The Graph structure of two-player games

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Introduction:

1. Fundamental question in Game theory: Representing preference
   - Von Neumann and Morgenstern's axiomatisation of utility
   - John Nash equilibrium concept
2. Non applicability of utility values to Ordinal Games
3. Non-convergence of Nash equilibria
4. Response Graphs

Response Graphs:

Graph $G = (N,A)$, $N=\{n1,\ldots,nk$ (finite set of nodes)$\}$, $A \subseteq N \times N$

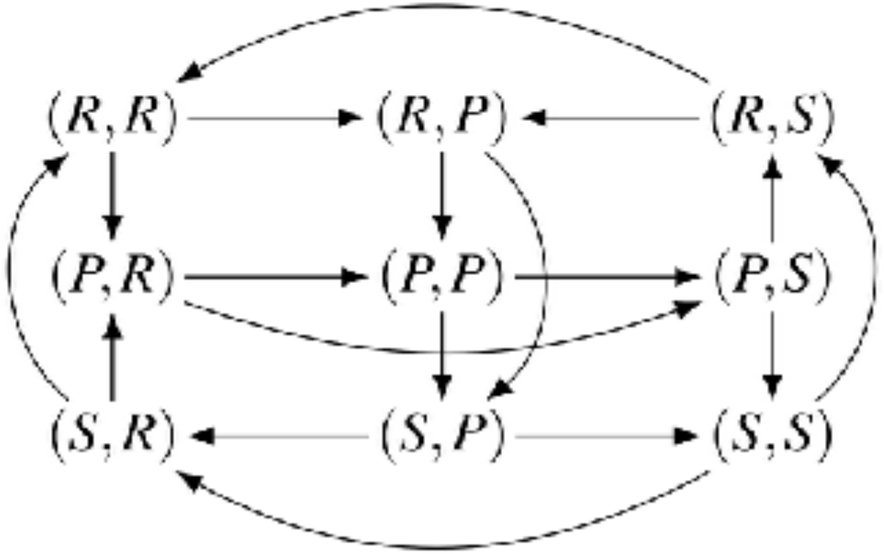
1. If arc $(x,y) \in A$ then $x \rightarrow y$
   →Preference goes from x to y

2. If arc $(x,y) \in A$ and arc $(y,x) \in A$ then x — y
   → Preference alloted equally to x and y.

The response graph of the game is the graph whose node set is $S1 \times S2$, with an arc $(s1,s2) \rightarrow (t1,t2)$ if the profiles $(s1,s2)$ and $(t1,t2)$ are i-comparable and $ui(t1,t2) \geq ui(s1,s2)$.

Example:

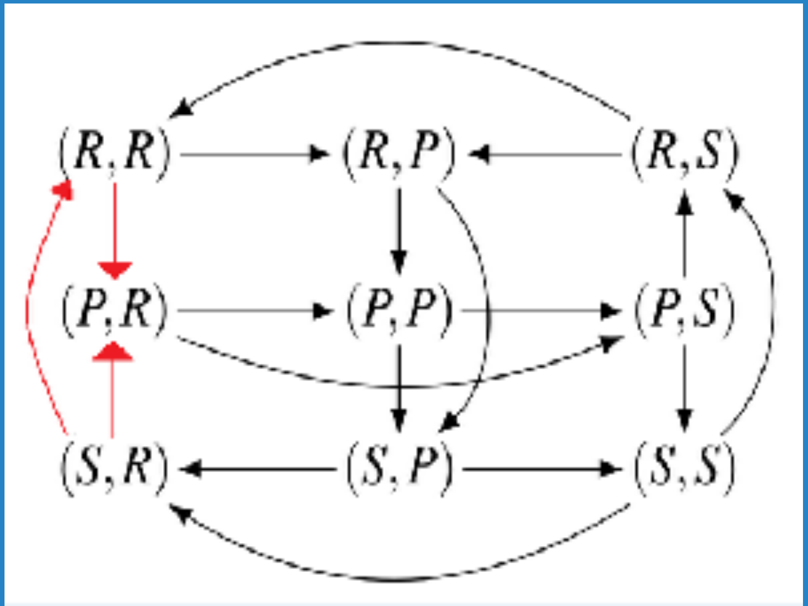
		Player 2		
		Rock	Paper	Scissors
	Rock	$0,0$	$-1,1$	$1,-1$
Player 1	Paper	$1,-1$	$0,0$	$-1,1$
	Scissors	$-1,1$	$1,-1$	$0,0$



Response Graphs:

1. Utility payoffs serve to instantiate preference orders, so cut the intermediary.
2. No need for labelling by profiles
3. ‘Sink strongly connected components’ in ‘sink chain components’

Sink Components



1. Solution concept generalizing pure Nash equilibria
2. Contained in Sink chain component
   → A topological concept emerged from Fundamental theory of Dynamical Systems.
3. Represent ‘long-run’ outcome of dynamic processes (learning or evolution of a game).
4. Dynamic and Predictive solution concept for games unlike Nash equilibria

Weighted Response Graphs:

Generalization of Response graphs for biased games.

- Arc weights equal payoff differences for associated player.

Decomposition to strategic equivalence.

- Depends on the payoff for mixed profiles
- (S,P) preferred over (P,R)

Modulo Isomorphism: preference equivalence.

The weighted response has a property that the arc (s1,s2) $\longrightarrow$ (t1,t2) is weighted by the non-negative number ui(t1,t2)− ui(s1,s2)

Preference Games:

1. Preference single dominance games
2. Preference double dominance games
3. Preference-zero-sum games
4. Preference-potential games

Preference Games:

(a) Double-dominance (b) Single-dominance (c) Coordination (d) Matching Pennies

The four non-isomorphic response graphs of generic 2×2 games.

Two-Player zero-sum and Potential Duality:

$$
\begin{tikzcd}
(c,-c) & (d,-d) \arrow[l, "c-d"'] \arrow[d, "(-b)-(-d)"] \\
(a,-a) \arrow[u, "(-c)-(-a)"] \arrow[r, "b-a"] & (b,-b)
\end{tikzcd}
$$

Two-Player zero-sum and Potential Duality:

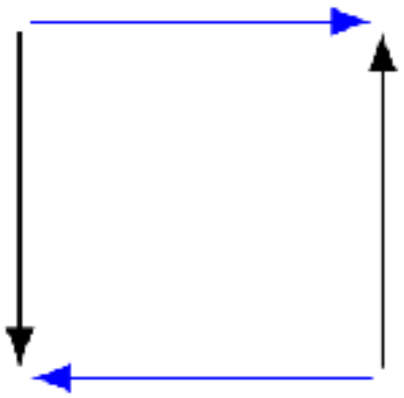


fig. CO

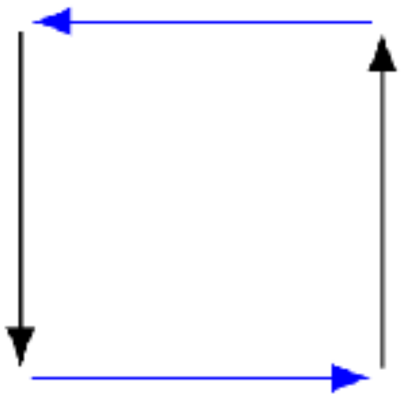
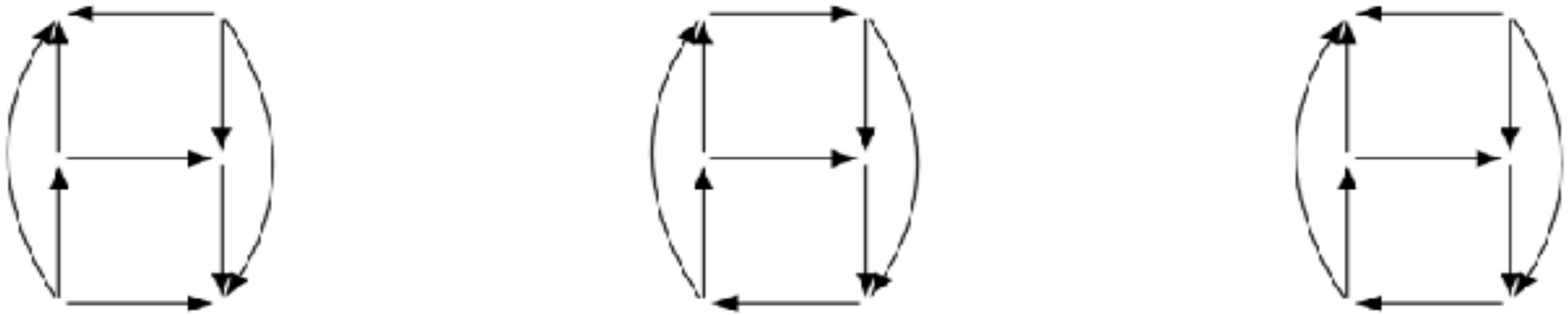


fig. MP

The CO graph on the left when reflected for second player we get graph for matching pennies

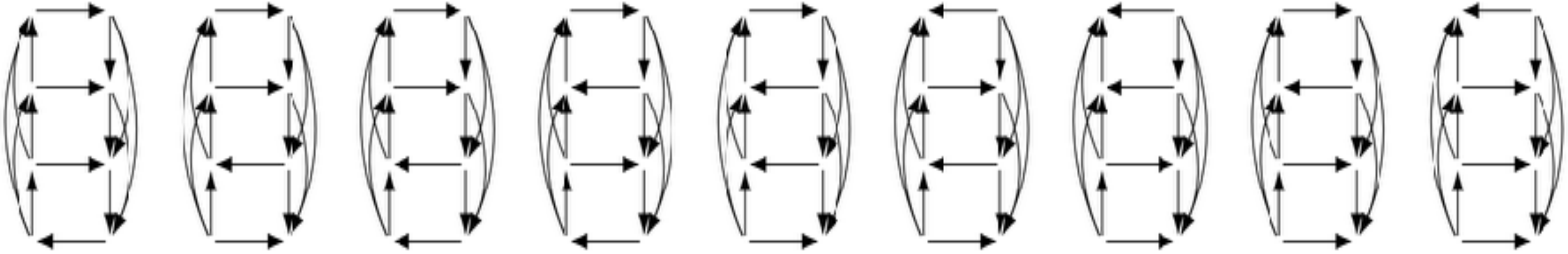
Applications: 2x3 Games



(a) 2x3 CO (b) 2x3 MP (c) 2x3 MP-CO

- Three fundamental non-isomorphic response graphs for 2x3 games
  - a) Preference-potential
  - b) Preference-zero-sum
  - c) Neither of them (unique minimal example)

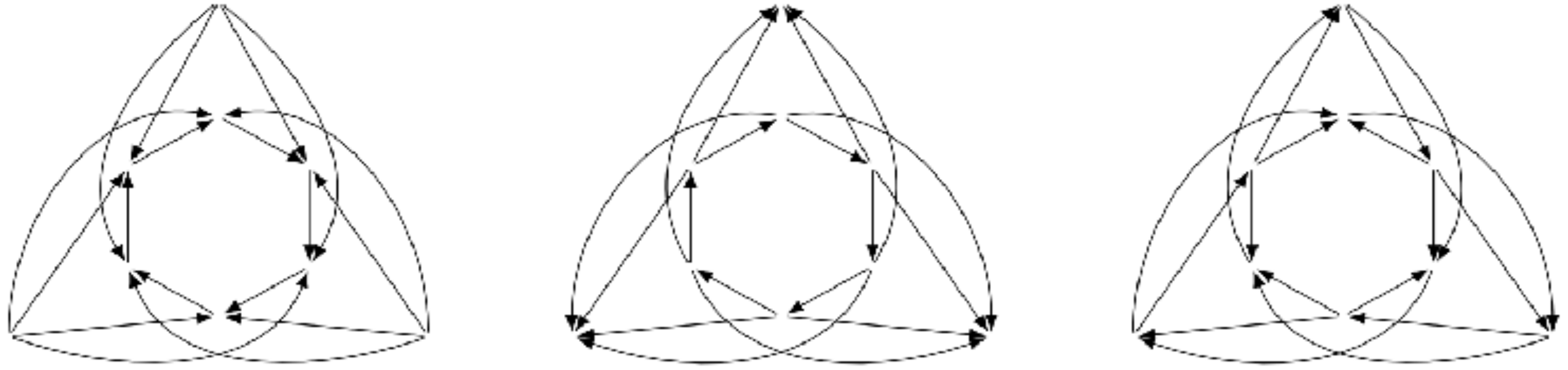
Applications: 2x4 Games



(a) 0001 (b) 0010 (c) 0011 (d) 0101 (e) 0110 (f) 1010 (g) 1100 (h) 0100 (i) 1000

- Binary encoding represents direction of arc for player 2
  - a) Preference-potential → g) and i).
  - b) Preference-zero-sum → a) and c).
  - c) Neither of them → rest of them

Applications: 3x3 Games



(a) 6-cycle-source graph (b) 6-cycle-sink graph (c) Reflected 6-cycle graph

- a) and its reversal b), do not contain MP and yet are not preference-potential.
- Reflection of player preferences in a) and b) gives c). It does not contain CO yet it is not preference-zero-sum.

Conclusion:

1. We defined a model which captures underlying notion of strategic preference without concerning about cardinal values for payoff.

2. This model can be used when access to or knowledge of cardinal payoffs is implausible.

3. We showed that two-player potential and zero-sum games have analogous properties of sink component.

References:

Biggar, Oliver, and Iman Shames. "The graph structure of two-player games." *Scientific Reports* 13, no. 1 (2023): 1833. (Main Paper)

Link: [2209.10182v2] The graph structure of two-player games (arxiv.org)

Thank You!