Spherical Mosaics with Quaternions and Dense Correlation

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Outline

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Automatic Spherical Mosaicing

Why is it called Spherical Mosaicing Algorithm ?

Because it allows any number of images to be merged into a single seamless view, simulating the image that would be acquired by a camera with a spherical field of view.

Automatic Spherical Mosaicing

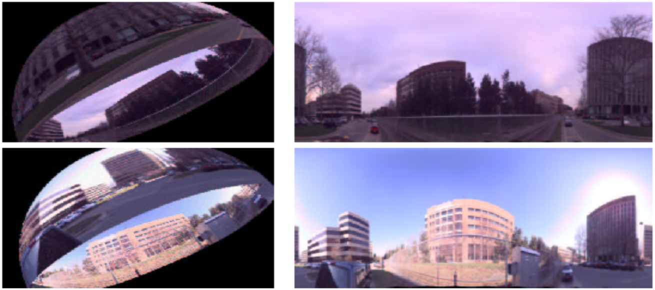


Figure: Two typical mosaics shown as spheres and cylinders

Automatic Spherical Mosaicing

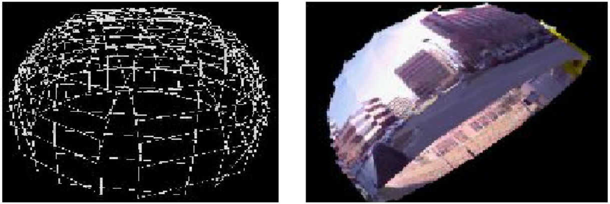


Fig: The roughly hemispherical tiling of a node of the dataset

Overview of the optimization technique

(a) (b) (c)

Figure 2: Part(a) shows one image of the hemispherical tiling blended with its adjacent images. Part (b) illustrates blurring due to incorrect pose estimates. Part (c) shows the same view after optimization.

- Figure 2-(a) – transformation maps pixels from adjacent images into a common 2-D space.

Overview of the optimization technique

- Incorrect transformations arising from inaccurate estimates of camera pose result in mismatches between pixels, causing the blurring and ghosting shown in Figure 2-(b).
- The optimization technique uses these pixel differences to refine pose estimates and eliminate the blurring artifacts, as shown in Figure 2-(c).

Overview of the optimization technique

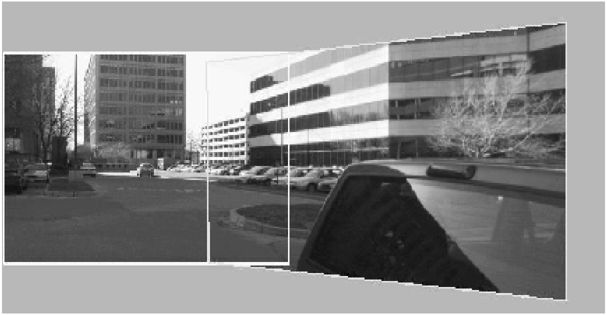
There are two methods for optimization of parameters for Spherical Mosaics:

- Local Optimization method
- Global Optimization method

Outline

Problems !!!

- The rotation computed by this method is not totally accurate and there by leaving a small/big gap between images depending on the estimates.



Problems !!!

- Thus, although this method generates visually consistent blending between adjacent images, they appear to be inadequate for recovering quantitative 3-D parameters, such as **relative rotations**
- Also, more fundamentally, relying on local pairwise warps to compute global quantities can lead to inconsistencies in the computed internal parameters and rotations
- Therefore we go for a global optimization method that addresses these problems!!

Outline

Why Spherical Mosaicing using Global Co-relation?

- Produces the "best" possible rotations for each image, given initial estimates
- Global consistency is guaranteed becoause aggregate rotation along any cycle in the image adjacency map is the identity
- Thus it avoids the possibility of "gaps" arising from inconsistent pairwise estimates
- It optimize a global correlation function defined for adjacent images w.r.t all orientations (represented as quaternions)

Rotations as Quaternions

- Quaternions represent rotations as four- dimensional unit vectors q $= [q_0, q_x, q_y, q_z]$ where ${q_0}^2 + {q_x}^2 + {q_y}^2 + {q_z}^2 = 1$
- Quaternion representation is compact in comparison to the orthonormal representation**(only four parameters instead of nine)**
- The following expression represents a quaternion as a 3×3 orthonormal matrix for rotation:

$$\begin{bmatrix} ({q_0}^2 + {q_x}^2 - {q_y}^2 - {q_z}^2) & 2(q_x q_y - q_0 q_z) & 2(q_x q_z + q_0 q_y) \\ 2(q_y q_x + q_0 q_z) & ({q_0}^2 - {q_x}^2 + {q_y}^2 - {q_z}^2) & 2(q_y q_z - q_0 q_x) \\ 2(q_z q_x - q_0 q_y) & 2(q_z q_y + q_0 q_x) & ({q_0}^2 - {q_x}^2 - {q_y}^2 + {q_z}^2) \end{bmatrix}$$

Inputs and Outputs of the Algorithm

- Describes an algorithm for generating spherical mosaics from a collection of images acquired from a common optical center.
- Inputs
  - Arbitrary number of partially overlapping images
  - An adjacency map relating the images
  - Initial estimates of the rotations relating each image to a specified base image
  - Approximate internal calibration information for the camera.
- Outputs
  - Is a rotation relating each image to the base image
  - Revised estimates of the camera's internal parameters.

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Features Of the Algorithm

- Image capture instrumentation provides both an adjacency map for the mosaic, and an initial rotation estimate for each image.
- It optimizes an objective function based on a global correlation of overlapping image regions.
- Representation of rotations significantly increases the accuracy of the optimization.
- This representation and use of adjacency information guarantees globally consistent rotation estimates.

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Spherical Mosaicing Algorithm

- The algorithm minimizes a **Global Error Function**: $E = \sum_{i,j} E_{ij} + E_{ji}$ where E_{ij} is the *SSD* error between luminance values of adjacent images i and j
- $E_{ij} = \sum_{x_i, y_i} (L_i(x_i, y_i) - L_j(P_{ij}(x_i, y_i)))^2$ and P_{ij} maps coordinates of image i to those of image j. It is nothing but a **2-D Projective Transformation matrix**.
- This error function is minimized by computing *derivatives* w.r.t each orientation and using **LM Nonlinear Optimization** starting from the initial orientations

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Steps Of Computation...

- For a single error term for images i and j of the form :

$\boxed{e^2_{x,y} = (L_i(x,y) - L_j(x'',y''))^2}$ with $\boxed{x'' = \frac{x'}{z'} \quad y'' = \frac{y'}{z'}}$ and

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = v' = P \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = KR'R^{-1}K^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

the *derivatives* are computed as: $\boxed{\frac{\partial v'}{\partial q} = KR'(\frac{\partial R^{-1}}{\partial q})v} \qquad (1)$

where

$$v = K^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Steps Of Computation...

- Then, the *derivative* of the term $e_{x,y}$ w.r.t the quaternion $\mathbf{q}$ is given by: $\boxed{\frac{\partial x"}{\partial q} = \frac{\frac{\partial x'}{\partial q} - x" \frac{\partial z'}{\partial q}}{z'} \quad \frac{\partial y"}{\partial q} = \frac{\frac{\partial y'}{\partial q} - y" \frac{\partial z'}{\partial q}}{z'}}$ (2)
- Using (1) and (2) we get: $\boxed{\frac{\partial e_{x,y}}{\partial q} = \frac{\partial L_j}{\partial x"} \frac{\partial x"}{\partial q} + \frac{\partial L_j}{\partial y"} \frac{\partial y"}{\partial q}}$
- These *derivatives* $\frac{\partial L_j}{\partial x"}$ and $\frac{\partial L_j}{\partial y"}$ are approximated using the following convolution matrices applied at $(x", y")$

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Steps Of Computation. . .

- The **Gradient** term corresponding to the quaternion q_i is computed over all terms that depend on q_i : $\boxed{G_i = \sum_{x_i, y_i} e_{x_i, y_i} \frac{\partial e_{x_i, y_i}}{\partial q_i}}$ It is computed in the coordinates of image i, w.r.t the quaternion q_i
- Similarly, the **Hessian** term corresponding to two adjacent images i and j is: $\boxed{H_{ij} = \sum_{x_i, y_i} \frac{\partial e_{x_i, y_i}}{\partial q_i} (\frac{\partial e_{x_i, y_i}}{\partial q_i})^T}$ It is computed in the coordinates of image i, w.r.t the quaternions associated with images i and j, respectively

Steps Of Computation...

- In an unconstrained optimization, the increments would be computed as $-H^{-1}G$
- Applying these increments directly to the q_i, however, would produce **Non-Unit Quaternions** which do not correspond to pure rotations !!
- To constrain the updated quaternions to be unit vectors, we enforce the following additional constraints on the increments
  $\boxed{\delta q_i : \forall i : q_i \dot{\delta} q_i = 0}$
- Applying these q_i moves the q_i tangentially to the unit four-sphere

Steps Of Computation. . .

- Using **Lagrange multipliers** λ_i to enforce these constraints, the equation for computing the increments becomes:

$$\begin{bmatrix} H & Q \\ Q^T & 0 \end{bmatrix} \begin{bmatrix} \Delta Q \\ \Lambda \end{bmatrix} = - \begin{bmatrix} G \\ 0 \end{bmatrix}$$

where

$$Q = \begin{bmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & q_n \end{bmatrix}, \Delta Q = \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \vdots \\ \delta q_n \end{bmatrix}, \Lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{4n} \end{bmatrix}$$

- The optimization solves the above equation for ΔQ and Λ
- Convergence is detected when the value of the objective function changes by less than some threshold

Internal Camera Parameters and its Relative Importance

- In addition to estimating orientations, the algorithm also performs an optimization on the internal camera parameters
- The overall optimization alternates between a step that updates all rotations, and a step that updates internal parameters of the camera
- The new parameters are computed using derivatives of v' w.r.t the camera focal length $\mathbf{f}$ and image principal point (c_x, c_y)

Internal Camera Parameters and its Relative Importance

- $$\frac{\partial v'}{\partial f} = (\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} R'R^{-1}K^{-1} + KR'R^{-1} \begin{bmatrix} -\frac{1}{f^2} & 1 & \frac{c_x}{f^2} \\ 0 & -\frac{1}{f^2} & \frac{c_y}{f^2} \\ 0 & 0 & 0 \end{bmatrix}) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- $$\frac{\partial v'}{\partial c_x} = (\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} R'R^{-1}K^{-1} + KR'R^{-1} \begin{bmatrix} 0 & 0 & -\frac{1}{f} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- $$\frac{\partial v'}{\partial c_y} = (\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} R'R^{-1}K^{-1} + KR'R^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{f} \\ 0 & 0 & 0 \end{bmatrix}) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Internal Camera Parameters and its Relative Importance

Are all internal parameters equally important for optimization?

A simplified analysis shown below tells us that determining the **focal length** accurately is more important than determining the coordinates of the image principal point

Sample Image

Image after optimization !!!

Outline

Discussion

- Two methods for optimization of the spherical mosaic formed from the initial estimates were described. The first method use the local optimization technique while the second used the global optimization technique.
- Let us see some of the merits and demerits of this approach !!

Merits Of the Approach

- Global Optimization technique provides better and accurate results than the local optimisation technique used previously.
- It provides robust automatic estimation of internal camera parameters as a by-product.
- The results are quite independent of the errors in estimating the initial principal-point estimate.
- It produces an image with an effectively super-hemispherical field of view, eliminating the ambiguity between camera translation and camera rotation found in narrow field-of-view images.

Merits Of the Approach

- The spherical mosaiced image formed can be of any desired effective resolution, subject to the choice of optics and number of raw images that are composited.
- Spherical mosaicing allows the resulting mosaic to be treated as a rigid, composite image.

Demerits Of the Approach

- Straightforward implementation of this algorithm fails where there are large textureless regions.
- High-pass filtering of images is necessary to remove the large textureless regions. But this introduces many discontinuities into previously smooth image regions, corrupting the derivative computations and preventing convergence. Thus their implementation must filter the images to remove textureless regions, while simultaneously preserving smoothness. Thus band-pass filtering is preferred.
- Due to traversing and mapping pixels in each image, and accumulating global derivatives, the computational costs increases.

Demerits Of the Approach

- The algorithm fails to converge for large errors in estimating focal length. Thus, fairly good camera calibration is required to provide initial focal length estimates.
- The algorithm uses a gradient based approach for optimization and may get stuck with the problem of local minima. Other optimization techniques like *Simulated Annealing* might provide better results.

Conclusion

- Paper describes two methods to recover relative rotations and internal camera parameters for the set of images acquired from a common optical center.
    - **First** - A closed-form solution using eigen-vectors of 8-parameter warps and local optimization of spherical mosaics parameters. But this method yields quantitatively inaccurate results.
    - **Second** - Solves the above problem by computing rotations and internal parameters directly from image-space correlation using a global optimization technique

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- There are several benefits in performing the spherical mosaicing optimization
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THANK YOU !!!