70 marks **CS602: Quiz 1** **55 Minutes**

Note. Write your answers clearly and succinctly. Feel free to ignore hints.

Que 1 [10 marks]. True or false? No explanation required.

1. The union of two convex sets is convex.
2. The intersection of two convex sets is convex.
3. An LP and its dual LP can both have unbounded optimum.
4. For $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, if there is no solution for $Ax = b$ then there must exist $y \geq 0$ such that $y^\top A = 0$ but $y^\top b \neq 0$.

Que 2 [10 marks]. Let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Then $\max\{w^\top x : Ax \geq b,\ x \geq 0\}$ is equal to which of the following. Justify your answer.

(a) $\min\{-b^\top y : y \geq 0,\ A^\top y \leq w\}$

(b) $\min\{b^\top y : y \leq 0,\ A^\top y \geq w\}$

(c) $\min\{b^\top y : y \geq 0,\ A^\top y \geq w\}$

(d) $\min\{b^\top y : y \leq 0,\ A^\top y \leq w\}$

(e) Unbounded.

(f) None of the above, then write your own answer.

Que 3 [10 marks]. For the convex hull of following three points in $\mathbb{R}^3$, write a description in terms of linear constraints in three variables x_1, x_2, x_3.

$$(0, 1, 0), \quad (2, 0, 1), \quad (1, -1, 1).$$

Please directly write the final answer. *Hint: one way to do it is via Fourier Motzkin elimination.*

Que 4 [20 marks]. Consider the following linear program.

$$\begin{array}{rcl} \max & -x_2 & \text{subject to} \\ 3x_1 + x_2 & \leq & 6 \\ 2x_1 - 3x_2 & \leq & 4 \\ x_1 + 2x_2 & \geq & -5 \\ x_2 - x_1 & \leq & 2 \end{array}$$

1. Find the optimal value and one of the optimizing points. No justification required, just directly write the answers.
2. Write the dual linear prgoram for the given LP.
3. Find a feasible solution for the dual program such that its dual value is same as the primal optimal value in part (1). *Remark:* This would confirm that your part (1) was indeed correct.

Que 5 [10 marks]. Give a really short proof for the fact that point u is not in the cone generated by p_1, p_2, p_3, where

$$p_1 = (1, 1, 1),\ p_2 = (-1, -1, 1),\ p_3 = (1, -1, -1),\ u = (-1, 1, -1)$$

Que 6 [10 marks]. Suppose there is an $\alpha \in \mathbb{R}^n$ such that $A\alpha \leq 0$ and $w^\top \alpha > 0$. Then show that $\max\{w^\top x : Ax \leq b\}$ is unbounded.