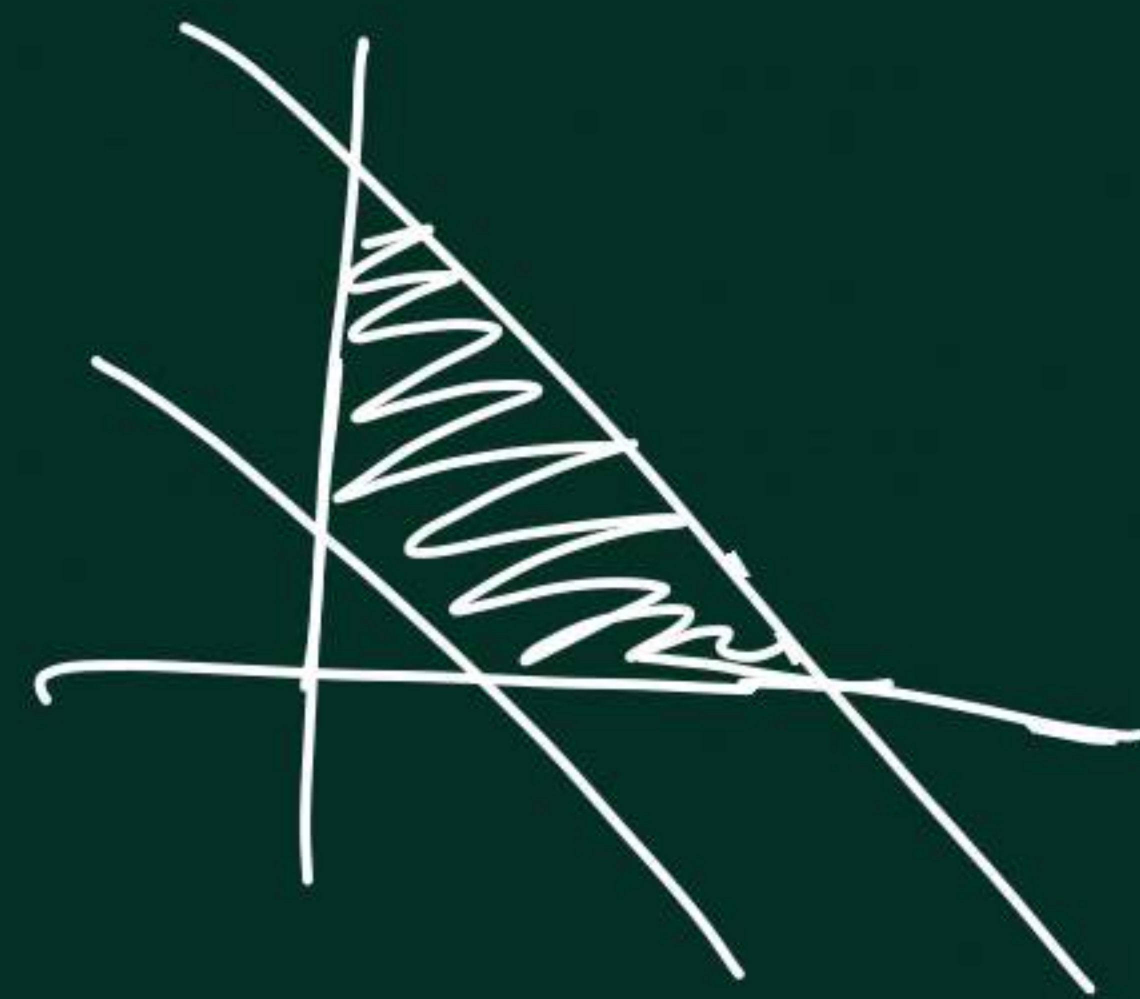
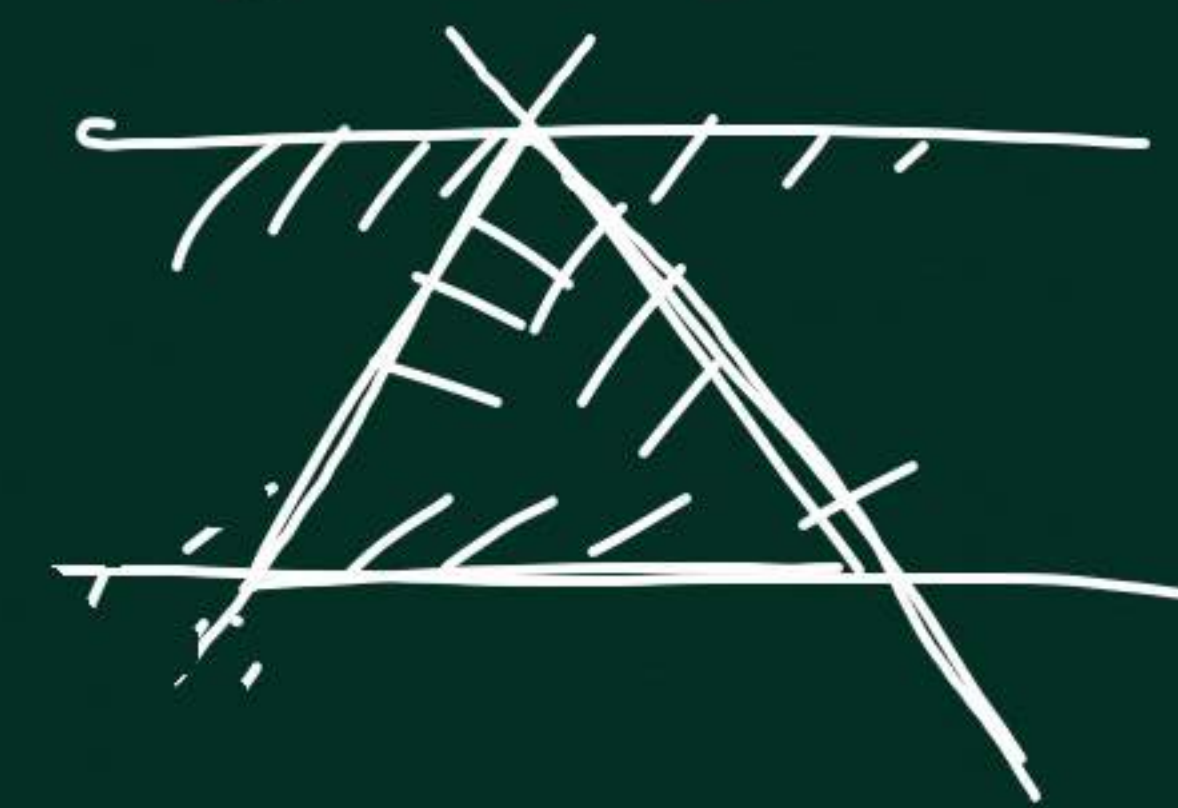
Simplex Algorithm (not known to be in P)

Recall : one of the **corners** is an optimal so

$$\begin{cases} 2x+3y \le 5 \\ x+y \geq 1 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



$x+y+z \geq 1$



Convex set

Polyhedron / Polytope

lution

Def (corner): For a system of linear inequalit
in n varibles, a corner is a feasible
point which satisfies n linearly
independent constraints with equality.

Def A corner is a feasible point
s.t. there exist a hyperplane H
s.t. $H \cap$ polyhedron is a unique poin

Simplex

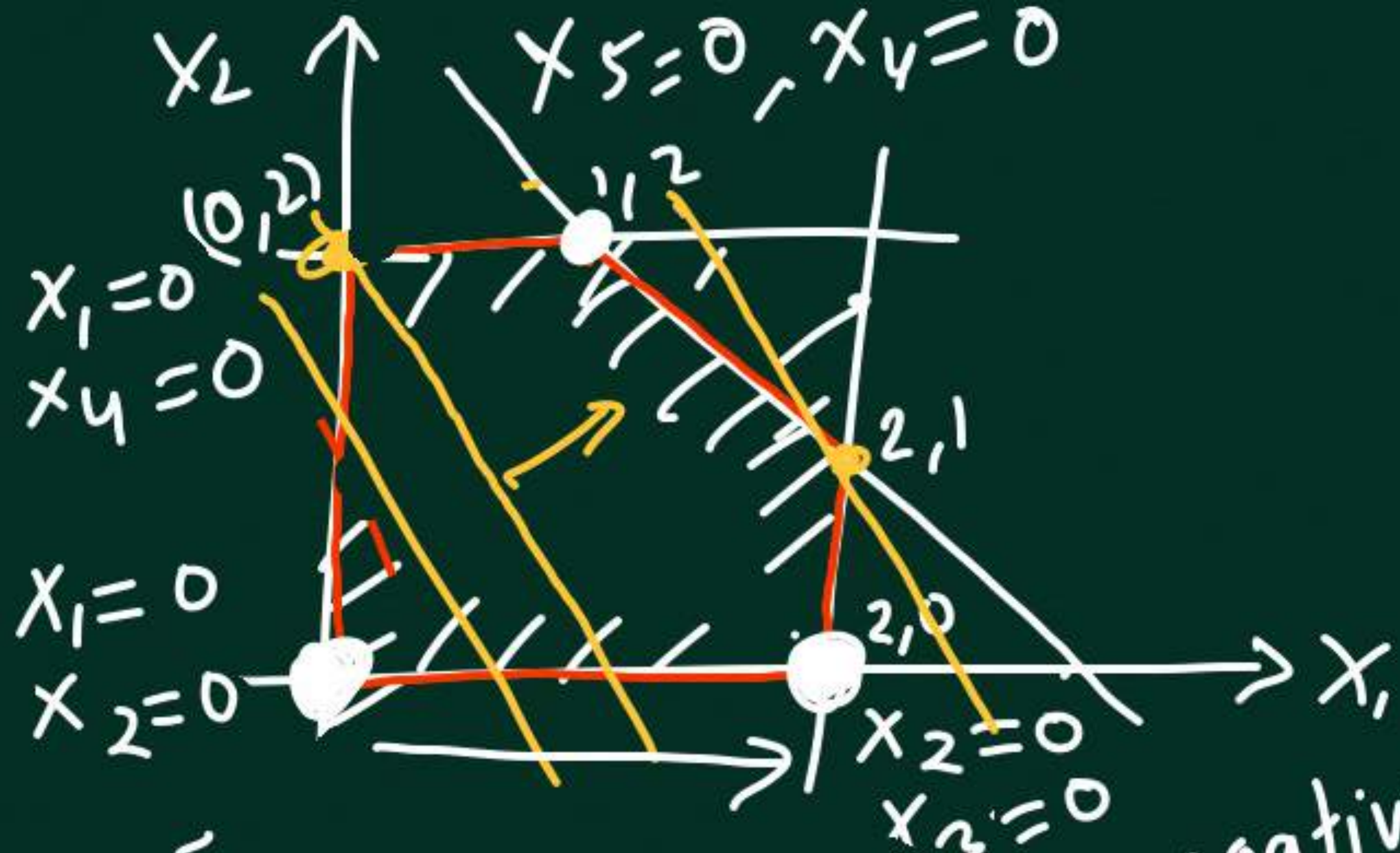
- Start from a corner
- Keep going to a neighboring corner such that the objective function increases.
- Stop when there is no neighoring corner with higher objective value.

n-dim hypercube

$$\left.\begin{array}{l} 0 \le x_1 \le 1 \\ 0 \le x_2 \le 1 \\ \quad\vdots \\ 0 \le x_n \le 1 \end{array}\right\} 2^n \text{ corners}$$

Hirsch Conjecture:

In n-dimensional polytope with m constraints there is always a path of length poly(n,m) between any two corners.



Initial basic feasible solution

corner

Assume: we have an Initial BFS

Standard Forms of linear Progra

$$\left.\begin{cases} x_1 \geq 0 \\ x_2 \geq 0 \end{cases}\right\} \text{Non-negativity Constraints.}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

$$x_1 \leq 2 \qquad x_1 = 2 - x_3, \quad x_3 \geq 0$$

$$x_2 \leq 2 \qquad x_2 = 2 - x_4, \quad x_4 \geq 0$$

$$x_1 + x_2 \leq 3 \qquad x_1 + x_2 = 3 - x_5 \quad x_5 \geq 0$$

$$\text{Max } 2x_1 + x_2$$

Start with

$x_1 = 0, x_2 = 0 \quad x_3 = 2 \quad x_4 = 2 \quad x_5 = 3$

s. $obj = 2x_1 + x_2$

ams Allow one variable to increase

Let me choose to increase x_1

x_2 will remain zero.

$x_3 \geq 0 \Rightarrow 2 - x_1 \geq 0 \Rightarrow x_1 \leq 2$

$x_5 \geq 0 \Rightarrow 3 - x_1 - x_2 \geq 0 \Rightarrow x_1 \leq 3$

$x_1 = 2 \quad x_3 = 0$

$x_2 = 0 \quad x_4 = 2$

$x_5 = 1$

Rewrite the obj function in terms of x_2, x_3

$obj = 2(2 - x_3) + x_2$

$= 4 - 2x_3 + x_2$

Should increase x_2

x_3 will remain zero

$2 - x_2 \geq 0$

$x_2 \leq 2$

$x_1 + x_2 \leq 3$

$2 - x_3 + x_2 \leq 3$

$\underline{x_2} \leq 1 + \underline{x_3}$

Change $x_2 \leftarrow 1$

$x_1 = 2$ $\quad x_4 = 1$

$x_2 = 1$ $\quad x_5 = 0$

$x_3 = 0$

rewrite obj

in terms

x_3, x_5

Obj $4 - 2x_3 +$ ___

$-$ x_5

Obj = 4 – 2x3 + x2
= 4 – 2x3 + (3–x1 –x5)
= 7 – 2x3 – x1 –x5
= 7 – 2x3 – (2–x3) – x5
= 5 – x3 – x5

The algorithm stops here because
both x3 and x5 have negative coefficients
in the objective function
Hence, we cannot increase any of the two.
The optimal value is 5 (as x3=x5=0)