CS602 Applied Algorithms **Spring 2021**

Homework 2 (Jan 21) No submission

1. Let p_1, p_2, p_3 be three points in $\mathbb{R}^n$. Suppose point q is a convex combination of p_1 and p_2. And point r is a convex combination of q and p_3. Show that point r can be written as a convex combination of p_1, p_2, p_3.

2. Consider the convex region defined by $x^2 + 4y^2 \leq 4$. Can you describe this region with a set of (infinitely many) linear constraints?

3. You want to test if a given set of linear constraints is feasible. Can you do it via Fourier Motzkin elimination procedure?

4. Consider the system of linear constraints given below with variables $x_1, x_2, \ldots, x_n$.

$$x_1 \leq a_{i,2}x_2 + a_{i,3}x_3 + \cdots + a_{i,n}x_n \text{ for } 1 \leq i \leq k, \quad (1)$$
$$x_1 \geq b_{j,2}x_2 + b_{j,3}x_3 + \cdots + b_{j,n}x_n \text{ for } 1 \leq j \leq \ell. \quad (2)$$

   Now, consider another system, obtained by eliminating x_1 via Fourier Motzkin procedure.

$$b_{j,2}x_2 + b_{j,3}x_3 + \cdots + b_{j,n}x_n \leq a_{i,2}x_2 + a_{i,3}x_3 + \cdots + a_{i,n}x_n \text{ for } 1 \leq i \leq k \text{ and } 1 \leq j \leq \ell \quad (3)$$

   Show that the set of feasible solutions for the second system is same as

$$\{(x_2, x_3, \ldots, x_n) : \exists x_1 \text{ such that } (x_1, x_2, x_3, \ldots, x_n) \text{ satisfies (1) and (2)}\}.$$

5. Suppose we want to maximize a linear function over m linear constraints with n variables. Assume that all the coefficients in the linear function and in the linear constraints are at most N (in absolute value). Show that the optimal value for this LP is upper bounded (in absolute value) by $\exp(n) \times \text{poly}(N)$.