CS602 Applied Algorithms **Spring 2021**

Homework 3 (Jan 29) No submission

We had proved a version of Farkas' lemma in the class (item 1 below). Using item 1, prove the other two versions of Farkas' lemma (item 2 and 3). For this, you might convert the given set of constraints into the original form ($Ax \leq b$) and then apply the lemma in item 1. Also we had not proved the converse of Farkas' lemma. Prove the converse for all three items.

1. For $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, if there is no $x \in \mathbb{R}^n$ satisfying $Ax \leq b$ then there exists a $y \in \mathbb{R}^m$ such that
$$y \geq 0, \quad y^T A = 0, \quad y^T b = -1$$
2. For $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, if there is no $x \in \mathbb{R}^n$ satisfying $Ax \leq b,\ x \geq 0$ then there exists a $y \in \mathbb{R}^m$ such that
$$y \geq 0, \quad y^T A \geq 0, \quad y^T b = -1$$
3. For $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, if there is no $x \in \mathbb{R}^n$ satisfying $Ax = b,\ x \geq 0$ then there exists a $y \in \mathbb{R}^m$ such that
$$y^T A \geq 0, \quad y^T b = -1$$