CS602 Applied Algorithms **Spring 2021**

Homework 1 (Jan 14) No submission

- Prove that any polyhedron (i.e., a set $\{x \in \mathbb{R}^n : Ax \leq b\}$ for some matrix $A \in \mathbb{R}^{m \times n}$ and vector $b \in \mathbb{R}^m$) is a convex set.
- Suppose we want to maximize a given function $w^T x$ over a polyhedron P. If z_1, z_2 are two maximizing points, then show that their mid-point $(z_1 + z_2)/2$ will also be a maximizing point.
- Suppose P is a polyhedron given by

  $$\{x \in \mathbb{R}^n : a_i^T x \leq b_i \text{ for } 1 \leq i \leq m\}$$

  and F is a face of P described by the tight constraints

  $$a_i^T x = b_i \text{ for } 1 \leq i \leq k,$$

  where $k \leq m$. Prove that there **exists** a function $w^T x$ such that F is the face maximizing $w^T x$ over P.

  Hint: you can try to express $w^T x$ in terms of the tight constraints for F.