A Comparison of Some Methods for Direct 2D Reconstruction from Discrete Projections

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Introduction

- Tomographic acquisitions can be described as mathematical transforms
- Direct reconstruction methods aim to compute an accurate inverse for such transforms
- **Aim**: To obtain a high-fidelity image of the original object from a limited set of measurements by applying a suitable inversion technique

Experiment

- Comparison of three existing direct inversion techniques for sets of discrete projections:
  - Radon inverse-Radon
  - Least squared error method
  - Filtered back-projection for Mojette inversion

Original images used for testing reconstruction algorithms

Radon inverse-Radon Reconstruction

- Radon projections taken along set of angles
- Each angle equals $\tan^{-1}(\frac{p}{q})$ corresponding to the discrete angle (p, q); p and q being co-prime integers
- Reconstruction using Shepp-Logan filtered back-projection

Reconstruction by Radon i-Radon technique

Least Squared Error Reconstruction

- The Mojette back-projected image m and the original image im are related by [1]:

$$m_{p,q}(x, y) = im(x, y) * h_{p,q}(x, y)$$

$*$ denotes convolution operator

- $h_{p,q}$: the PSF corresponding to the Mojette angle set (p, q) along which the projections are taken

- In frequency domain: $M(u, v) = IM(u, v) \cdot H(u, v)$
- $H(u, v)$ is estimated using a least squared error technique
- **Reconstruction Process**: Direct deconvolution of $H(u, v)$ with the Mojette back-projected image

Reconstruction by least squared error technique

Filtered Back-Projection Reconstruction

- $H_{p,q}(u, v)$- regularized with a weighted filter
- **Reconstruction Process**: Direct deconvolution of the filtered Point Spread Function (PSF) with the Mojette back-projected image
- Equivalent to regularizing the back-projected image in the frequency domain and then taking the inverse Fourier transform
- Extremely low values (below a fixed threshold) in the FFT of the filtered PSF are replaced by the mean evaluated over neighborhoods of fixed size

Reconstruction by filtered back-projection technique

- **This method performs better than the first two**

Filtration

- The PSF filtration [2]

$$PSF_{\text{modified}} = \begin{cases} PSF. \times \texttt{wnp} & K \geq 1 \\ PSF. \times \texttt{tnp} & K < 1 \end{cases}$$

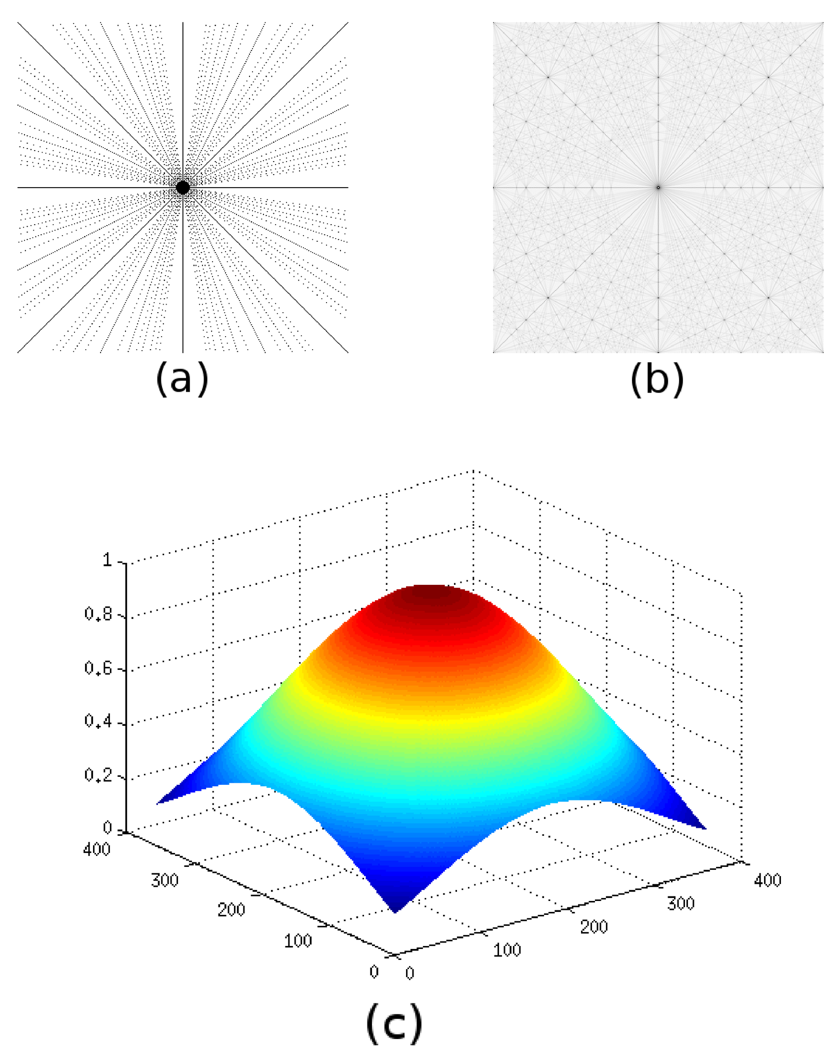
$$\texttt{wnp} = (\texttt{wp} \star \texttt{wn}) \star (D \star D)$$

$$\texttt{tnp} = (\texttt{wp} \star \texttt{wn})$$

$.\times$ denotes 'point-to-point' multiplication

$\star$ denotes 'cross-correlation'

- wnp (the filter) is given by cross-correlation of the back-projected images wp and wn
- wp is given by back-projection of delta image along the set of angles that was used while taking measurements
- wn is given by back-projection of delta image along complementary set of angles
- D denotes the circular region of interest
- $K = k/N$; k being the Katz number given by: $k = 1 + max(\sum_i |p_i|, \sum_i |q_i|)$ [3] and $N \times N$ being the dimension of the image.
- The Katz number is dependent on:
  - the number of views
  - the specific set of angles (p, q) chosen.



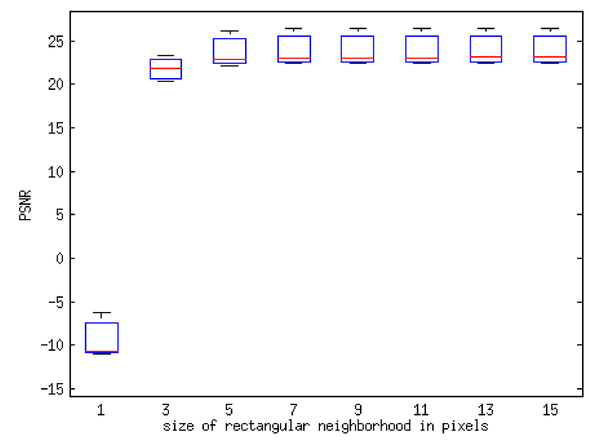
(a) Raw PSF (b) FFT of raw PSF (c) Filter

Analysis

- We observed the quality of reconstruction for different values of threshold

Analysis

- The sensitivity of image reconstructions to the selected value of the threshold is quite weak over a wide range of image sizes and Katz values.



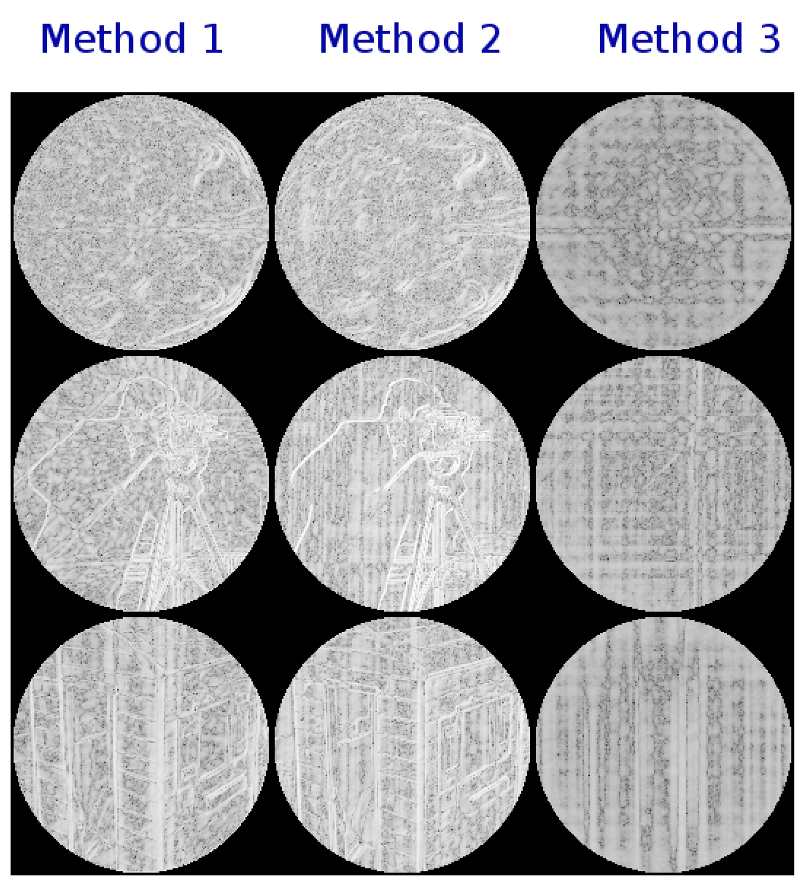
Quality of reconstruction as the size of neighborhood over which mean is evaluated is varied

Katz ratio	Image size: 179x179 # shortest angles	Peak PSNR	fval	Image size: 89x89 # shortest angles	Peak PSNR	fval	image size: 43x43 # shortest angles	Peak PSNR	fval
0.46	33	16.6	13	21	18.13	8	13	17.65	8
0.7	44	20.52	16	28	21.36	10	17	20.12	7
0.88	52	22.39	15	32	22.26	11	20	22.2	8
1.10	60	23.58	14	38	23.63	11	24	23.36	7
1.30	64	24.24	15	42	24.69	11	25	24.34	8
1.75	81	26.51	13	50	26.75	10	31	26.44	9

Table 1: Variation of optimal threshold (fval) with image size and Katz ratio

Comparison and Inferences

- The reconstruction errors in filtered back-projection method are least dependent on the image structure and orientation



Absolute error (in log scale) on applying different reconstruction techniques using the shortest 200 projection angles

- The optimal threshold is slightly dependent on image size [larger sized PSF $\implies$ higher frequency content $\implies$ requirement of relatively higher threshold]
- **Future Work:** Modify the filter such that there is no need for a final threshold

References

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[3] Katz, M.: Questions of Uniqueness and Resolution in Reconstruction from Projections. Volume 26 of Lecture Notes in Biomathematics. (1978)

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