CS601 Algorithms and Complexity **Jul-Nov 2021**

Homework (No submission)

Lecture 25 (Oct 11)

- Say $1 \geq \epsilon > 0$. Find $x \in (0,1)$ where the function $\frac{((1-\epsilon)+x(1+\epsilon))^2}{4x}$ is minimized.
- Say $1 \geq \epsilon > 0$. Say a coin gives heads with probability $(1-\epsilon)/2$. Show that the probability that there will be at least $k/2$ heads out of k coin tosses is upper bounded by $e^{-\epsilon k}$.
- Given n values, we want to output a value whose rank among the n values is in the range

$$(n(1-\epsilon)/2, n(1+\epsilon)/2)$$

  with probability at least $1-\delta$. Show that it suffices to take a random sample of $O(\frac{1}{\epsilon}\log(1/\delta))$ many values.
- Show the following inequality for $j \geq k/2$

$$\frac{\binom{n/4}{j}\binom{3n/4}{k-j}}{\binom{n}{k}} \leq \binom{k}{j}(1/4)^j(3/4)^{k-j}.$$

Lecture 26-27 (Oct 12-14)

- Recall the algorithm for minimum cut seen the class and the success probability bound of $2/(n(n-1))$. Can you argue that the number of minimum cuts in any graph with n vertices is at most $n(n-1)/2$.
- Let G be an undirected graphs with given weights on the edges. Weight of a cut is the sum of the weights of the cut edges. We want to find a cut with minimum weight. Modify the algorithm discussed in the class to obtain an algorithm for this. Show that the probability of success is at least $2/(n(n-1))$.