50 marks **CS 408 Makeup Quiz** **8:30-9:25, 18/4/11**

Solve any 2 from the first 3 problems. Problem 4 is compulsory.

Problem 1:[20 marks] Suppose $n = 2^k$. Suppose $G = Q_{3k}$ is embedded into $H = P_n \Box P_n \Box P_n$, with load 1.

(a) Show that the dilation must be at least $\Omega(n/\log n)$.

(b) Show that the congestion must be at least $\Omega(n)$.

(c) Consider a specific embedding as follows. Vertex $x = x_{3k-1} \ldots x_0$ of G is placed on vertex $(x_{3k-1} \ldots x_{2k}, x_{2k-1} \ldots x_k, x_{k-1} \ldots x_0)$ of H. What is the dilation and congestion of this embedding?

(d) Suppose instead that H is embedded into G with load 1. What is the minimum congestion and dilation possible?

Problem 2:[20 marks] Show that there exists an n vertex graph that neither contains a clique of size $\log n$, nor an independent set of size $\log n$.

Problem 3:[20 marks] Suppose IIT adopts the system of using BTech/DD students as TAs. Some students from years 2,3,4,5 have volunteered to serve as TAs. We are also given a list of courses, and the number of TAs needed by each course. A student from year x can be a TA for any course meant for years $1, \ldots, x-1$, and say the courses for the fifth year are not included in the list. We would like to ensure that from each hostel at least one TA gets allocated. Formulate this as a max flow problem. Assume that you have all information about the students who have volunteered as TAs.

Problem 4:[10 marks] Suppose a graph has a perfect elimination order. Show that if it contains a cycle v_1, v_2, v_3, v_4, then it must also contain either the edge (v_1, v_3) or the edge (v_2, v_4).