Learning 3D Human Pose from Structure and Motion Supplementary Material

1 Illegal Angle Loss ($\mathcal{L}_a$)

We repeat the loss definition (for the sake of continuity) and then explain the backpropagation of gradients from the loss.

Defining the loss: Let subscripts n, s, e, w, k denote neck, shoulder, elbow, wrist and knee joints in that order, and superscripts l and r represent left and right body side, respectively. We define $\mathbf{v^r_{sn}} = J^r_s - J_n$, $\mathbf{v^r_{es}} = J^r_e - J^r_s$ and $\mathbf{v^r_{we}} = J^r_w - J^r_e$ as the collar-bone, upper-arm and the lower-arm, respectively. Now, $\mathbf{n^r_s} = \mathbf{v^r_{sn}} \times \mathbf{v^r_{es}}$ is the normal to the plane defined by the collar-bone and the upper-arm. For the elbow joint to be legal, $\mathbf{v^r_{we}}$ must have a positive component in the direction of $\mathbf{n^r_s}$, i.e. $\mathbf{n^r_s} \cdot \mathbf{v^r_{we}}$ must be positive. We do not incur any penalty when the joint angle is legal and define $E^r_e = \min(\mathbf{n^r_s} \cdot \mathbf{v^r_{we}}, 0)$ as a measure of implausibility. Note that this case is opposite for the right knee and left elbow joints (as shown by the right hand rule) and requires E^r_k and E^l_e to be positive for the illegal case. For the knee case, neck, shoulder and wrist are replaced by pelvis, hip and foot respectively. We exponentiate E to strongly penalize large deviations beyond legality. Our angle loss can now be defined as:

$$\mathcal{L}_a = -E^r_e e^{-E^r_e} + E^l_e e^{E^l_e} + E^r_k e^{E^r_k} - E^l_k e^{-E^l_k} \tag{1}$$

All the terms of the loss are functions of bone vectors which are defined in terms of the output poses. The loss is, therefore, differentiable with respect to the joint positions involved.

Computing gradients: The backprop for E^r_e is given below. Similar analysis can be performed for the left/right knee and left elbow joints as well. Let $J^r_s(x)$ denote the x-coordinate of joint s.

$$E^r_e = ((J^r_s - J^r_n) \times (J^r_e - J^r_s)) \cdot (J^r_w - J^r_e)$$

$$\frac{\partial E^r_e}{\partial J^r_n(x)} = ((-1,0,0) \times (J^r_e - J^r_s)) \cdot (J^r_w - J^r_e)$$

$$\begin{aligned}\frac{\partial E^r_e}{\partial J^r_s(x)} &= ((1,0,0) \times (J^r_e - J^r_s)) \cdot (J^r_w - J^r_e) \\ &\quad + ((J^r_s - J^r_n) \times (-1,0,0)) \cdot (J^r_w - J^r_e)\end{aligned}$$

$$\begin{aligned}\frac{\partial E^r_e}{\partial J^r_e(x)} &= ((J^r_s - J^r_n) \times (1,0,0)) \cdot (J^r_w - J^r_e) \\ &\quad + ((J^r_s - J^r_n) \times (J^r_e - J^r_s)) \cdot (-1,0,0)\end{aligned}$$

$$\frac{\partial E_e^r}{\partial J_w^r(x)} = ((J_s^r - J_n^r) \times (J_e^r - J_s^r)) \cdot (1, 0, 0)$$

Similarly, derivatives for the y and z coordinates can be found too. Note again that we backpropagate only when the joint is illegal. For the legal case, the loss is zero and hence we backpropagate a zero gradient for all the four joints.