CS601 Algorithms and Complexity **Jul-Nov 2021**

Homework (No submission)

Lectures 30-32

- Reduce the independent set problem to the vertex cover problem.
- Suppose the following version of knapsack problem is NP-complete. Given a set of integer weights $w_1, w_2, \ldots, w_n$ and target weights W_1, W_2, is there a subset S of the weights whose sum is between W_1 and W_2, i.e., $W_1 \leq \sum_{i \in S} w_i \leq W_2$?

  Using this fact, prove that the following load balancing problem is NP-complete. Given a set of integer loads $t_1, t_2, \ldots, t_n$ and a target makespan T, is there a way to distribute all the loads to two machines so that the maximum load on any machine is at most T?
- *Integer programming*: Given a set of linear inequalities in variables $x_1, x_2, , x_n$, decide if there is an integer solution satisfying all of them simultaneously. For example the set

$$\begin{aligned} 0 \leq x_1, x_2 \leq 1 \\ x_1 - x_2 \geq 0. \end{aligned}$$

  has an integer solution $(1, 0)$ (and also $(1, 1)$, $(0, 0)$).

  Show that Integer Programming is NP-hard. You can try a reduction from SAT to this problem. Given a CNF Booelan formula ϕ, you need to generate a set S of linear inequalities and prove that ϕ is satisfiable if and only if the set S has an integer solution.