50 marks **CS 408 Quiz 1** **8:00-9:25, 28/1/11**

A brief explanation is always expected, even for yes/no questions.

Problem 1: Let f denote an automorphism for Q_5. Suppose $f(10110) = 10001$.

(a)[6 marks] How many different automorphisms f can there be satisfying $f(10110) = 10001$?

The 5 directions can be chosen in 5! ways, i.e. that many possiblities.

(b)[6 marks] Find $x, y \neq 10110, 10001$ such that $f(x) = y$ for every f satisfying $f(10110) = 10001$.

The vertex at a distance 5 from 10110 must be embedded at a distance 5 from 10001, and there is only one choice for this. Hence x,y=01001,01110.

(c)[6 marks] How many possible values can $f(11111)$ have for f satisfying $f(10110) = 10001$?

11111 is at distance 2 from 10110. Thus any node at distance 2 from 10001 is fine. There are $\binom{5}{2} = 10$ possibilities.

Problem 2: Let K_n denote the complete graph on n nodes. For even integers $p < q$ what is the bisection width of $K_p \square K_q$?

(a)[5 marks] Describe your upper bound, drawing a picture if possible.

Draw the product in the usual manner, p rows and q columns. The possible cuts are $pq^2/4$ or $qp^2/4$, hence the latter is a better upper bound.

(b)[15 marks] Describe your lower bound.

Embed the complete directed graph on n=pq nodes. Embedding of $((i,j),(i',j'))$ uses the canonical path with the intermediate node being (i',j). 5 marks.

Edge $((i,j),(i',j)$ will receive paths starting at (i,j) going to (i',k) for all k. Thus q paths. Similarly edge $((i,j'),(i',j'))$ will receive p paths. 4 for the first and 2 for stating the second.

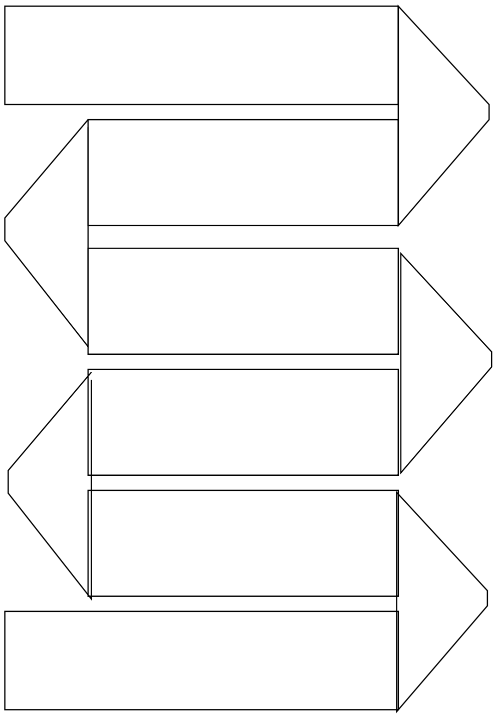
Thus the congestion is q. Thus the bound is $B \geq n^2/4c = p^2q^2/4q = p^2q/4$. 4 marks.

Problem 3:[6 marks] The cube connected cycles network CCC(n) on $n2^n$ nodes is defined as follows. Each vertex is numbered (i,j), where $0 \leq i < 2^n$ and $0 \leq j < n$, . Vertex (i,j) connects to vertex $(i, j+1 \bmod n)$ and $(i, j-1 \bmod n)$ using edges called "cycle edges", and to vertex $(i \oplus 2^j, j)$ using edges called "cube edges". Show that the butterfly network $B(n)$ on $(n+1)2^n$ nodes can be embedded into CCC(n) with load 2 and dilation 2.

Place (i,j) of B onto $(i, j \bmod n)$ of CCC. This gives load 2 because columns $n, 0$ are placed on the same set. 2 marks

The cycle edges are actually present directly, except for the edges $((i,n),(i,0))$ both endpoints of which are placed on the same node. 1 mark

Edge $((i,j),(i \oplus 2^j, j))$ is connected through the intermediate vertex $(i \oplus 2^j, j+1)$ which is adjacent to both of the endpoints. 2 marks (b)[6 marks] Show that $P_m \square P_{n^2 m}$ can be embedded into $P_{mn} \square P_{mn}$ with load 2 and dilation 1. Hint: Paper folding. A pictorial and non-algebraic but precise answer is fine.



Extra credit: [5 marks] Let $N(X), B(X)$ denote the number of vertices and bisection width of a graph X. Give an example of graphs G, H such that $B(G \square H) < \min(N(G)B(H), B(G)N(H))$.

Let G be 3 copies of K_n, and H be P_2. The best bisection is: leftmost two copies of K_n from the top row, and the leftmost from the bottom row on one side of the bisection. Clearly this requires removing $n/3$ edges, better than cutting down the middle horizontally or vertically.