We had proved a version of Farkas’ lemma in the class (item 1 below). Using item 1, prove the other two versions of Farkas’ lemma (item 2 and 3). For this, you might convert the given set of constraints into the original form \((Ax \leq b)\) and then apply the lemma in item 1. Also we had not proved the converse of Farkas’ lemma. Prove the converse for all three items.

1. For \(A \in \mathbb{R}^{m \times n}\) and \(b \in \mathbb{R}^{m}\), if there is no \(x \in \mathbb{R}^{n}\) satisfying \(Ax \leq b\) then there exists a \(y \in \mathbb{R}^{m}\) such that
\[
y \geq 0, \quad y^T A = 0, \quad y^T b = -1
\]

2. For \(A \in \mathbb{R}^{m \times n}\) and \(b \in \mathbb{R}^{m}\), if there is no \(x \in \mathbb{R}^{n}\) satisfying \(Ax \leq b, x \geq 0\) then there exists a \(y \in \mathbb{R}^{m}\) such that
\[
y \geq 0, \quad y^T A \geq 0, \quad y^T b = -1
\]

3. For \(A \in \mathbb{R}^{m \times n}\) and \(b \in \mathbb{R}^{m}\), if there is no \(x \in \mathbb{R}^{n}\) satisfying \(Ax = b, x \geq 0\) then there exists a \(y \in \mathbb{R}^{m}\) such that
\[
y^T A \geq 0, \quad y^T b = -1
\]