Midsemester Exam

CS709

23-Feb-2017 (2pm-5pm)

Note: Please answer the questions using rigorous and succinct mathematical arguments. Simplify expressions as much as possible. Always clearly justify your Yes/No answers.

Total Marks 45. Total time 3hrs.

Problem 1. Is the function \( f_r : \mathbb{R}^n \mapsto \mathbb{R} \), defined below, convex for any given natural number \( r \) between 1 and \( n \)?

\[
 f_r(x) \equiv \text{sum of } r \text{ largest components/entries of } x.
\]

[2 Marks]

(Appeared 15-Sep-2012 Midsem)

Problem 2. Consider the set of all symmetric matrices of size \( n \) (denoted by \( S_n \)) endowed with the usual +, ·, \( \langle \cdot, \cdot \rangle_F \) to form an inner-product space. Let \( C \subset S_n \) be the following:

\[
 C \equiv \{ M \in S_n \mid M \succeq 0, \text{trace}(M) = 1 \}.
\]

In the context of this space associated with \( S_n \), what is the dimensionality of \( C \) and \( C^o \)? Provide a simplified expression for \( C^o \).

[8 Marks]

(Appeared 19-Aug-2011 Quiz-1)

Problem 3. Is the Log-sum-exp function \( f \), defined below, convex?

\[
 f : \mathbb{R}^n \mapsto \mathbb{R}, \quad f(x_1, \ldots, x_n) \equiv \log \left( \sum_{i=1}^{n} e^{x_i} \right).
\]

[5 Marks]

(Solved example in Boyd pg74)
Problem 4. Is the function $f : \mathbb{R}^{m \times n} \mapsto \mathbb{R}$, defined below, convex for any $p, q \in [1, \infty]$?

$$f(X) \equiv \max_{u \in \mathbb{R}^m, v \in \mathbb{R}^n} e^{u^\top X v}$$

s.t. $\|u\|_p \leq 1$, $\|v\|_q \leq 1$,

[5 Marks]

Appeared 16-Sep-2011 Midsem

Problem 5. Prove the following statement: If a cone admits a finite primal description then it will also admit a finite dual description.

[5 Marks]

Discussed in Tutorial

Problem 6. Is the negative harmonic mean function $h : \mathbb{R}^n \mapsto \mathbb{R}$, defined below, convex?

$$h(x_1, \ldots, x_n) \equiv \frac{-n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}, \forall x_i > 0.$$  

[5 Marks]

Appeared 15-Sep-2012 Midsem

Problem 7. Consider $K = \{(u, v) \in V \times \mathbb{R} \mid \|u\| \leq v\}$, where $V$ is a given (but arbitrary) set of vectors forming an inner-product space $V$. Note that $\| \cdot \|$ used in defining $K$ may NOT be the norm induced by the inner-product in $V$. Write down a simplified expression for the dual cone $K^*$ (in the space that is direct-sum of spaces $V$ and $\mathbb{R}$).

[5 Marks]

Solved example in Boyd E.g. 2.25

Problem 8. Let $P$ be an arbitrary (but given) polyhedron in $\mathbb{R}^n$. Pose the problem of finding the largest $\| \cdot \|_p$-norm ball\(^1\) (here, $p \geq 1$) lying inside the polyhedron as a convex problem. Further, do you think this problem can be posed as a convex program with finite number of linear inequality constraints\(^2\)?

[10 Marks]

Appeared 16-Sep-2011 MidSem

\(^1\)Needless to say, the center of the p-norm ball need not be the origin.

\(^2\)In this case the domain of the convex program you write needs to be a vector space. Else it is trivial to answer this question.