CS 740: Problem Set 1: Due: 06:50 AM 06-Aug

- Please write (only if true) the honor code. If you used any source (person or thing) explicitly state it. This is an individual assignment.
- Submit two files in folder name hw01.zip on moodle
  1. readme.txt (case sensitive name. This text file contains identifying information, honor code, links to references used and reflection essay – more on this later)
  2. hw01.pdf (case sensitive and don't add other characters) this file should have the solution.
  3. Do not include any group name or identification in submission files.

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1. Construct a matrix with the required property or say why that is impossible:

   (a) Column space contains $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   Answer: Not possible.

   $$\mathbf{n}^\top \mathbf{v}_1 = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot (-3) = 0$$
   $$\mathbf{n}^\top \mathbf{v}_2 = 1 \cdot 2 + 1 \cdot (-3) + 1 \cdot 5 = 4 \neq 0$$

   Marking scheme: 4 marks. No partial credit.

   (b) Row space contains $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   (c) $\mathbf{A}x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a solution and $\mathbf{A}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) Every row is orthogonal to every column (A is not the zero matrix)

(e) Columns add up to a column of zeros, rows add to a row of 1's

2. If $\mathbf{AB} = 0$ then the columns of $\mathbf{B}$ are in the ________________ of $\mathbf{A}$. The rows of $\mathbf{A}$ are in the ________________ of $\mathbf{B}$. Why can't $\mathbf{A}$ and $\mathbf{B}$ be 3 by 3 matrices of rank 2?

3. This system of equations $\mathbf{A}\mathrm{x} = \mathrm{b}$ has no solution (they lead to $0 = 1$):

$$\begin{aligned} x + 2y + 2z &= 5 \\ 2x + 2y + 3z &= 5 \\ 3x + 4y + 5z &= 9 \end{aligned}$$

Find numbers y_1, y_2, y_3 to multiply the equations so they add to $0 = 1$. You have found a vector $\mathbf{y}$ in which subspace? Its dot product $\mathbf{y^T b}$ is 1, so no solution $\mathbf{x}$.