Problem 1:[20 marks] Consider a processor scheduling problem as follows. We are given $P[1..T]$ where $P[t]$ is the number of processors available at time step t. We are also given $A[1..n]$ and $D[1..n]$, where $A[j]$ is the time step at the beginning of which job j arrives, and $D[j]$ is the time step at the end of which job j must depart. We are also given $L[1..n]$ where $L[j]$ is the length of job j, i.e. the number of steps for which job j must be assigned some processor between $A[j]$ and $D[j]$, not necessarily consecutively. Any processor can be assigned any job, and at any instant a job can be given at most one processor, and a single processor can be given to at most one job. Formulate this as a flow problem and show how to determine whether an assignment respecting all the conditions exists. Clearly state how the flow assignment indicates which processors process which job at what time, and also why every condition is respected.

Problem 2:[10 marks] In the Ford-Fulkerson algorithm we used any path from s to t in the residual graph in order to augment the flow. Instead, suppose that we use the shortest path. Show that length of the shortest path found in consecutive iterations cannot increase.

Problem 3:(a)[7 marks] Show that Q_4 is not planar. A crisp arugment without too many cases is expected. (b)[8 marks] Usually, you expect that only borders of at most 3 countries will meet in a single point. Consider a map in which the number of boundaries meeting in a point is always even. The outer face is also a country, and its boundaries also satisfy the above. Show that such a map can be coloured using 2 colours.