CS602 Applied Algorithms **Spring 2021**

Homework 4 (Jan 29) No submission

Q1. Write complementary slackness conditions for various standard forms of an LP and its dual LP. Show that the conditions are necessary and sufficient for optimality.

Q2. Show that if a pair of primal and dual feasible solutions satisfy approximate complementary slackness (see lecture slides for the definition), then the solutions are approximately optimal.

Q3. Complete all the details for the primal dual schema for the shortest path LP.

Q4. We wrote an LP for the minimum weight perfect matching problem in bipartite graphs. We claimed that there is always an integral optimal solution for this LP. We gave a proof outline for this. Complete all the proof details.

Q5. Suppose we write a similar LP for the minimum weight perfect matching problem in general graphs (not necessarily bipartite).

$$\begin{aligned} \min \sum_{e \in E} w_e x_e \qquad & \text{subject to} \\ x_e \geq 0 \qquad & \text{for } e \in E, \\ \sum_{e \in \delta(v)} x_e = 1 \qquad & \text{for } v \in V. \end{aligned}$$

Here $\delta(v)$ is the set of edges incident on the vertex v. Give an example of a graph with weights on the edges so that the optimal value of this LP is less than the minimum weight of a perfect matching. Note that this example has to be non-bipartite.

Q6. Suppose we have an LP ($\min w^T x$ subject to $Ax = b,\ x \geq 0$), its dual LP ($\max y^T b$ subject to $y^T A \leq w^T$). Let y be a feasible solution for the dual. If there is no primal feasible solution x that satisfies complementary slackness conditions with y, then we must be able to construct a new feasible dual solution z with greater dual objective value. Argue that such a z exists. Note that to find z algorithmically, you might need to solve another LP. Here we are only asking to show existence. We have already discussed the rough idea in the lecture on Feb 11.