A Finitary Analogue of the Downward Löwenheim-Skolem Property

Abhisekh Sankaran

IIT Bombay

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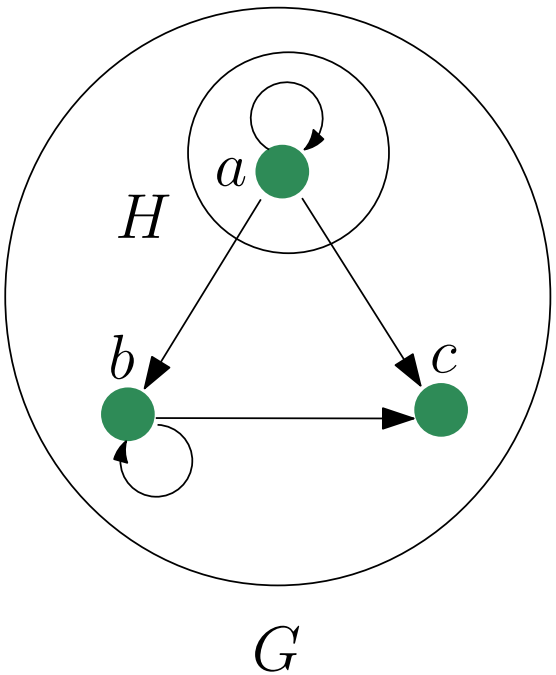
Similarity of structures



Let $\phi = \exists x \forall y E(x, y)$

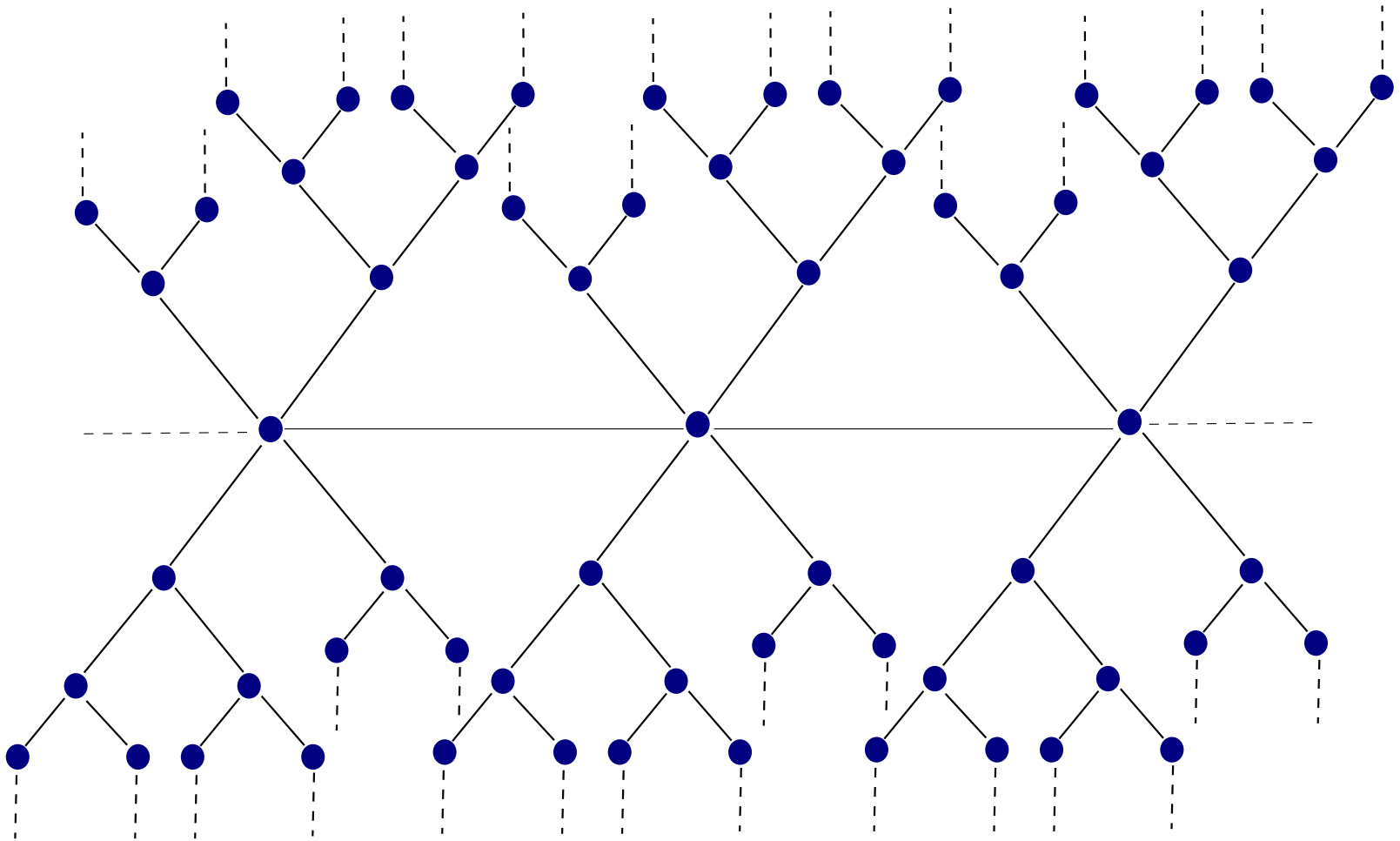
Then $G \models \phi$.

Observe that $H \models \phi$ as well.

Then G and H are "similar" w.r.t. ϕ.

- We say two graphs G and H are similar w.r.t a logic (like first order logic), or simply "similar", if G and H agree on all properties expressible in the logic.

The Downward Löwenheim-Skolem Property (DLSP)



- G of infinite size

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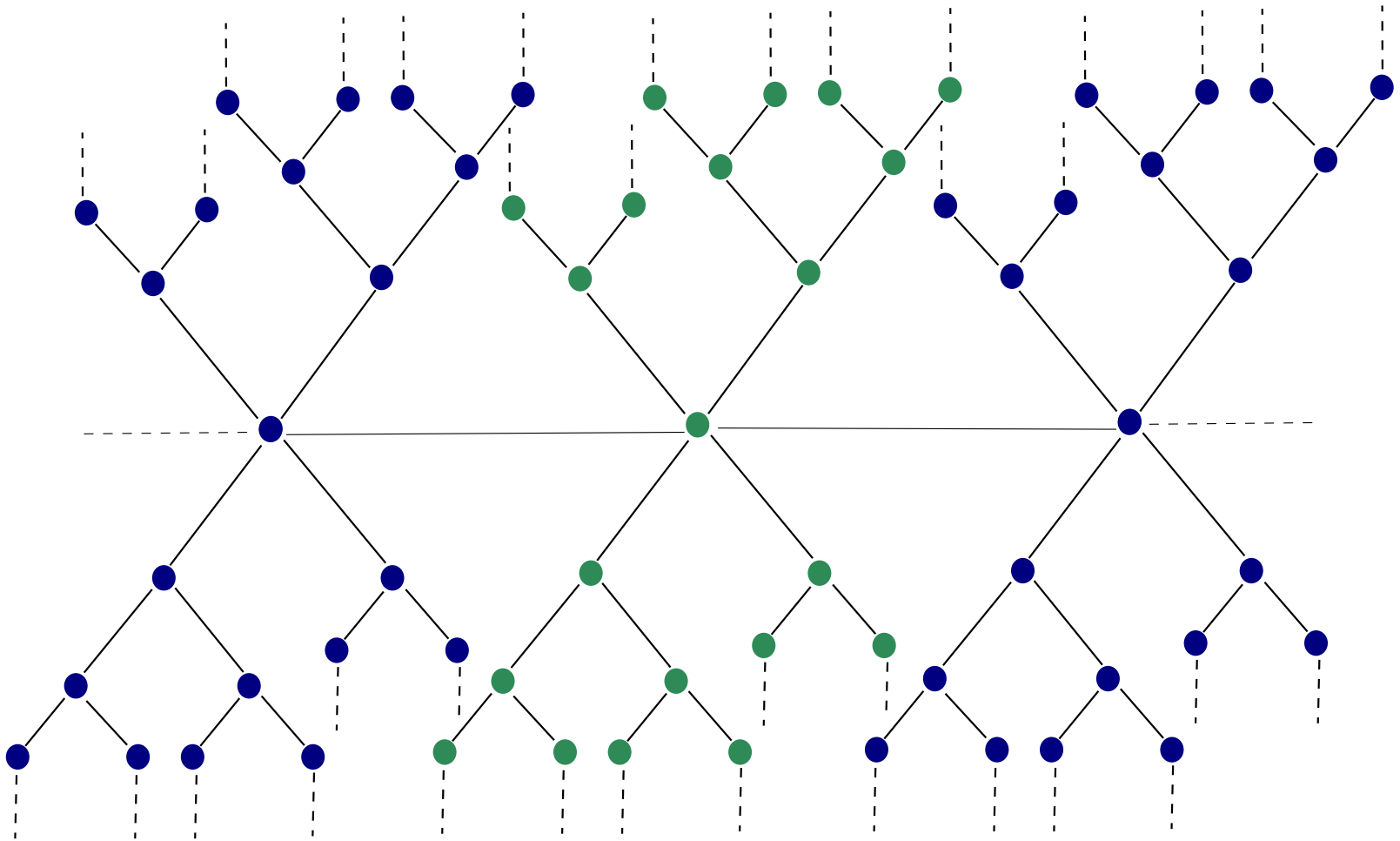

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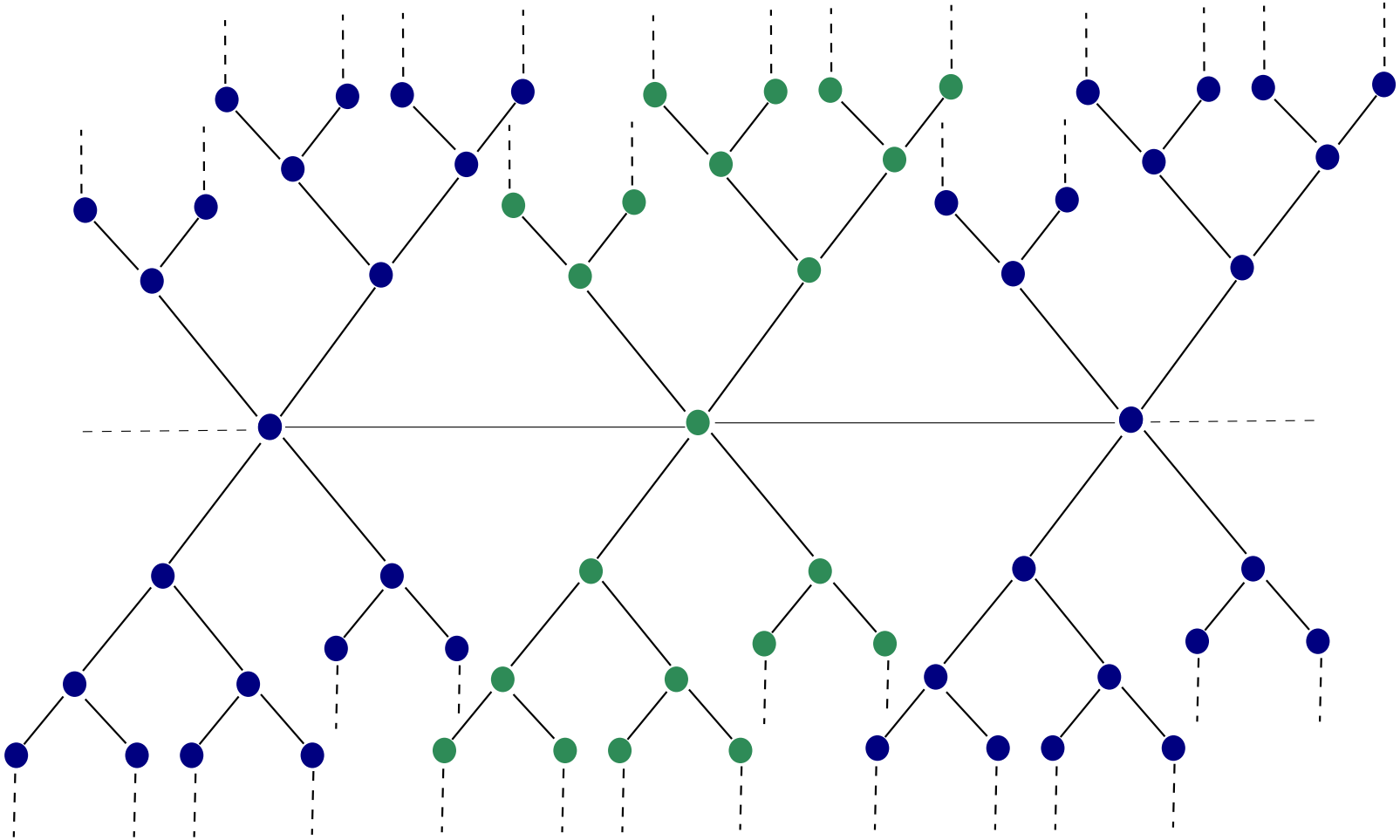
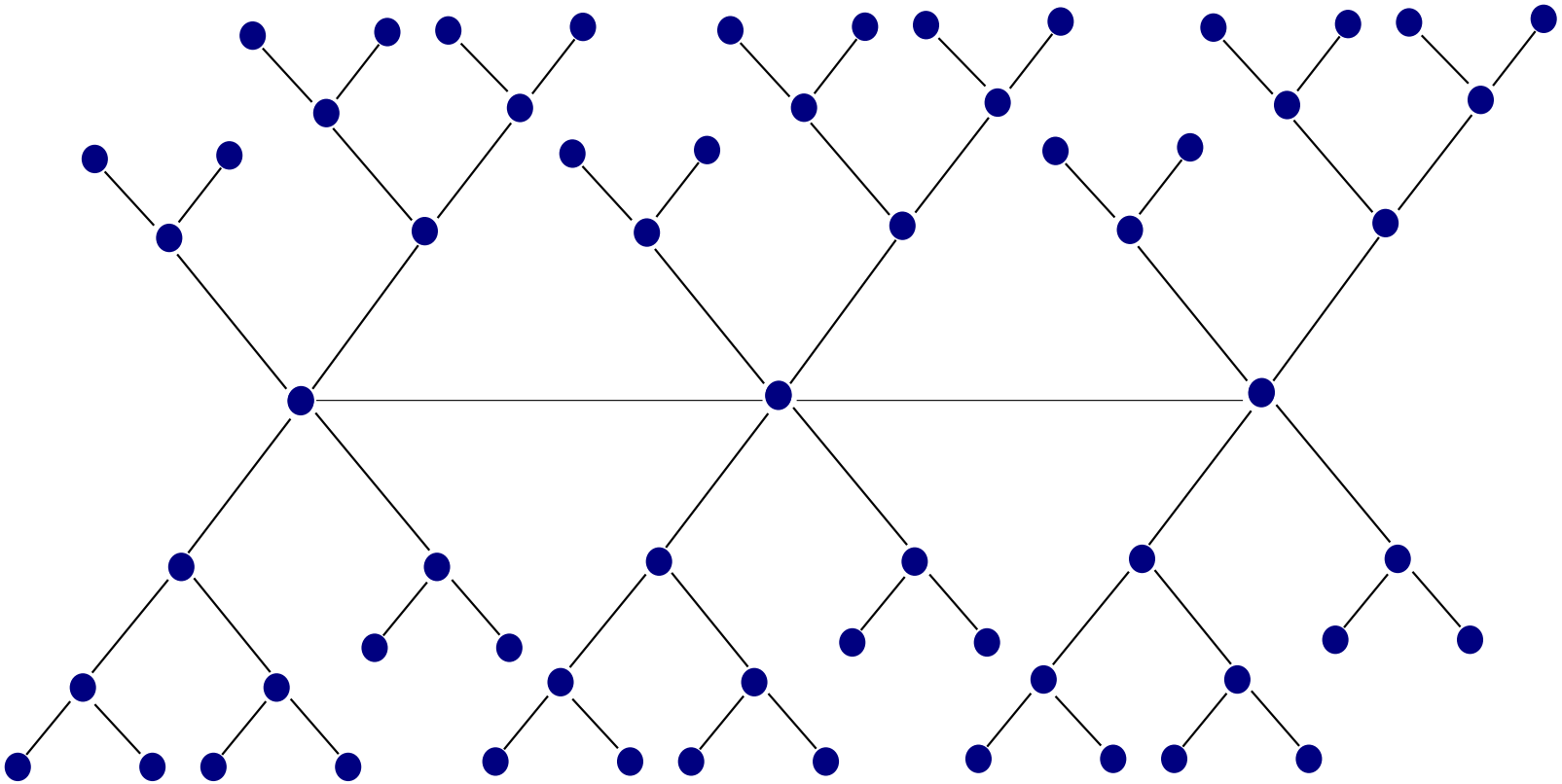


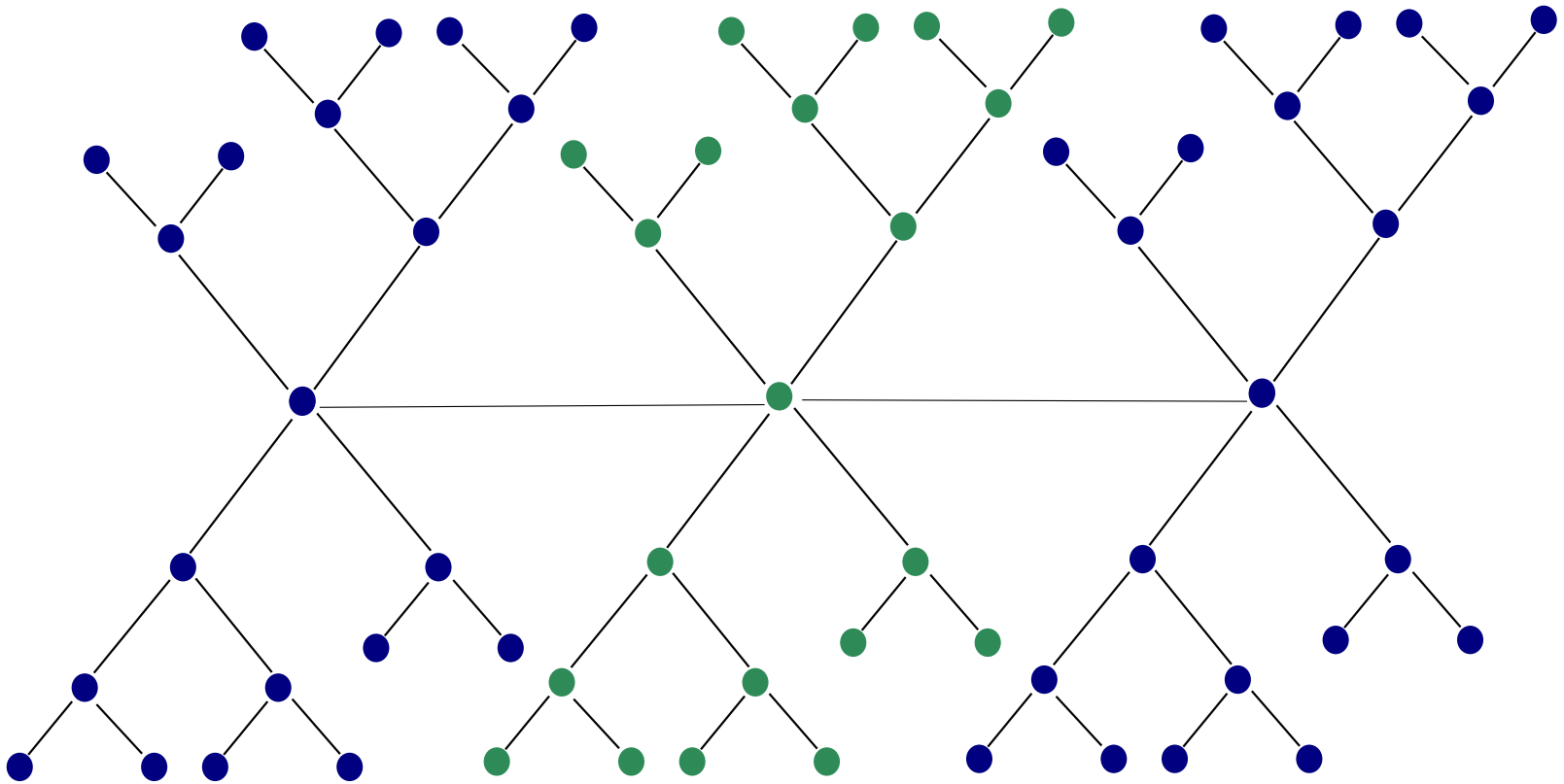
"A large (infinite) structure contains a similar small (infinite) substructure"

A finitary analogue of DLSP



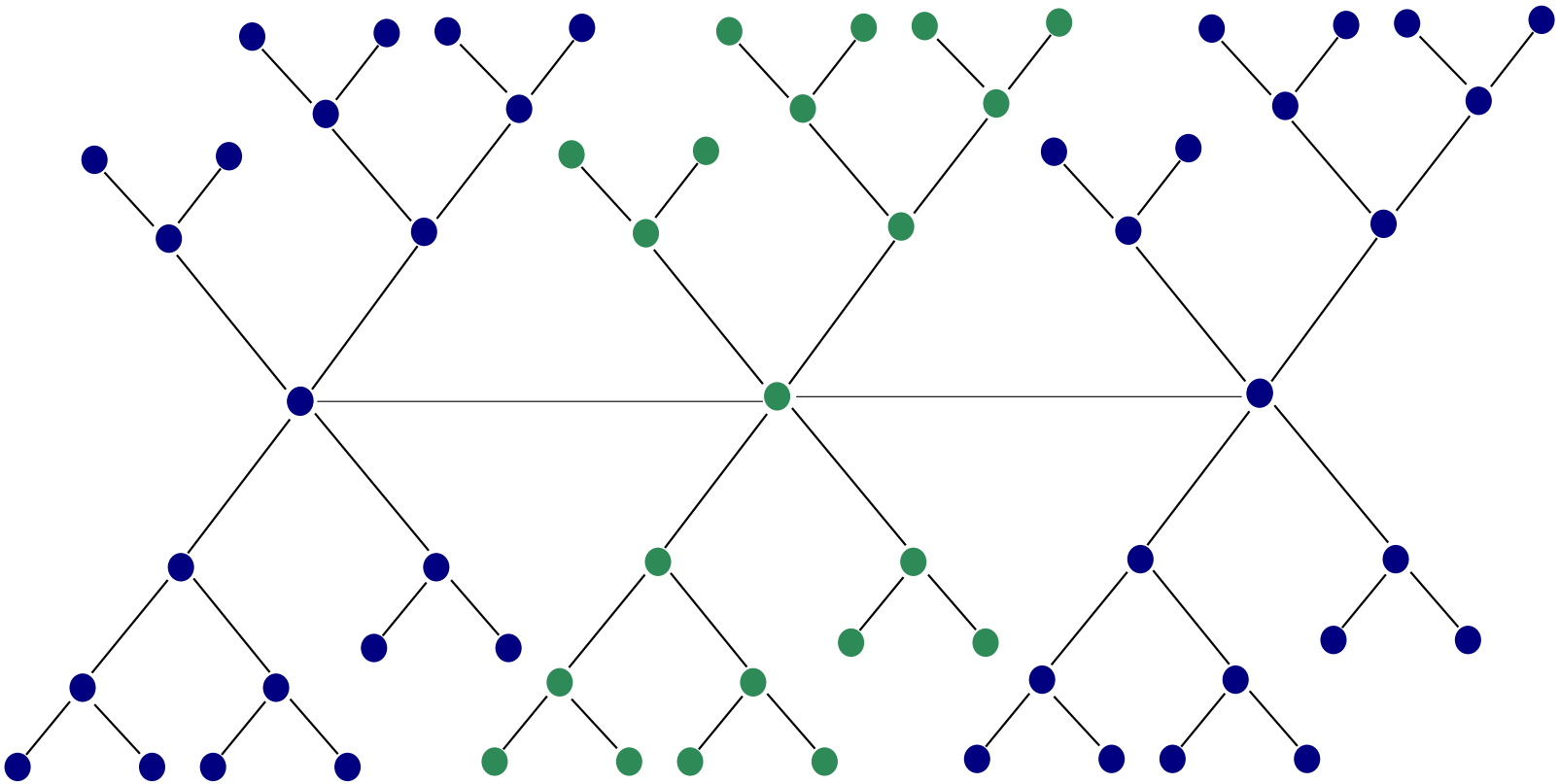
- G of finite size

A finitary analogue of DLSP



● G of finite size G "similar" to H ● H of bounded size

A finitary analogue of DLSP



"A large (finite) structure contains a similar small (finite) substructure"

Relevance to computer science

- Classical math. structures used in CS: words/strings, trees (unordered, ordered, ranked), grids
- Graph theory: cliques, n-partite graphs, cographs, hamming graphs, graphs of bounded tree-depth
- Compilers: nested words
- Databases: DATALOG, CSPs, conjunctive queries
- Poset theory: well-quasi-ordering

Philosophical import:
A finitary analogue of a beautiful "infinitary" idea can be widely useful in the finite world!