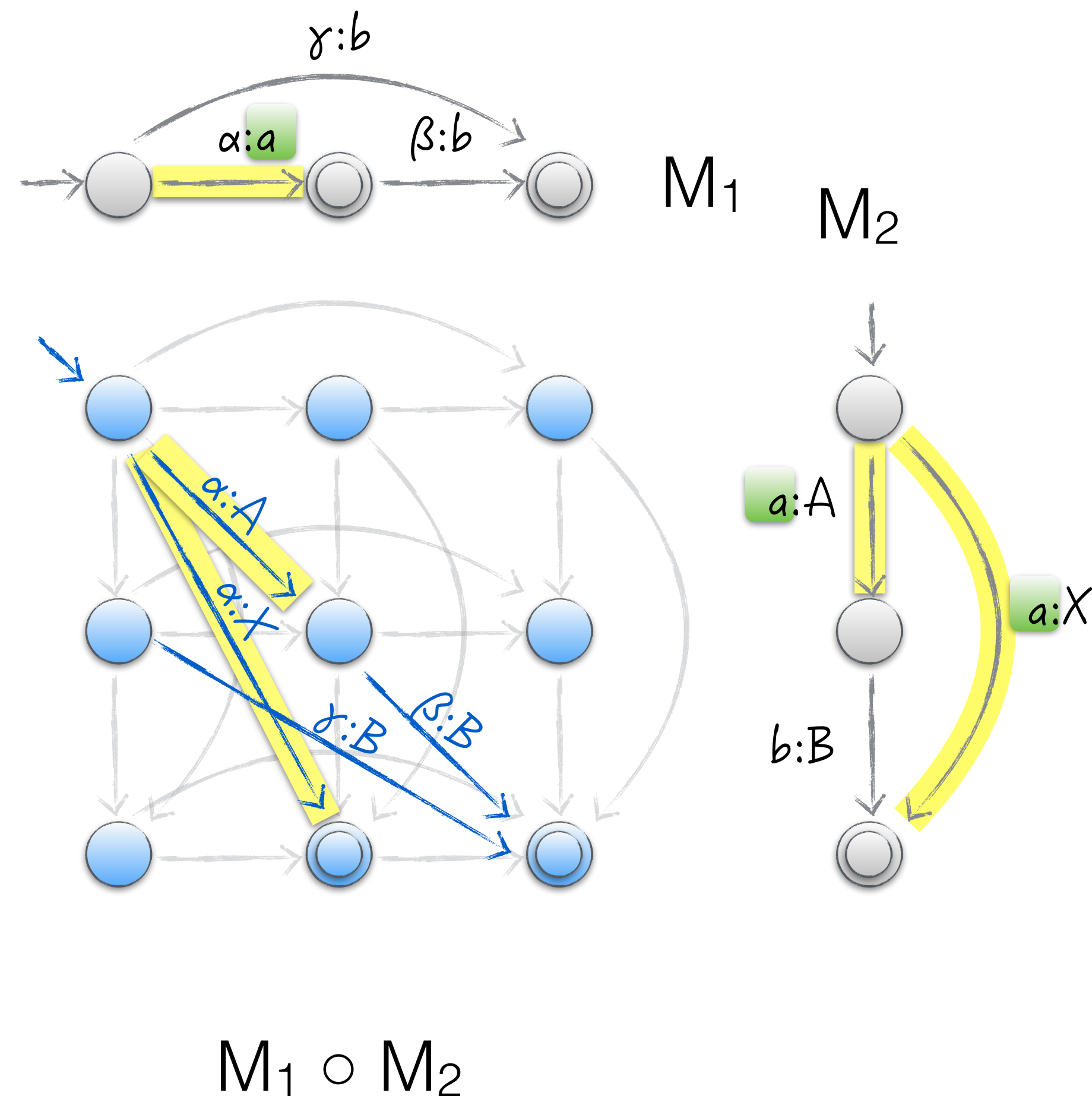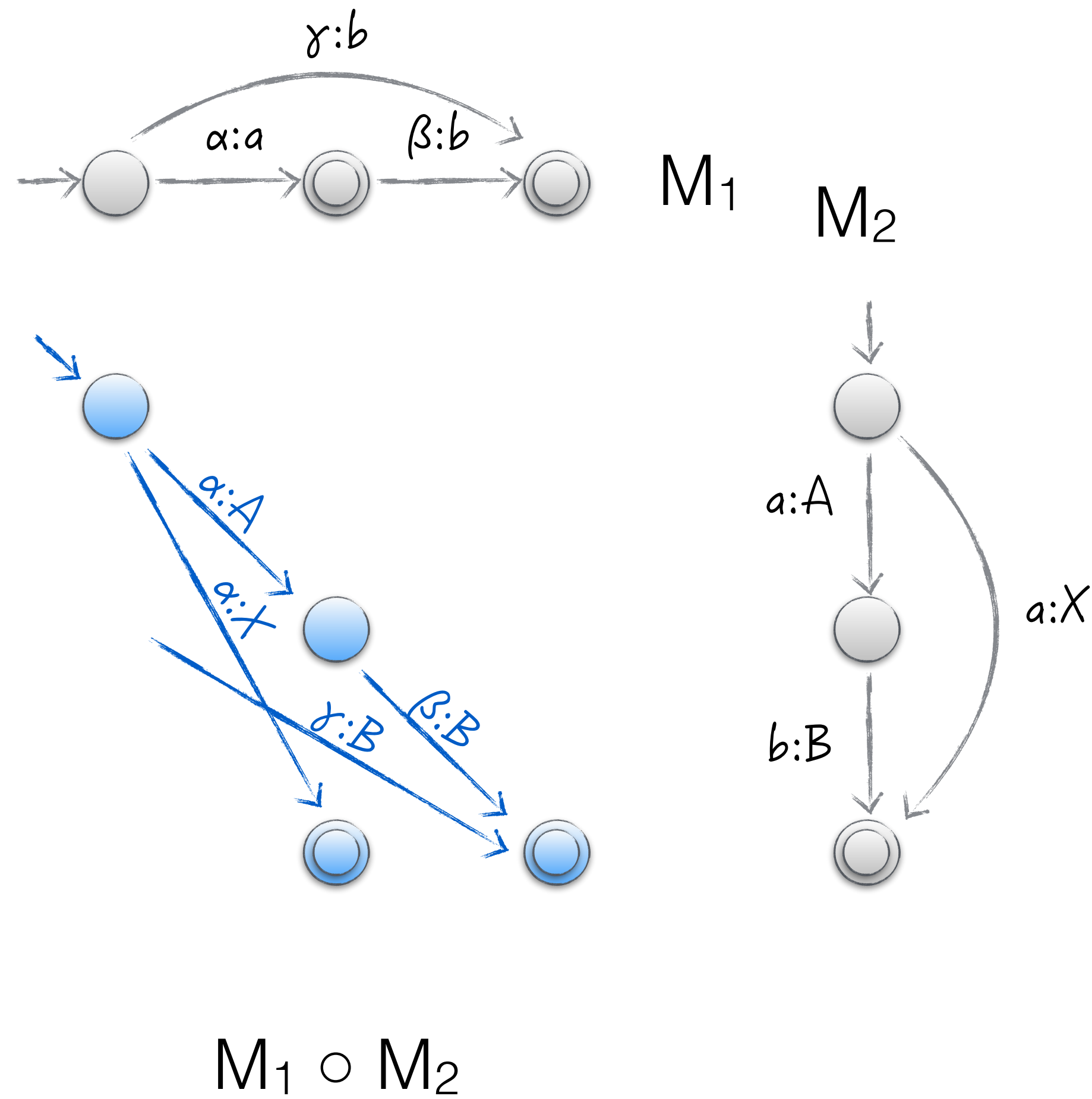
- If T_1 transduces x to z, and T_2 transduces z to y, then $T_1 \circ T_2$ transduces x to y

- $(T_1 \circ T_2)(x, y) = \bigoplus_z T_1(x, z) \otimes T_2(z, y)$
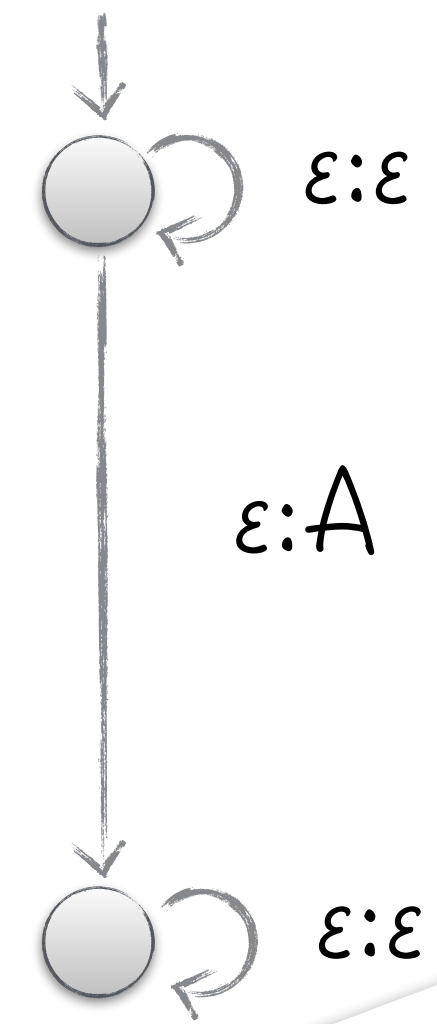
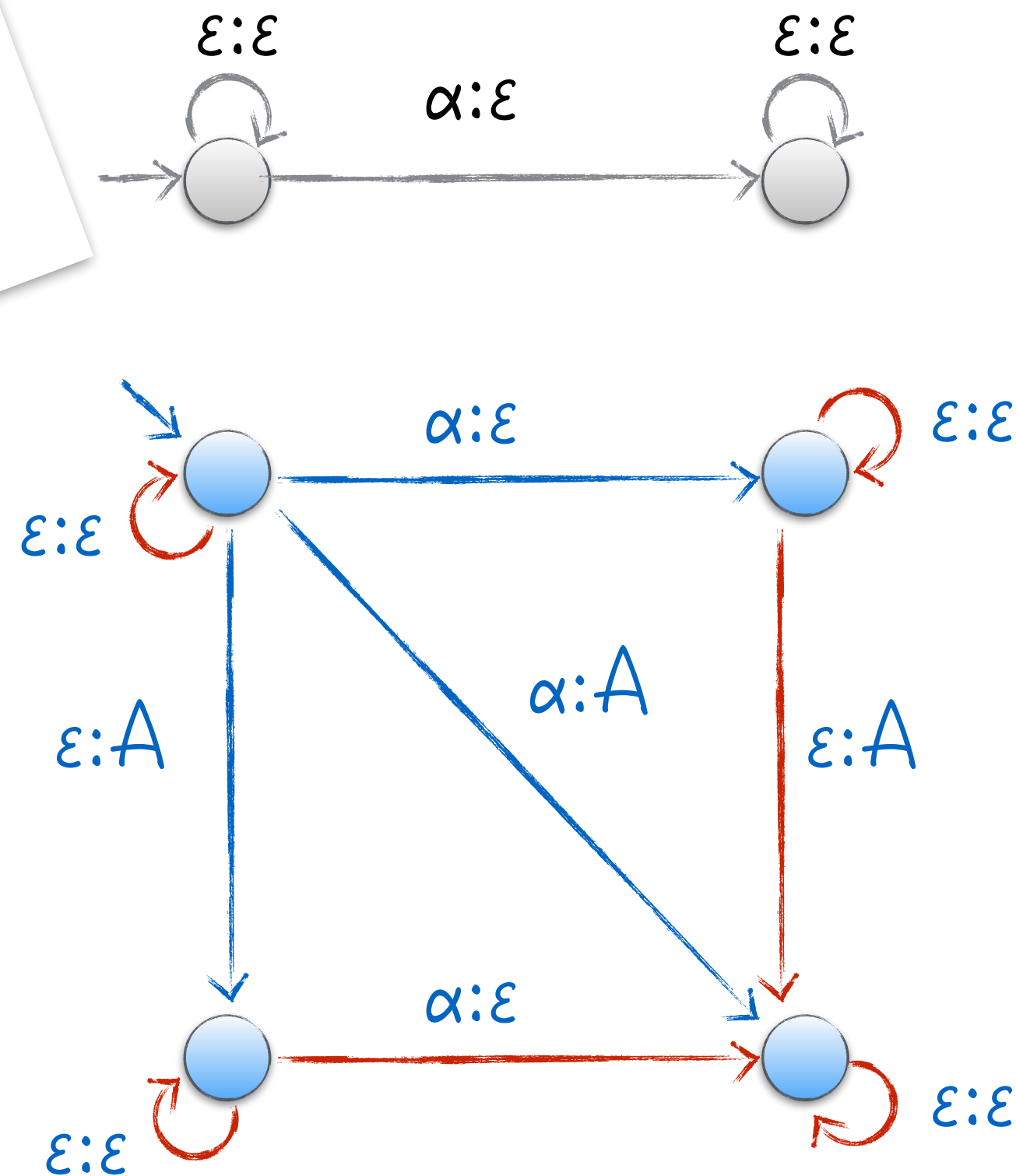# Composition: Construction

M_1 M_2

$M_1 \circ M_2$

Composition

$\gamma{:}b$

$\alpha{:}a$ $\beta{:}b$

M₁   M₂

$\alpha{:}A$

$\alpha{:}X$

$\gamma{:}B$ $\beta{:}B$

$a{:}A$

$a{:}X$

$b{:}B$

M₁ ○ M₂

Composition: Handling epsilons

Composition: Filters

Problematic paths:
$\varepsilon_1\varepsilon_2$ output from M'_1
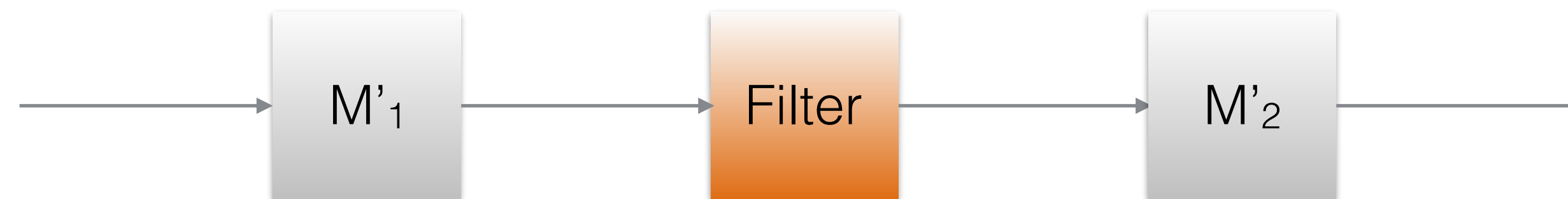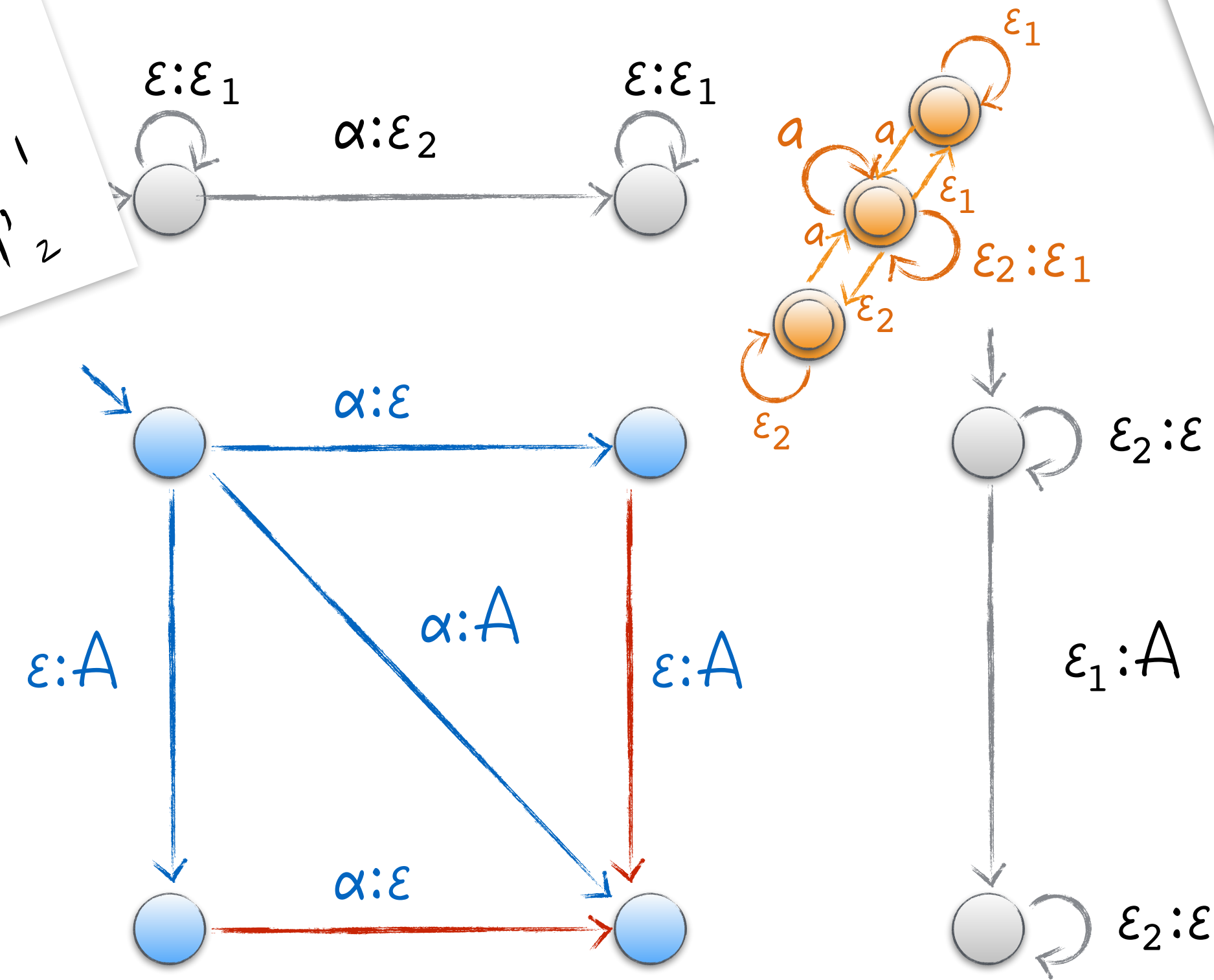and $\varepsilon_2\varepsilon_1$ input to M'_2
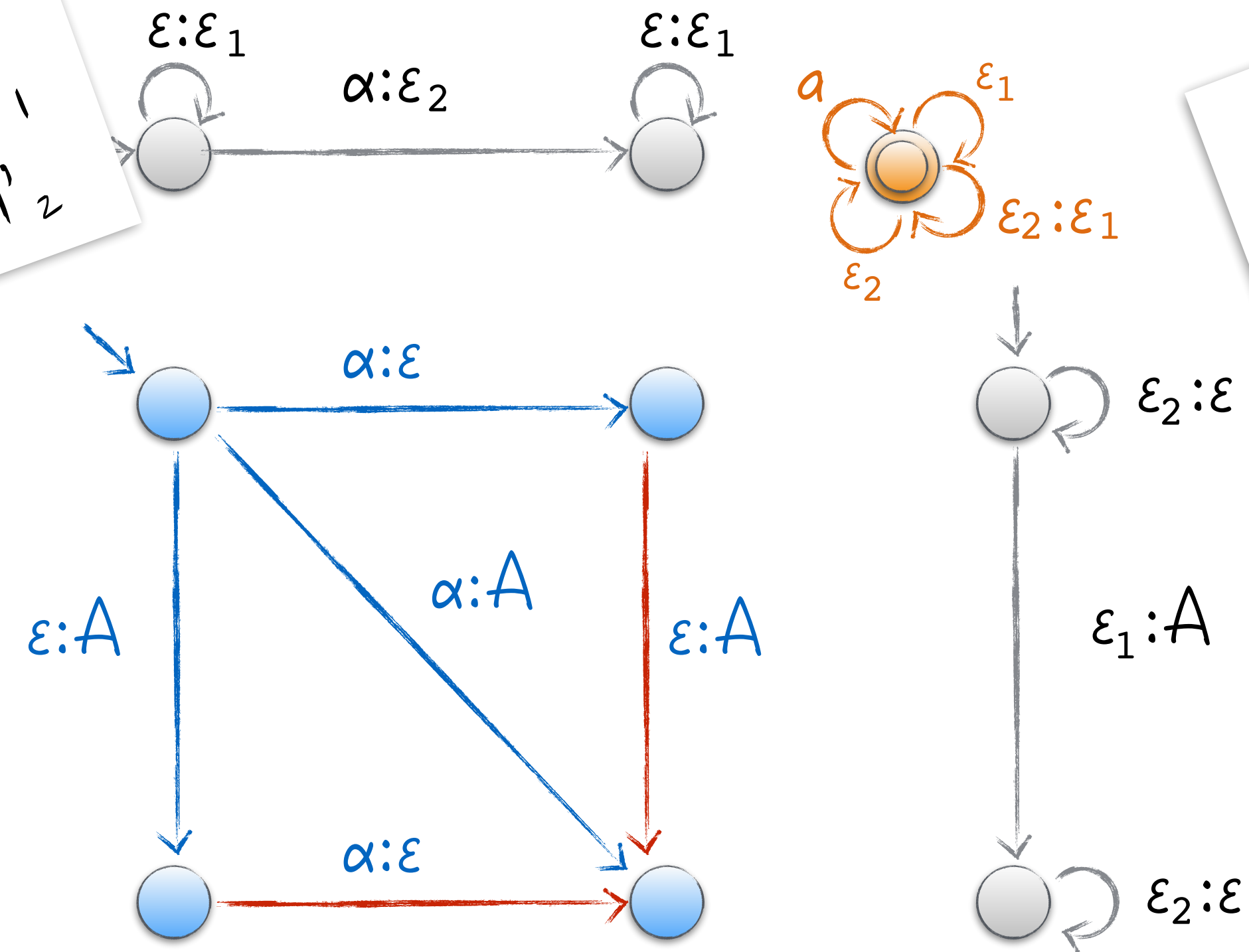
Gets rid of the
$\varepsilon{:}\varepsilon$ self-loops

Composition: Filters

Composition: Filters

Filter that doesn't take input $\varepsilon_1 \varepsilon_2$ or produce output $\varepsilon_2 \varepsilon_1$
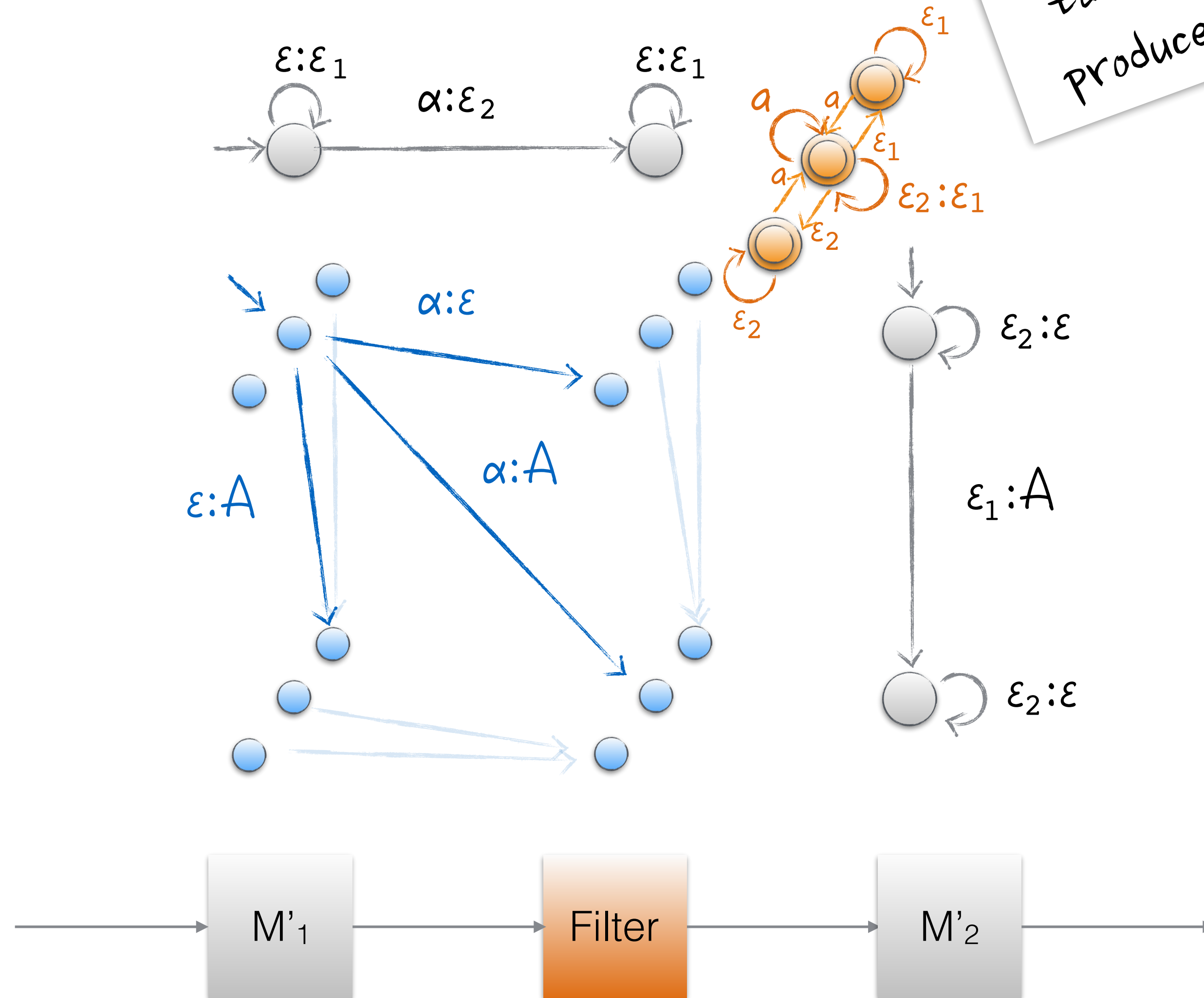
The Telephone Game

Given what D said,
can we infer the message A started with?

The Telephone Game

ransack
pet shop

Model the errors made by each player using an FST

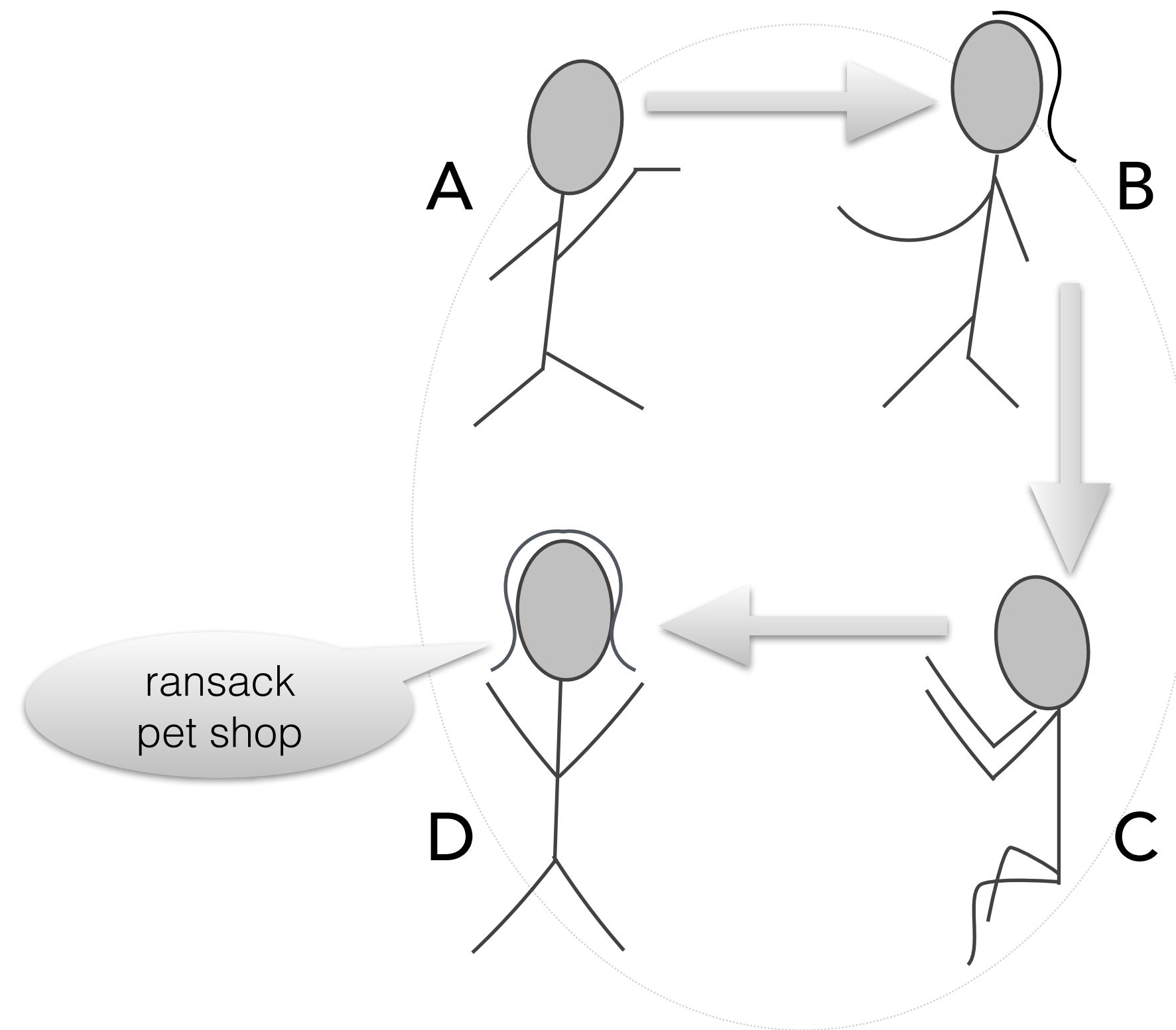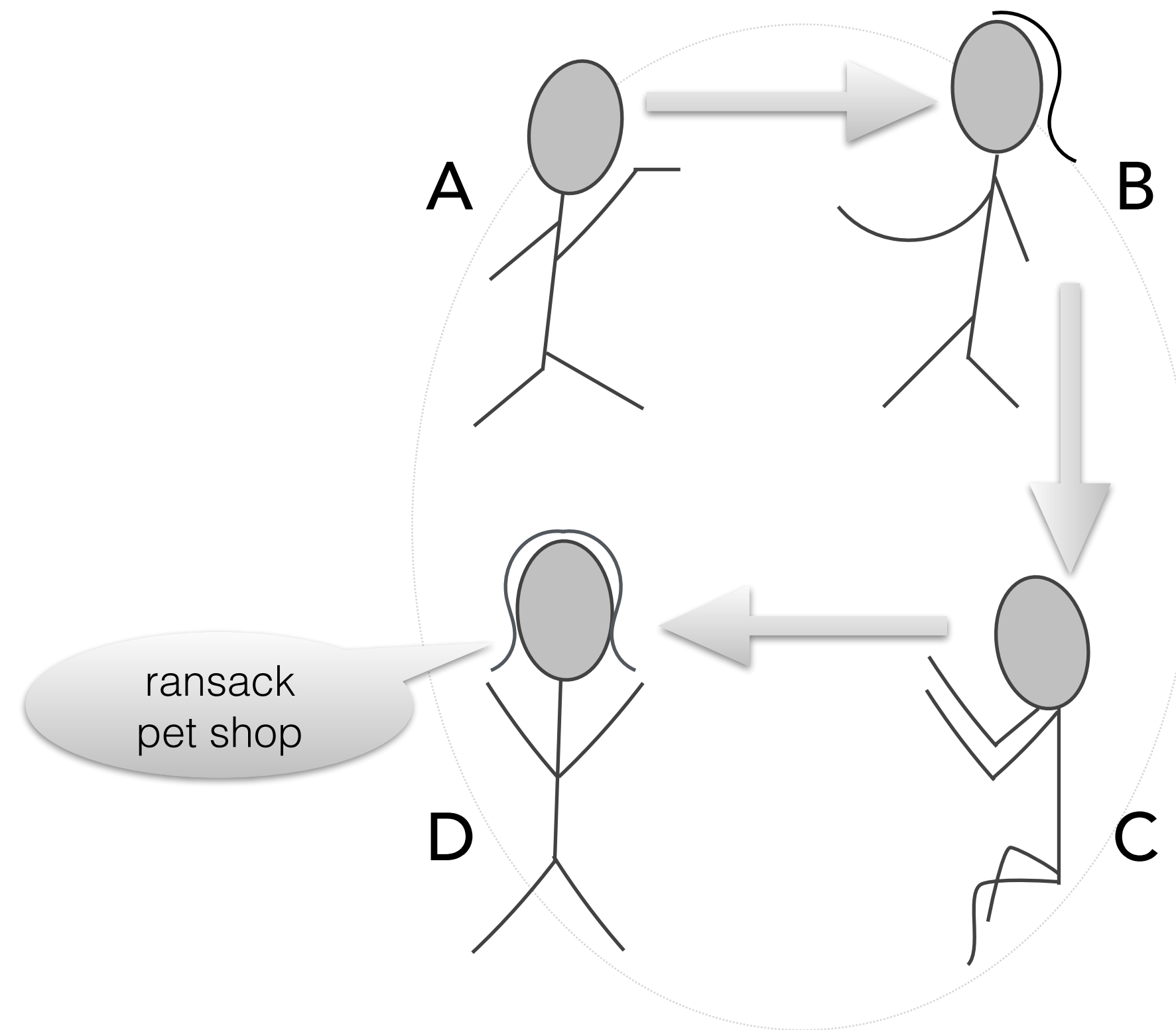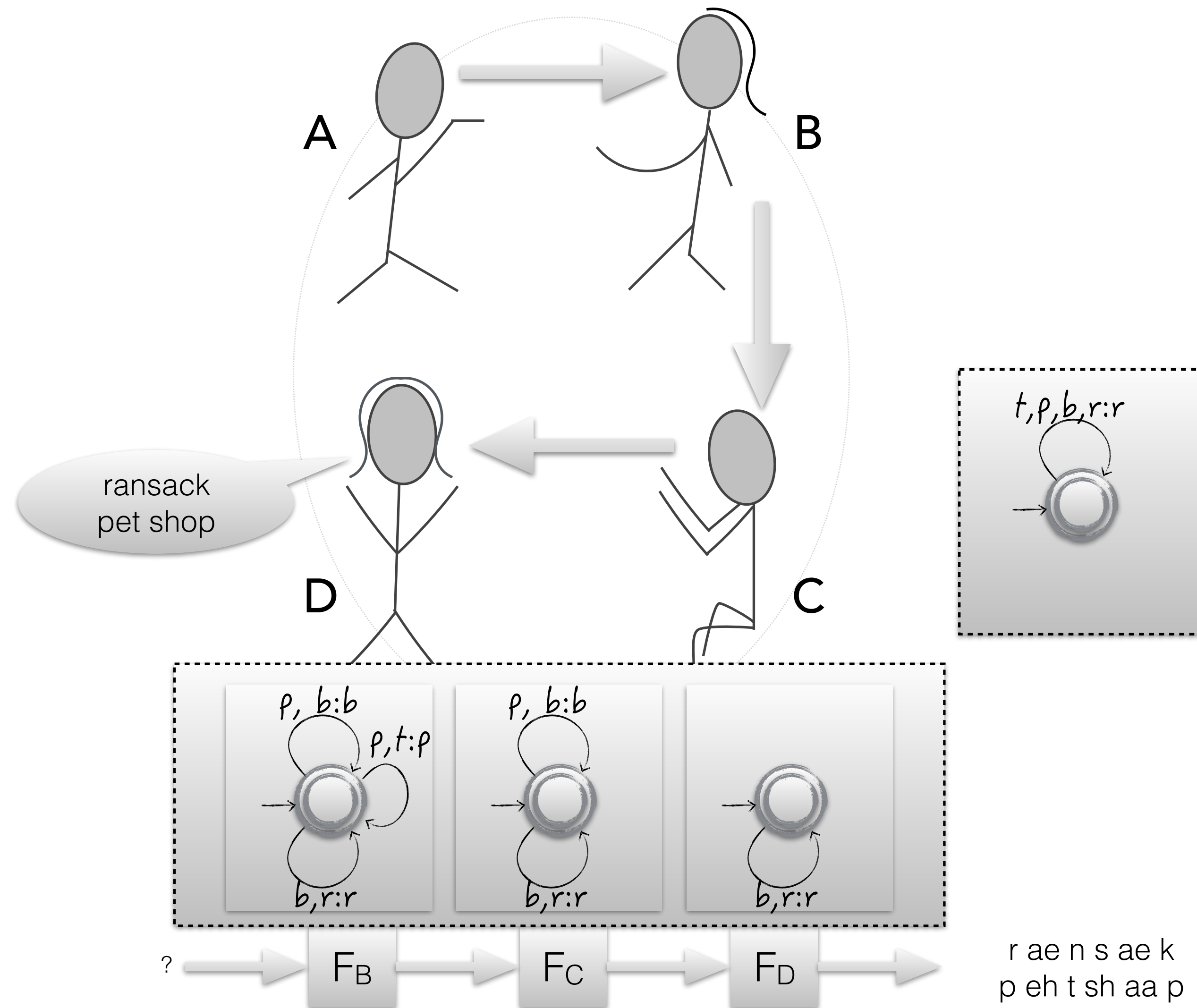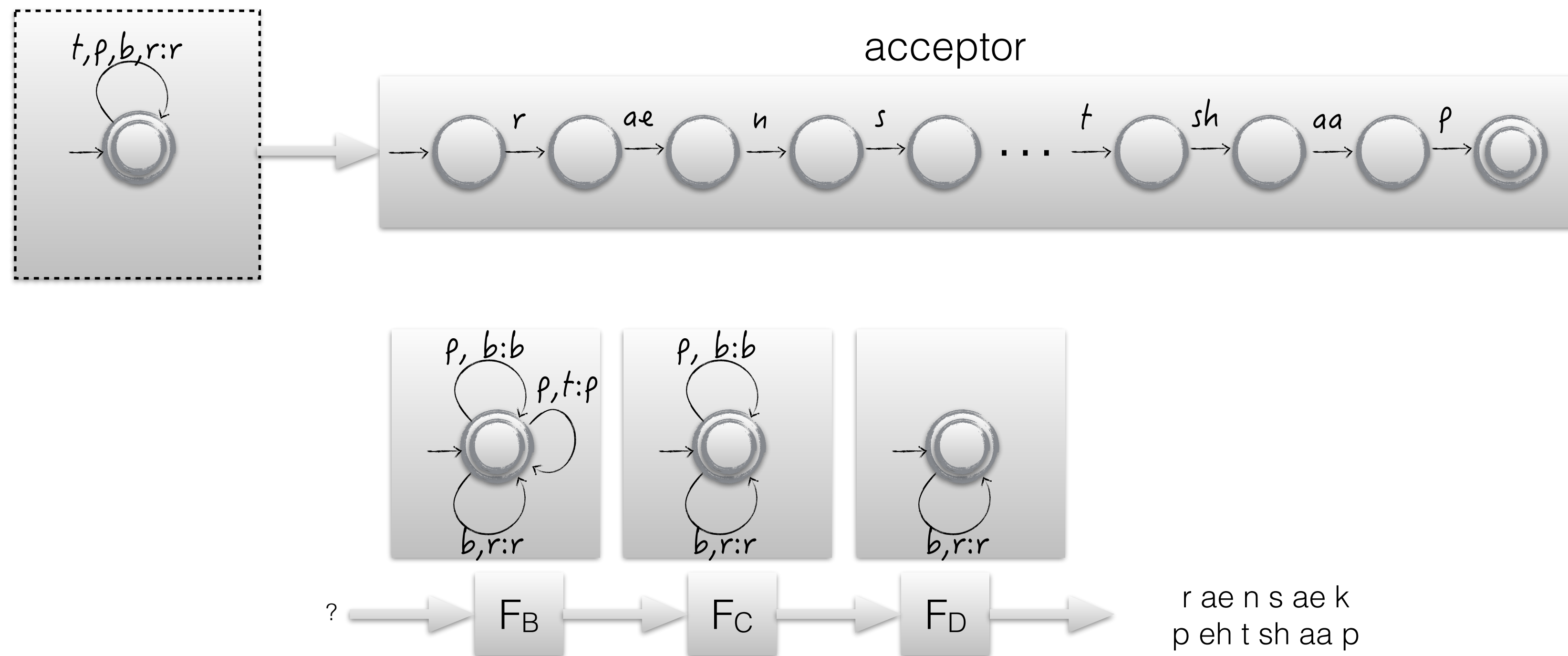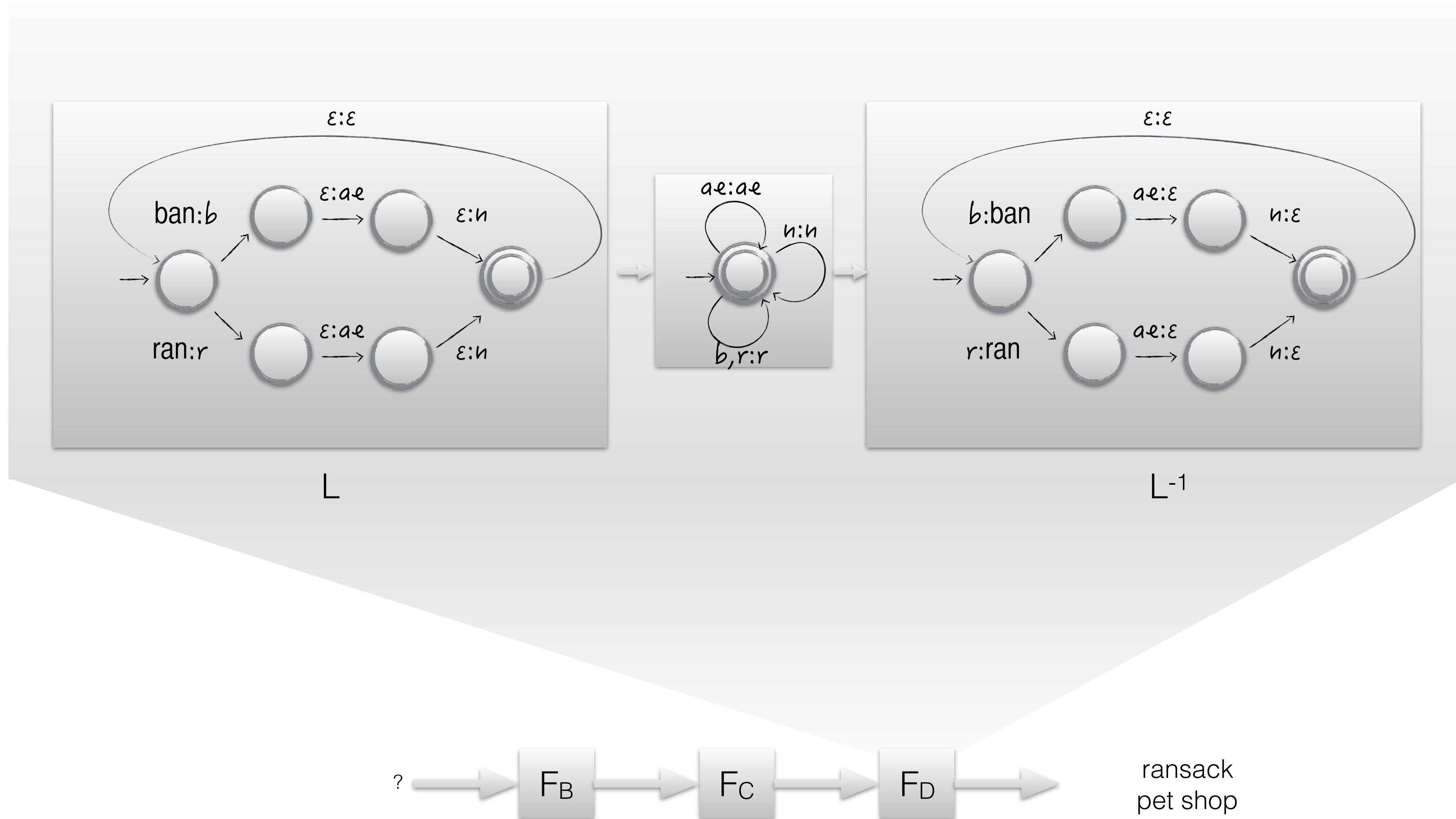# The Telephone Game

The Telephone Game

acceptor

The Telephone Game

The Telephone Game
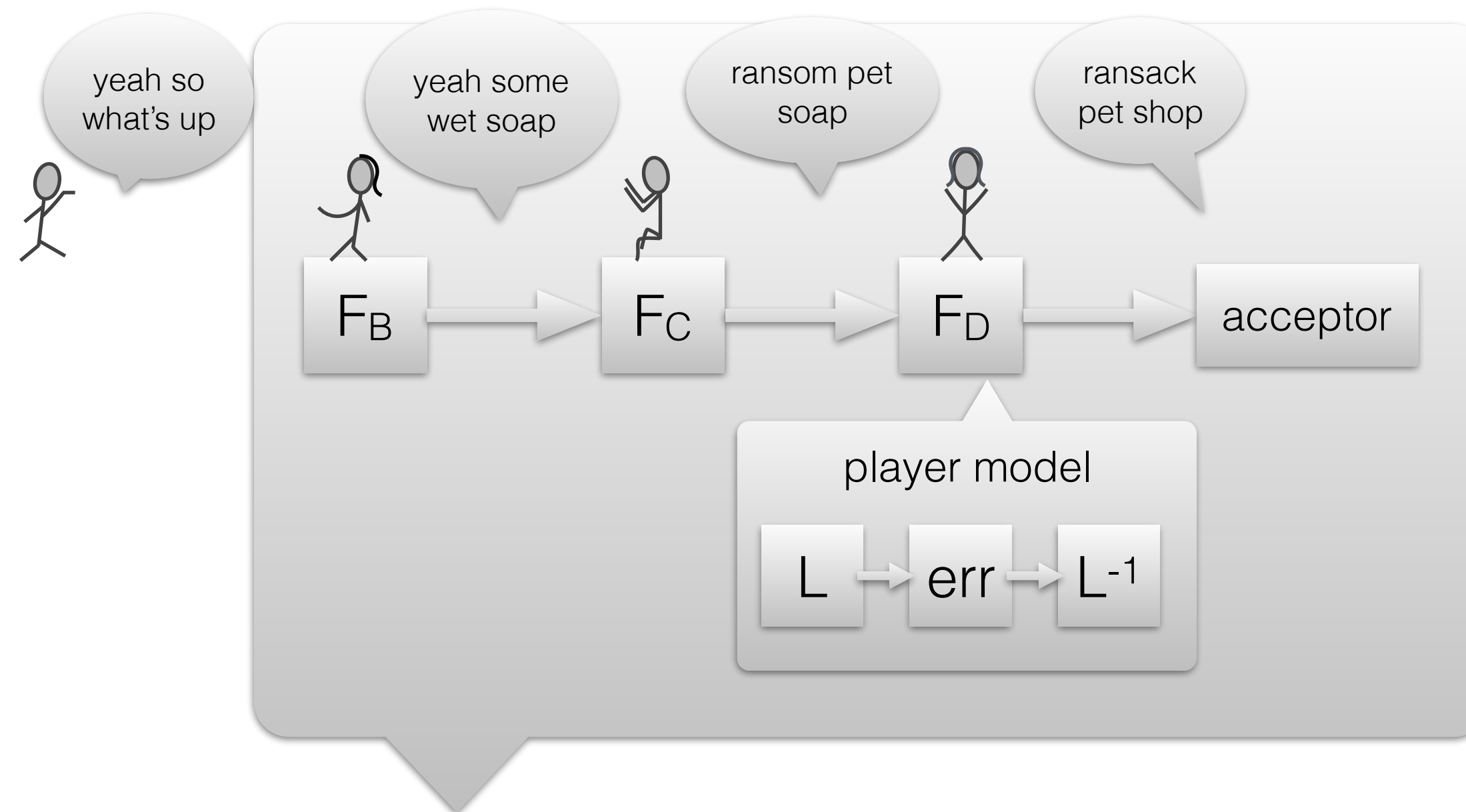
Find the best path in this FST
Read off the input words on the arcs
Can also find the best combination of paths in each player FST

Composition: Recap

- If T_1 transduces x to z,
  and T_2 transduces z to y,
  then $T_1 \circ T_2$ transduces x to y

$\alpha\ \beta\ \gamma \longrightarrow$ $a\ b\ c$ $\longrightarrow A\ B\ C \longrightarrow$
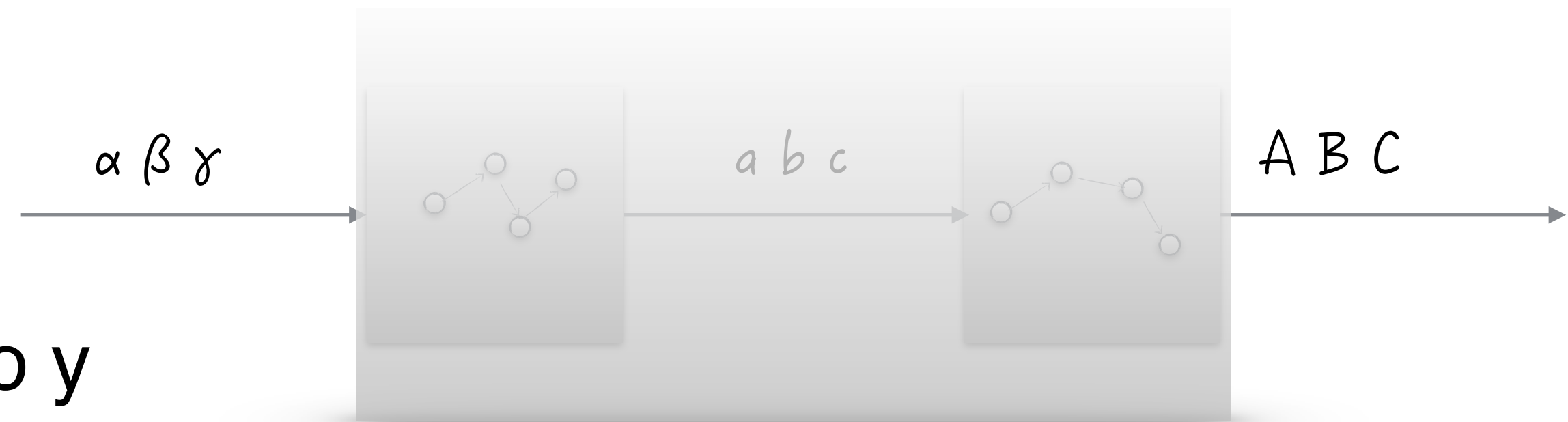
- Note: output alphabet of $T_1 \subseteq$ input alphabet of T_2

- E.g. If T_1 removes punctuation symbols from a string, and T_2 changes uppercase letters to lowercase letters, then $T_1 \circ T_2$ brings about both changes

Determinization and Minimization

- WFSTs constructed using various operations (or designed by hand) may have several redundancies

  - Affects the efficiency of subsequent operations

- Determinization and minimization seek to remove redundancies

  - Determinization can expand a WFST, but makes it faster to process an input string

  - Minimization results in the smallest number of states

- Will discuss WFSAs here. Extends to WFSTs.

Deterministic FSAs

- An FSA is deterministic if:

  - Unique start state

  - No two transitions from a state share the same label

  - No epsilon labels

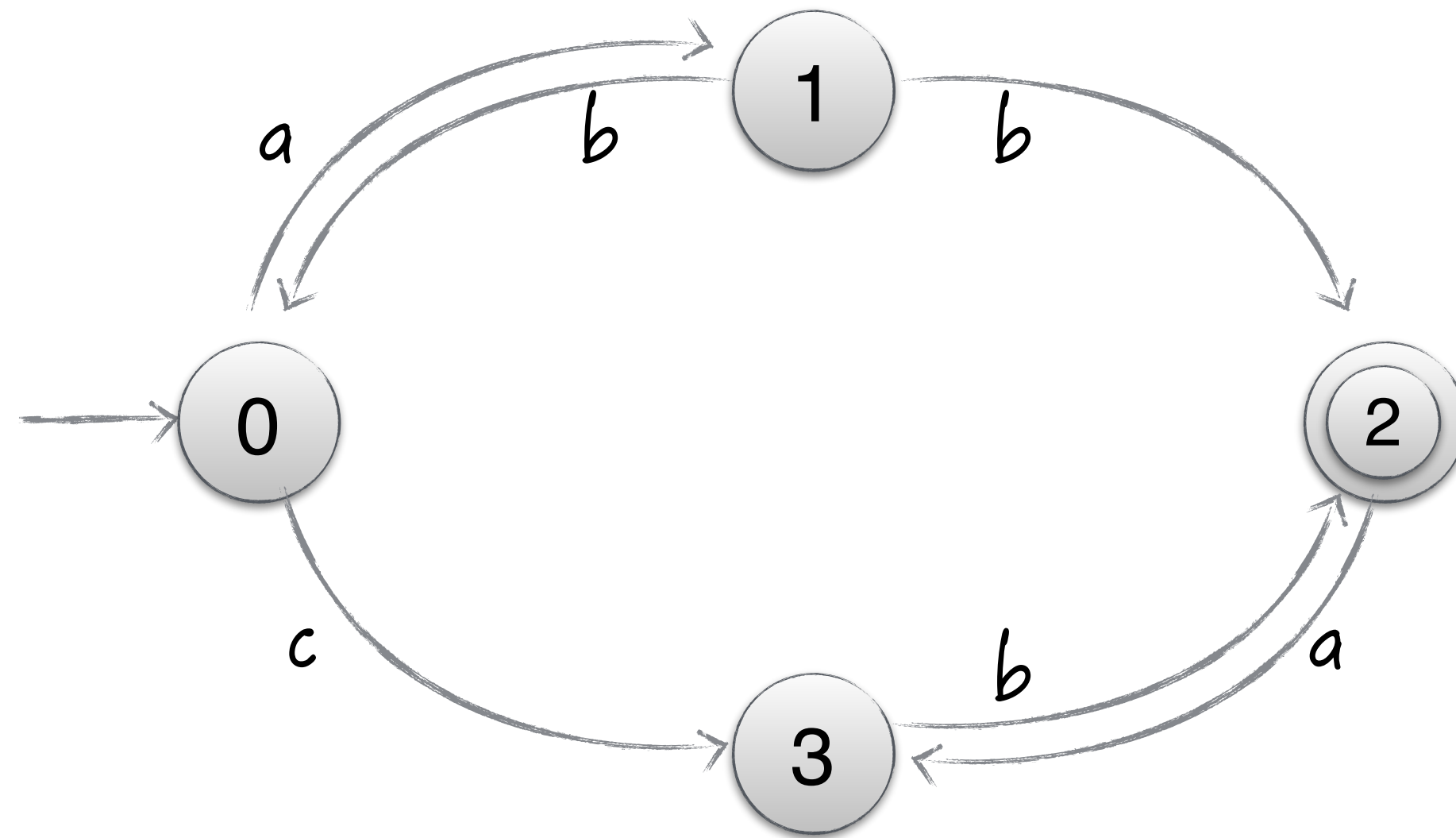Any input sequence yields a unique path (if at all)
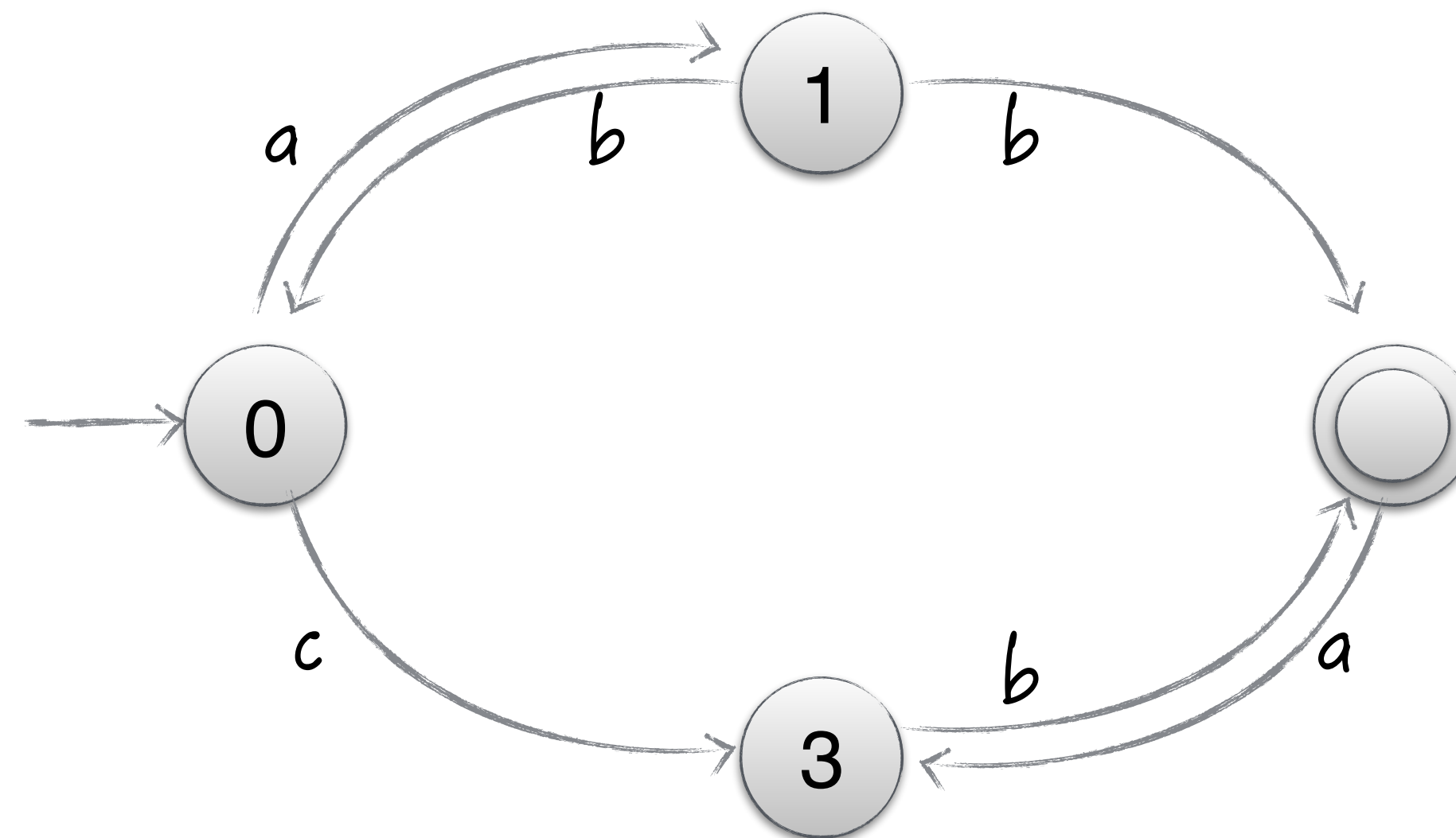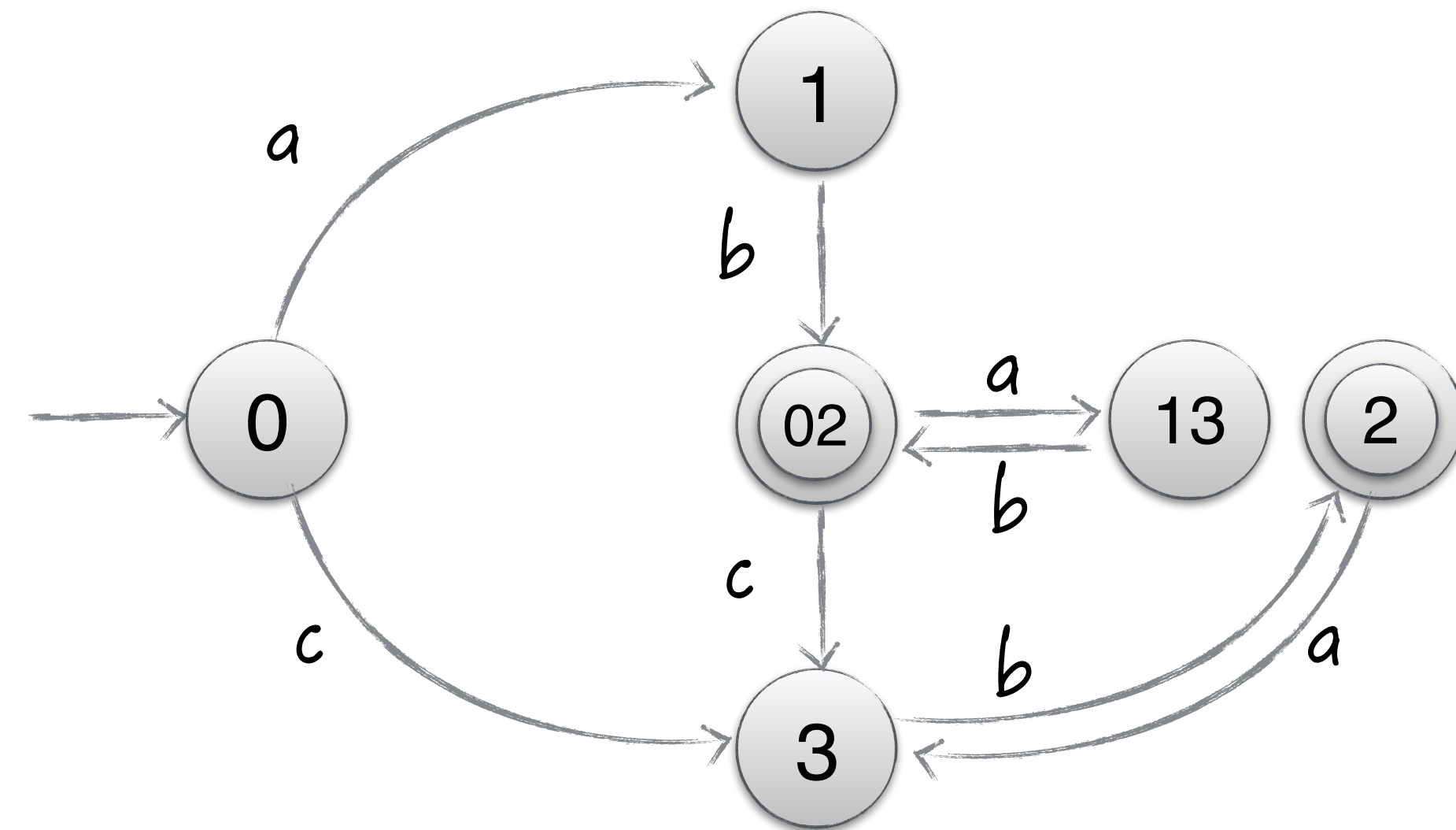

Deterministic or non-deterministic?

Determinization

States correspond to subsets of states in the original FSA
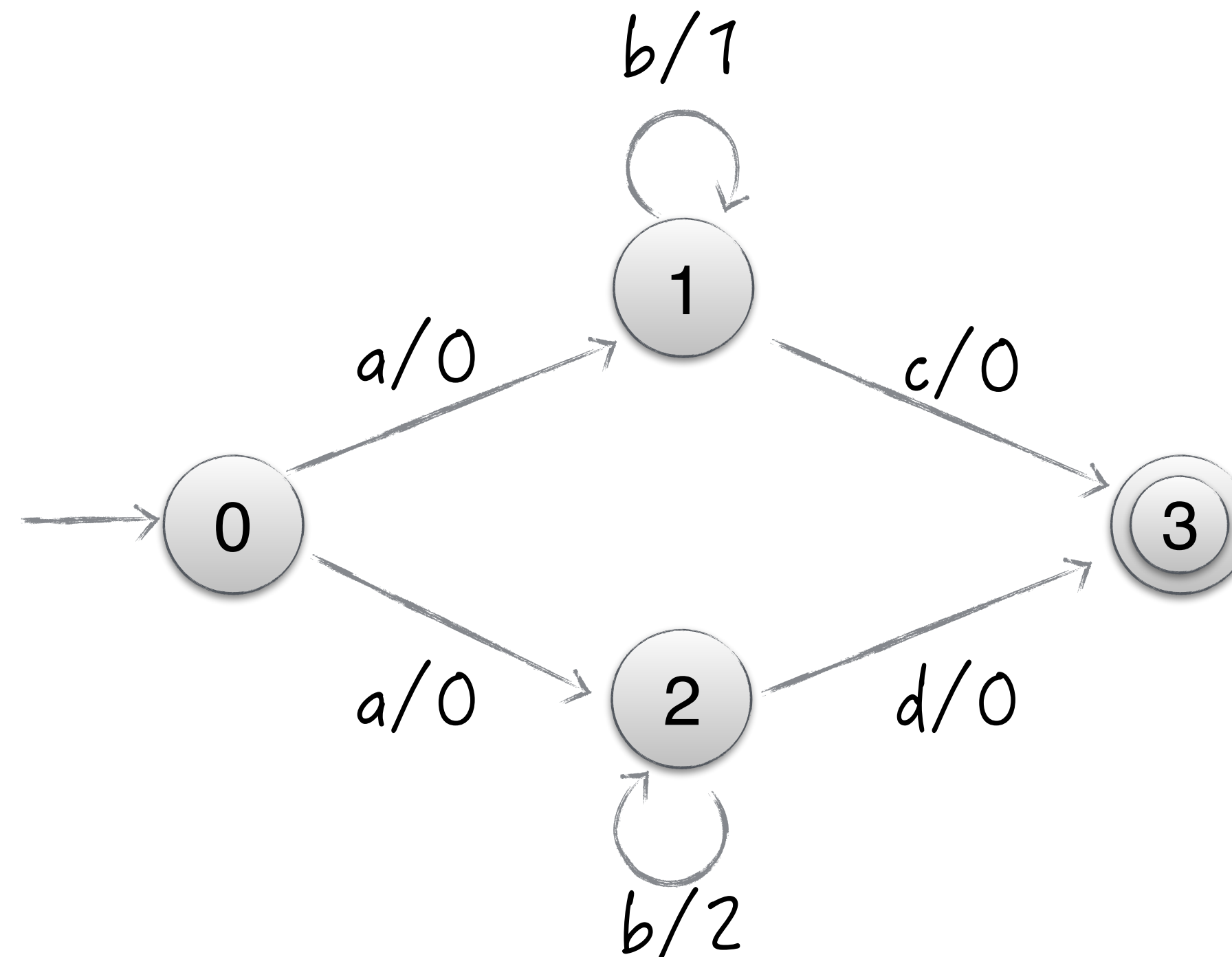
Construct an equivalent deterministic FSA

non-deterministic FSA

equivalent deterministic FSA

Determinization: Weighted FSA

Some Weighted-FSAs are not determinizable! [M97]

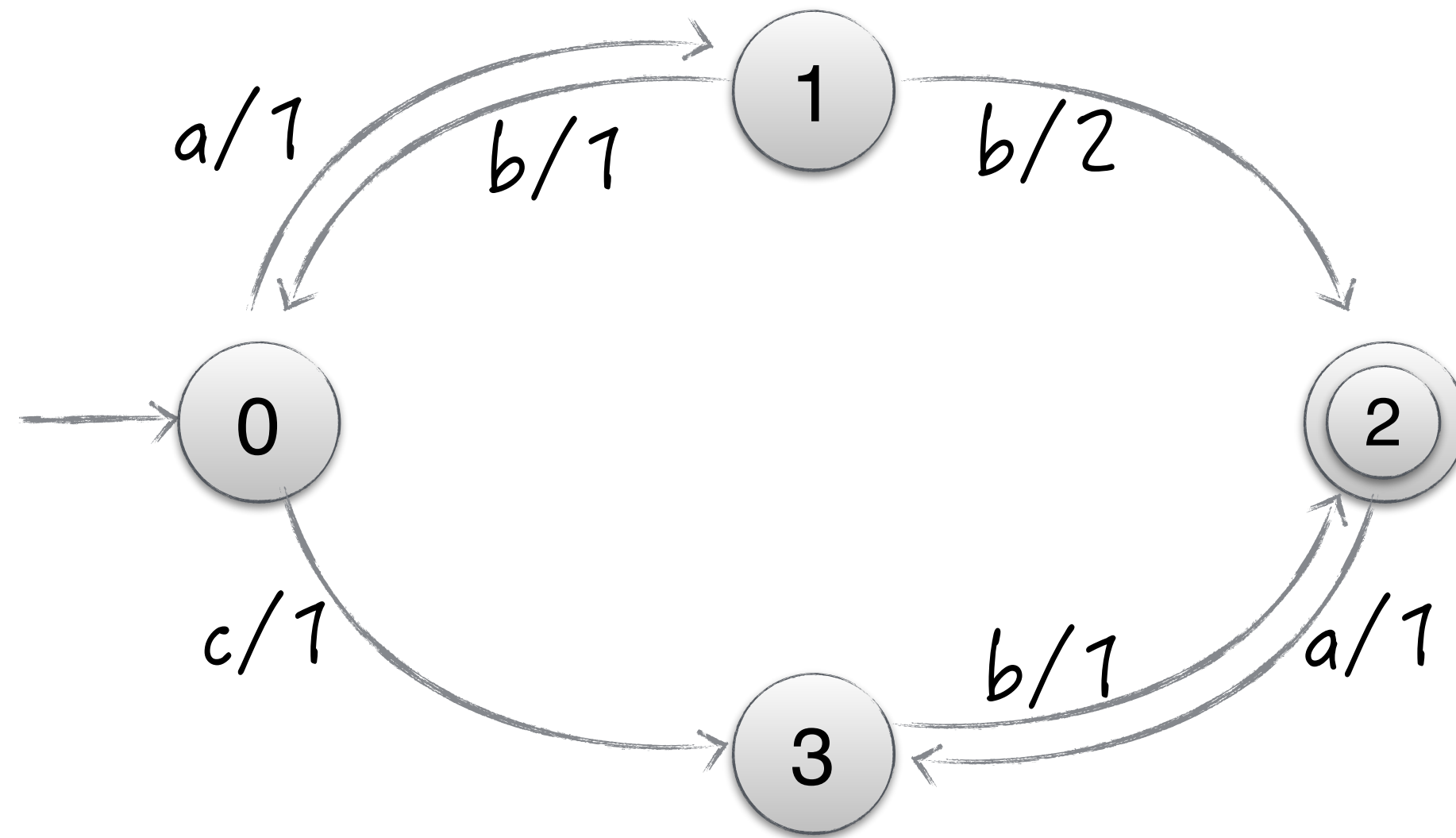

Weight of string $ab^n c = n$ and weight of $ab^n d = 2n$

After seeing ab^n an FSA can't remember n

[M97] M. Mohri. Finite-State Transducers in Language and Speech Processing. Computational Linguistics, 23(2), 1997

Determinization: Weighted FSA

Two WFSAs are equivalent if they associate the same weight
to each input string

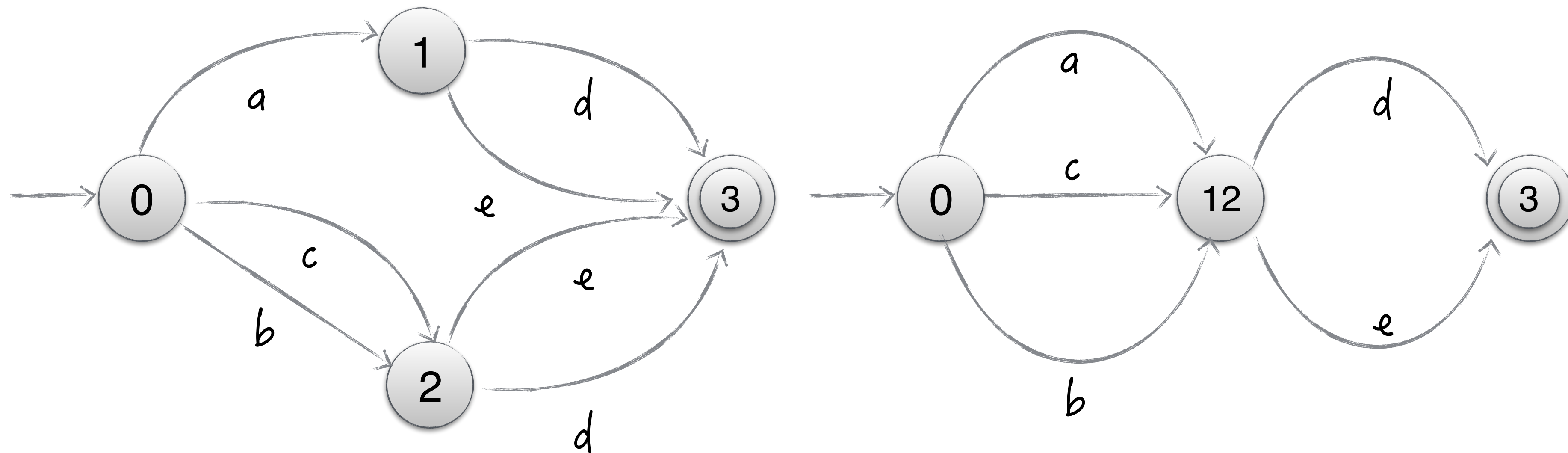non-deterministic WFSA   equivalent deterministic WFSA
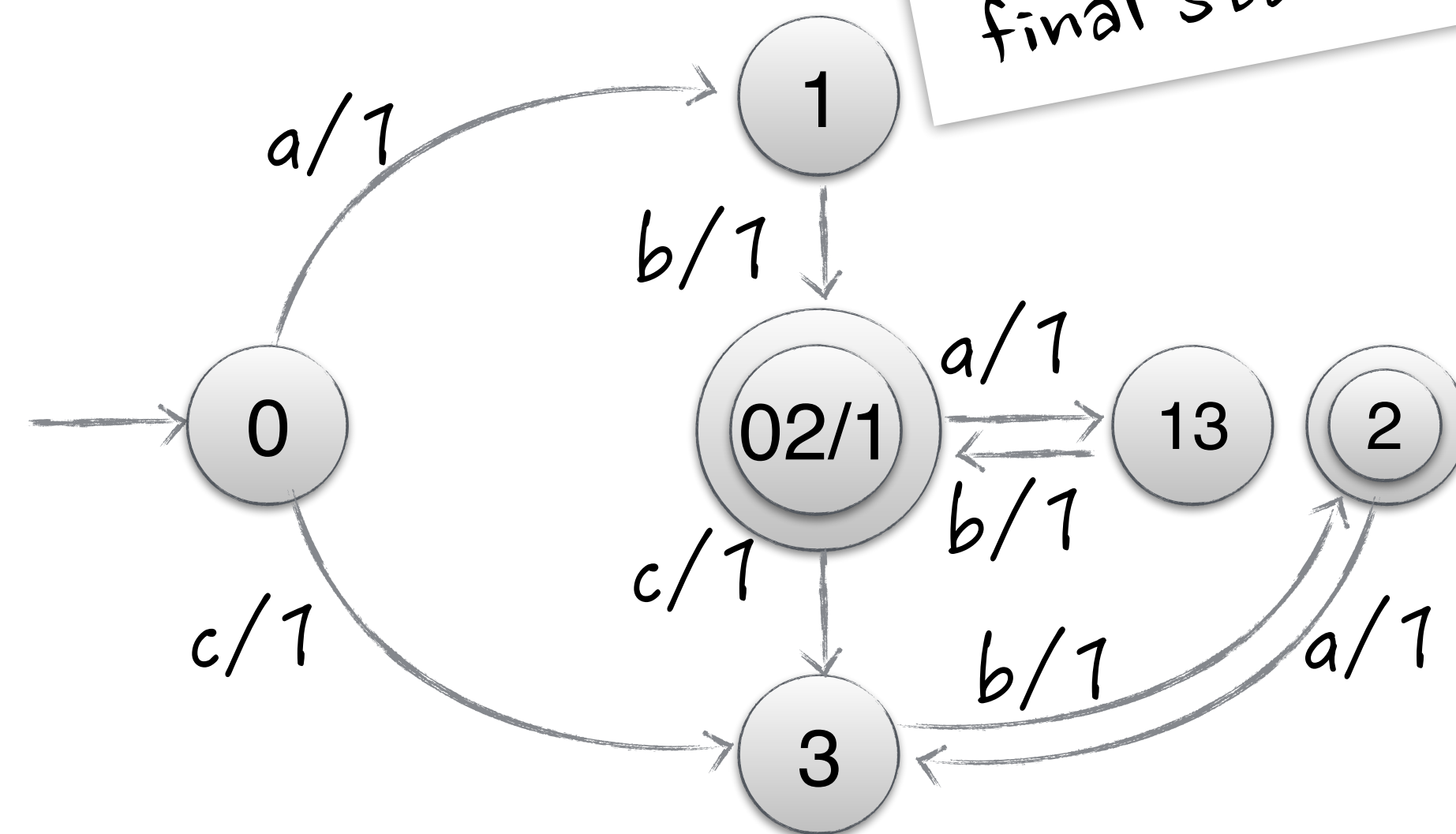
Minimization

Minimization: find an equivalent deterministic FSA with the least number of states (and transitions)

Unweighted FSAs have a unique minimal FSA [Aho74]

Obtained by identifying and merging equivalent states

Alfred V. Aho, John E. Hopcroft, and Jeffrey D. Ullman. The design and analysis of computer algorithms. Addison Wesley, 1974.

Minimization: Weighted FSA

Two states are equivalent only if for every input string, the outcome — weight assigned to the string, if accepted — starting from the two states are the same
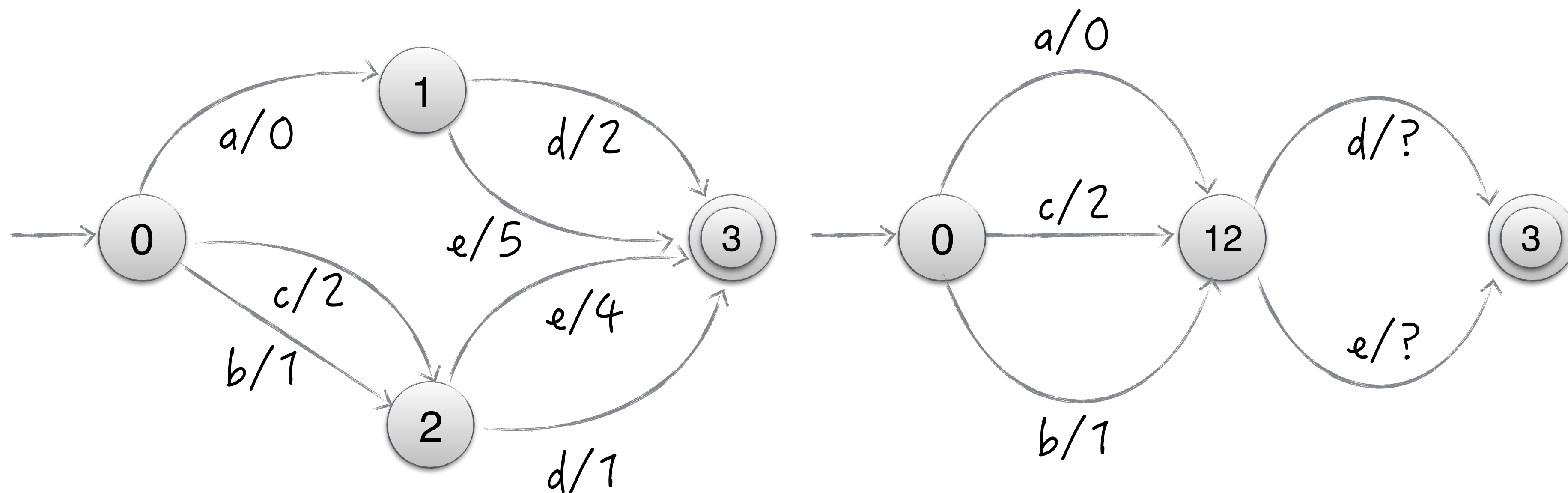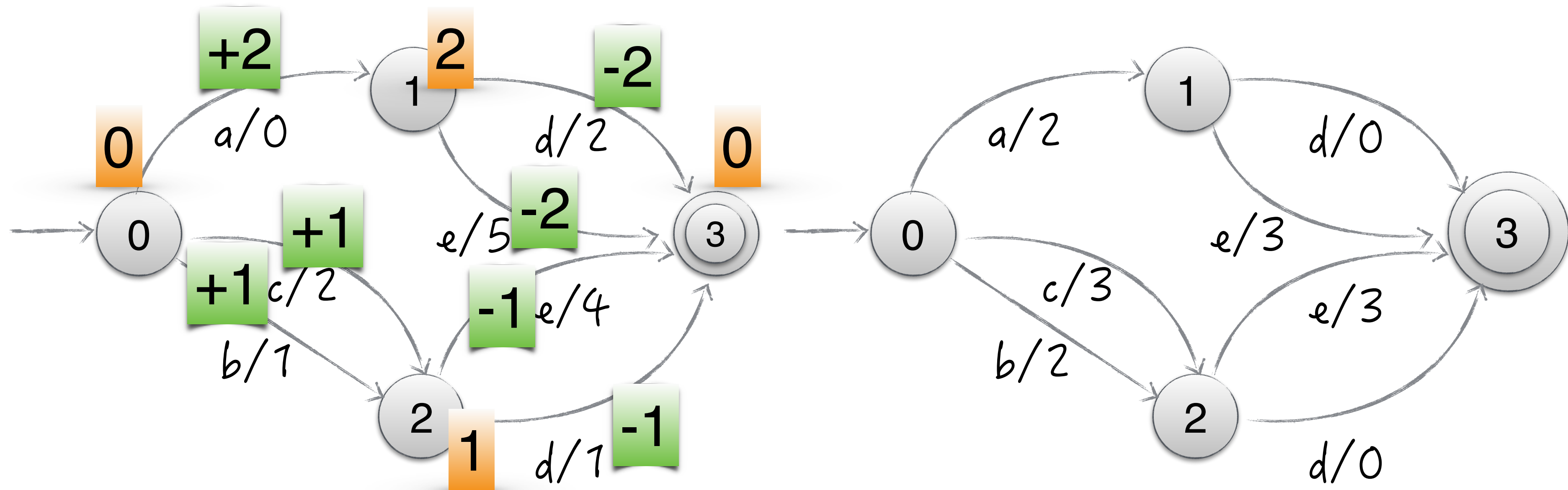

Redistribute weights before identifying equivalent states

Minimization: Weighted FSA

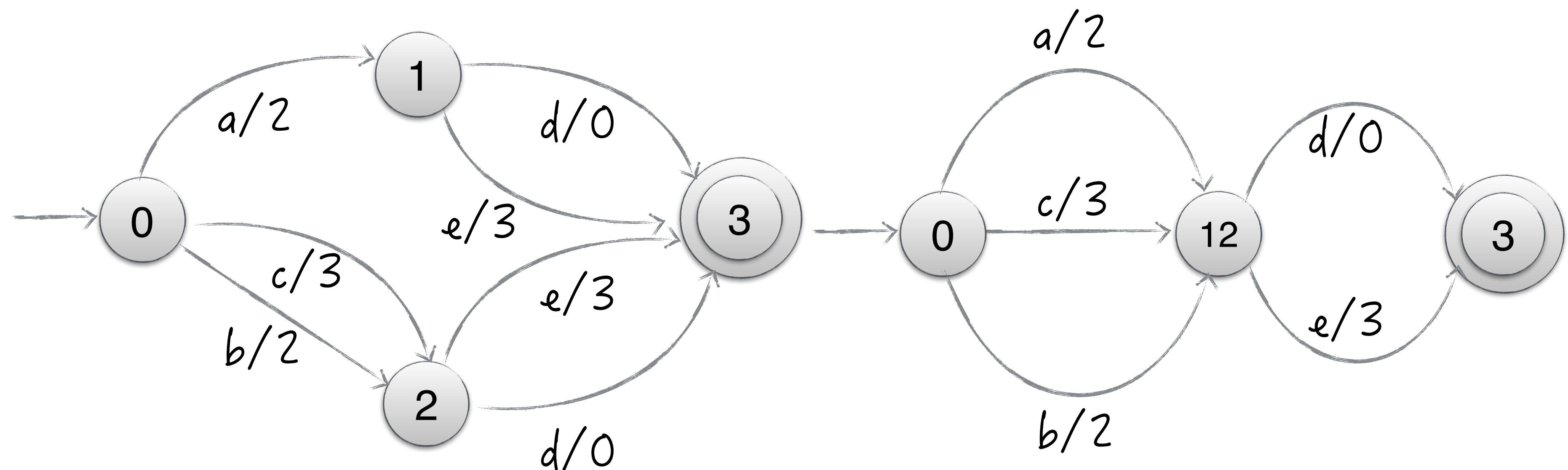Reweighting OK as long as resulting WFSA is equivalent

Can reweight using a "potential function" on states

"Weight pushing": Reweighting using a potential function that optimally moves weights towards the start state
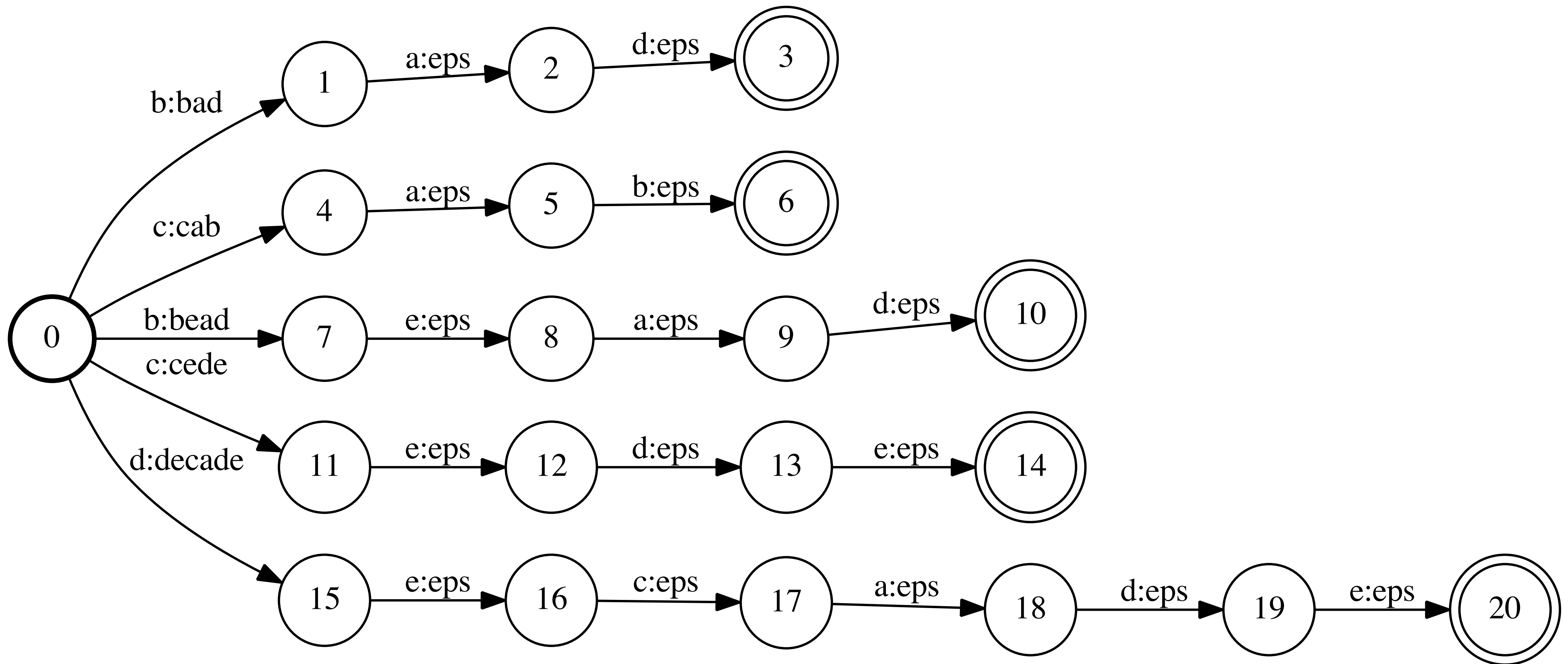
Minimization: Weighted FSA

After weight-pushing, can simply apply unweighted FSA minimization (treating label/weight as label)
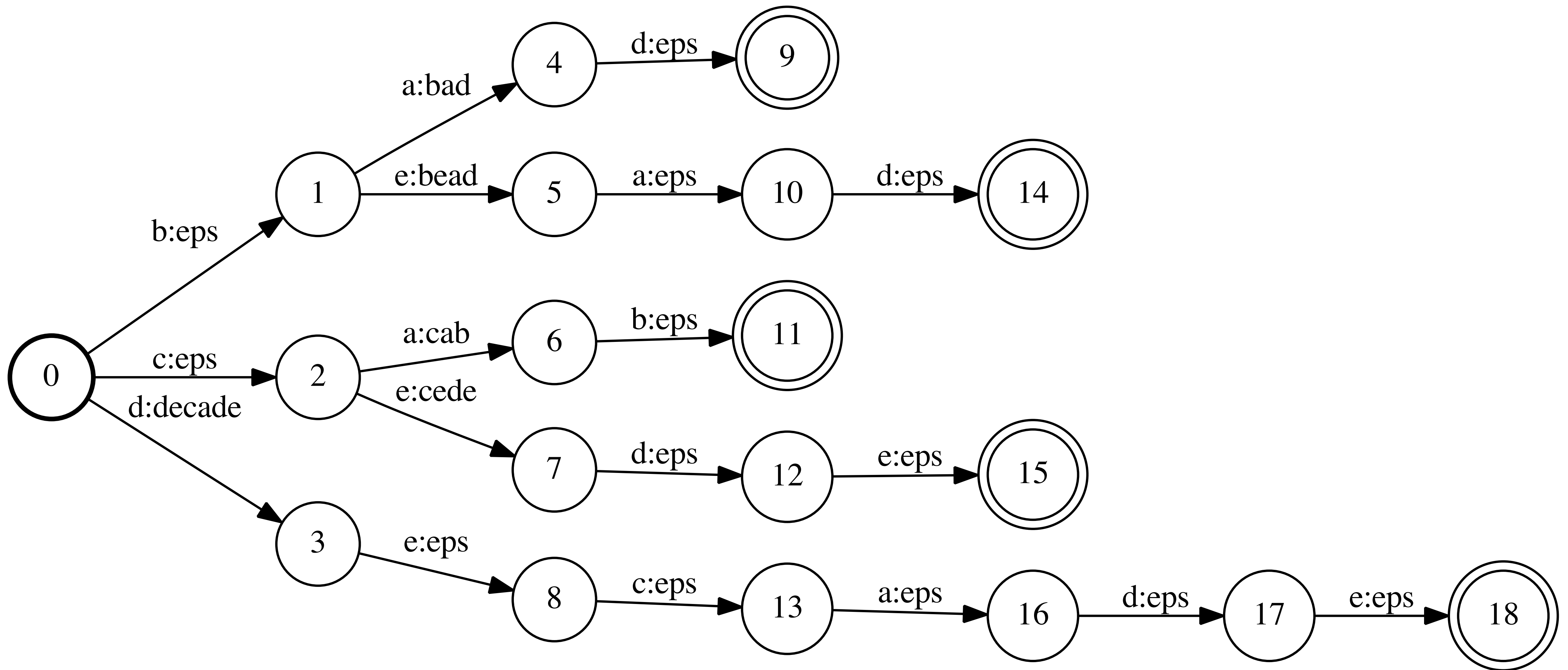

Guaranteed to yield a minimal WFSA (under some technical conditions required for weight-pushing)

Example: Dictionary WFST

Determinized Dictionary WFST

Minimized Dictionary WFST