Reduction from SAT to IND-SET

$$\text{SAT} \le \text{IND-SET}$$

This will prove that IND-SET is NP-hard.

<u>IND-SET</u>: Given a graph G and number k, does G have an independent set of size k?

Reduction: Boolean formula $\phi \longrightarrow$ Graph $G(\phi)$, number $k(\phi)$

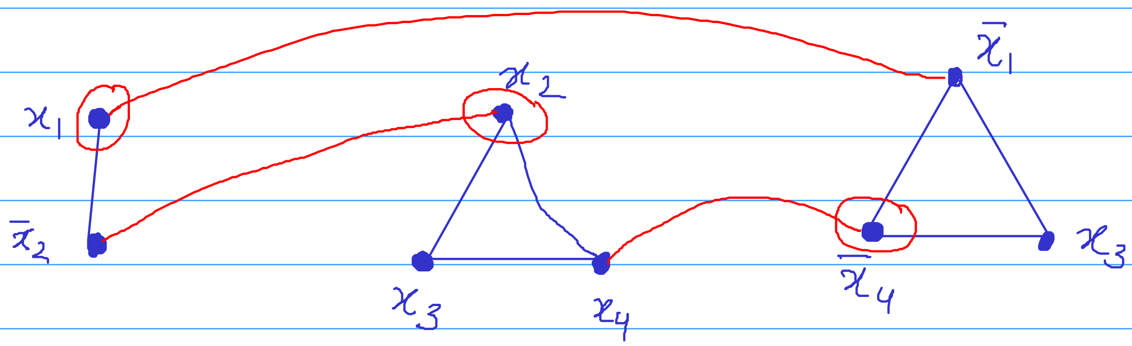
such that ϕ is satisfiable
if and only if
$G(\phi)$ has an independent set of size $k(\phi)$

Without loss of generality, we will assume that ϕ is in Conjunctive normal form ((CNF)

AND of OR clauses

$$\phi = (x_1 \vee \bar{x}_2) \wedge (x_2 \vee x_3 \vee x_4) \wedge (\bar{x}_1 \vee x_3 \vee \bar{x}_4)$$

$G(\phi)$



$k = 3$ $\quad$ $x_1 = T \quad x_2 = T \quad x_4 = F$

$x_3 = T/F$

Construction of $G(\phi)$

1. For each clause with p literals, create a complete graph with p vertices, with the literals as labels.

2. For any two vertices labeled x_i and $\overline{x_i}$, connect them with an edge.

$k(\phi)$ = number of clauses

Claim: ϕ is satisfiable
if and only if
$G(\phi)$ has an independent set of size $k(\phi)$

Proof: ($\Rightarrow$) Say ϕ is satisfiable with some assignment $x_1, x_2, \cdots, x_n \in \{\text{True}, \text{False}\}$

Each clause must have at least one literal set to True.
$\rightarrow$ Take the corresponding vertex from each clause graph.

This set is an independent set because the edges across different clauses are of the kind $(x_i, \overline{x_i})$ and both $x_i, \overline{x_i}$ could not have been true.

($\Leftarrow$) Say there is an independent set S in $G(\phi)$ with size $k(\phi)$.

S must have exactly one vertex from each clause graph.

Set the corresponding literals to true.
There is doable because an independent set cannot have both x_i and $\overline{x_i}$
It gives us a satisfying assignment.

HW

1. IND-SET $\leq$ Vertex Cover

2. Vertex Cover $\leq$ Feedback vertex set