Local Contrast Enhancement of Grayscale Images using Multiscale Morphology

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Abstract

This paper presents a method for locally enhancing a gray-level image using multiscale morphology. Though the basic underlying concept of the work is an extension of usual contrast enhancement techniques, the method emphasizes on nonlinear enhancement of selectively exracted features from the image based on shape as well as size or scale. Multiscale tophat and bottomhat transformations are employed so as to extract intensity profile of scale-specific features present in the image. A number of morphological towers are constructed to stack the feature-images so formed. An iterative weighted combination of the images stacked in different towers gives rise to an image enhanced locally. The proposed algorithm has been executed on a raw image for testing its efficacy.

Keywords: Mathematical morphology, multiscale morphology, morphological towers, tophat transformation, local contrast enhancement.

1 Introduction

The visual quality of an image depends on various factors and any subsequent processing on the image for its recognition, interpretation or description depends on it. An image of an object or a scene, passes through several intermediate steps like recording, digitization, coding, transmission etc in course of its formation. The image, so formed, might be found to have undergone degradation and hence unable to present the exact replica of the object or the scene. A poor illumination of the object or the scene to be imaged, for example, causes a low contrast in the images whereas, the recording device or the intervening medium may introduce noise in the image data. The technique usually adopted for improving the visual quality of such degraded images is broadly termed as *image enhancement*. Image enhancement is an ad-hoc process [28, 8, 12] of improving the visual quality of a degraded image subject to a pre-requisite quality for subsequent application specific processing. An enhancement technique performing is thus highly context sensitive and a given enhancement technique suitable for enhancing biomedical images may not be identically efficient in enhancing satellite images. There exist two broad categories of image enhancement techniques *viz.* the *spatial domain* techniques and the *frequency domain* techniques.

Quite often a recorded image suffers from a common degradation like poor contrast. The range of intensity i.e. the difference between the highest and lowest intensity values in an image gives a measure of its *contrast*. There are standard techniques like *histogram stretching* [9, 28], *histogram equalization* [28, 8] for improving the poor contrast of the degraded image. A few variations of histogram equalization technique e.g. histogram modification [6, 7] also serve the purpose. The speciality of conventional histogram equalization technique is that it treats the image globally. However, there is a need for devising a context-sensitive technique based on local contrast variation since the image characteristics differ considerably from one region to another in the same image and also the local histogram does not necessarily follow the global histogram. Dorst [2] adapted histogram stretching method over a neighborhood around the candidate pixel for local contrast stretching. A numerous modifications [27, 24, 22] of histogram equalization (or modification) are suggested based on adapting the same over a subregion of the image. Contrast stretching methods using local statistics are also reported [14, 15, 23].

The application of *mathematical morphology* [19, 29] to image processing and analysis has initiated a new approach for solving a number of problems in the related field. This approach is based on set theoretic concepts of shape. In morphology objects present in an image are treated as sets. The identification of objects and object features through their shape makes mathematical morphology become an obvious approach for various machine vision and recognition processes. Hardware implementation of morphological processors include Golay logic processor [5], Leitz Texture Analysis System TAS [13], the CLIP processor arrays [3],and the Delft Image Processor DIP [17]. The entension of concepts of morphological operations like dilation and erosion (also known as Minkowski addition and subtraction [21] respectively) of binary images to arena of gray level images has proved to be reasonably efficient. Natural extension of morphologic transformations from binary image processing to gray scale processing using max and min operations is done by Sternberg [30] and Haralick et.al. [11]

A number of researchers in various fields of image processing use morphologic techniques. Peleg and Rosenfield [26] use it to generalize medial axis transform, Peleg et.al. [25] use it to measure changes in texture properties as a function of resolution, Werman and Peleg [31] used it for feature extraction, and Favre [4] used it for the detection of platelet thrombosis in cross sections of blood vessels. Lee et.al. [16] and Chanda et.al. [1] have employed gray scale morphology for edge detection.

This paper presents a local contrast enhancement technique using multiscale morphology. The usual notations of digital image processing and mathematical morphology [10] have been used in the following description. We have given a brief discussion on conventional local contrast enhancement methods based on local statistics and local histogram equalization in section 2. Section 3 gives a discussion on mathematical morphology, multiscale morphology and tophat transfomation. In section 4.1 we have first presented the theoretical formulation of local contrast enhancement using multiscale morphology. An application of the formulation in one dimension has been presented in section 4.2. Section 4.3 presents elaborately various steps of the implementational aspects of the proposed alogorithm. Finally section 5 gives a discussion on the result and its comparison with that of other well-known techniques.

2 Local contrast stretching

In a low contrast image, the entire range of the gray-scale is not exhaustively occupied by its pixels. A poor ambience of light illuminating the object or scene to be imaged may be a possible reason for such low contrast. However, even if an image exhaustively utilizes its entire range of gray scale, the contrast over different smaller regions of the image may suffer from low contrast and may require contrast enhancement. Usual methods of contrast enhancement treat the image globally and does not pay special attention to smaller regions of low contrast. A relatively smaller number of pixels from such areas are insufficient to have any significant influence on the computation of global transformation. So the conventional histogram stretching or histogram equalization technique fails to serve the purpose. Such images need local enhancement and the technique by which this can be achieved is termed as *local contrast stretching*.

A local contrast stretching method as suggested by Lee [14, 15] makes use of local statistics of a predefined neighborhood in modifying the graylevel $g(r,c)$ of a pixel. The difference between graylevel $g(r,c)$ and mean graylevel $\overline{g}(r,c)$ over a predefined neighborhood surrounding the pixel (r,c) is amplified so that the modified graylevel $\tilde{g}(r,c)$ is given by

$$\tilde{g}(r,c) = \overline{g}(r,c) + k[g(r,c) - \overline{g}(r,c)] \quad (1)$$

where k is a global amplification factor and is greater than one. Narendra and Fitch [23] in another approach, have considered the amplification factor too to be a function of (r,c) based on the local graylevel statistics over the same neighborhood as is used to define the mean graylevel $\overline{g}(r,c)$. The factor $k(r,c)$ is defined in terms of the graylevel variance $\sigma^2(r,c)$ over the neighborhood as

$$k(r,c) = \gamma \frac{\overline{g}}{\sigma^2(r,c)}, \quad 0 < \gamma \leq 1$$

where $\overline{g}$ is the global mean of image graylevel and γ is a user defined parameter.

The conventional histogram equalization technique may be adopted to enhance the local contrast of the image by modifying the intensity of each pixel through a local histogram equalization over a small region of the image around that pixel [22, 23, 24, 27].

However, this kind of transformation using only local graylevel statistics cannot distinguish between consistent variation in intensity over a region and the variation in intensity due to presence of a feature (bright or dark) within a region. So it may stretch contrast evenly in both the cases. As a result, undesired contrast intensification (in supposedly smooth region) takes place at some regions of the image, which may require further processing such as *de-enhancement* [22]. On the other hand, if local contrast is stretched based on the presence of spatial features, then this problem can be avoided completely.

As mathematical morphology is an appropriate tool for dealing with spatial features or shapes, we modify equation (1) in terms of mathematical morphological operators.

3 Multiscale Morphology

Mathematical Morphology is a powerful tool in the field of image processing and computer vision and is used for extracting, modifying and combining image components that are useful in the representation and description of region shapes. In morphology, the objects in an image are considered as set of points and operations are defined between two sets: the object and the structuring element (SE) [29, 10]. The shape and the size of SE is defined according to the purpose of the associated application. Basic morphological operations are erosion and dilation. Other operation like opening (closing) is sequential combination of erosion (dilation) and dilation (erosion). We adopt, here, *function- and set-processing* (FSP) system[18]. FSP dilation of a graylevel image $g(r,c)$ by a two dimensional point set B is defined as

$$(g \oplus B)(r,c) = max\{g(r-k, c-l)|(k,l) \in B\} \quad (2)$$

Similarly, FSP erosion of $f(x,y)$ by B is defined as

$$(g \ominus B)(r,c) = min\{g(r+k, c+l)|(k,l) \in B\} \quad (3)$$

The shape of the structuring element B plays a crucial role in extracting features or objects of given shape from the image. However, for a categorical extraction of features or objects from the image based on shape and size we must incorporate a second attribute to the structuring element which is its *scale*. A morphological operation with a scalable structuring element can extract features based not only on shape but also on size. Also features of identical shape but of different size are now treated separately. Such a scheme of morphological operations where a structuring element of varying scale is utilized is termed as *multiscale morphology* [29, 18]. Multiscale opening and closing are defined, respectively, as

$$\begin{aligned} (g \circ nB)(r,c) &= ((g \ominus nB) \oplus nB)(r,c) \quad (4) \\ (g \bullet nB)(r,c) &= ((g \oplus nB) \ominus nB)(r,c) \quad (5) \end{aligned}$$

where B is a point set representing convex structuring element of a definite shape while n is an integer representing the scale factor of the convex structuring element. So, we obtain nB by dilating B recursively $n-1$ times with B itself as shown below.

$$nB = B \underbrace{\oplus B \oplus B \oplus \cdots \oplus B}_{n-1 \text{ times}} \quad (6)$$

By convention $nB = \{(0,0)\}$ when $n = 0$.

3.1 Multiscale Tophat Transformation

The *tophat transformation* originally proposed in [20] provides an excellent tool for extracting bright (respectively, dark) features smaller than a given size from an uneven background. It relies on the fact that by gray-scale opening, one can remove from an image the brighter areas, i.e. features, that cannot hold the structuring element. Subtracting the opened image from the original one yields an image where the features that have been removed by opening clearly stand out. Similar thing is true for closing operation also. That means using a *closing* in stead of an *opening* and subtracting the original image from the closed one helps us extract dark features from a brighter background. Let us call it a *black tophat* transformation as opposed to *white tophat* transformation in case of opening. Suppose the structuring element used in both opening and closing is a disk, or, more specifically, a discrete approximation of disk. Therefore, the bright tophat tranformation decomposes an image into two parts as given by

$$g(r,c) = \underbrace{(g \circ B)(r,c)}_{\text{part 1}} + \underbrace{[g(r,c) - (g \circ B)(r,c)]}_{\text{part 2}} \quad (7)$$

where B is a disk in discrete domain. Fig 1 shows an example of bright tophat transformation for an one-dimensional signal. Let us call part 1 of equation (8) the *base image* with respect to $B(r,c)$. And let us call part 2 of equation (8) the *feature image* at size of B as it contains all the features of $g(r,c)$ that are smaller than the size of B and are brighter than the base image over a region of size (also of shape) of B surrounding (r,c).

An ordered sequence of morphological tophat filtering through opening (closing) of the image with a disk structuring element at different scales extracts scale-specific bright(dark) features from the image. These scale-specific features resulting from the multiscale tophat transformation of the image can be amplified selectively to achieve local contrast stretching. The proposed method is described elaborately in the following section.

4 Proposed method

4.1 Local Contrast Enhancement using Morphology

As mentioned in the previous section, the bright tophat tranformation decomposes an image into two parts. This may be expressed in terms of gray scale morphological operators as

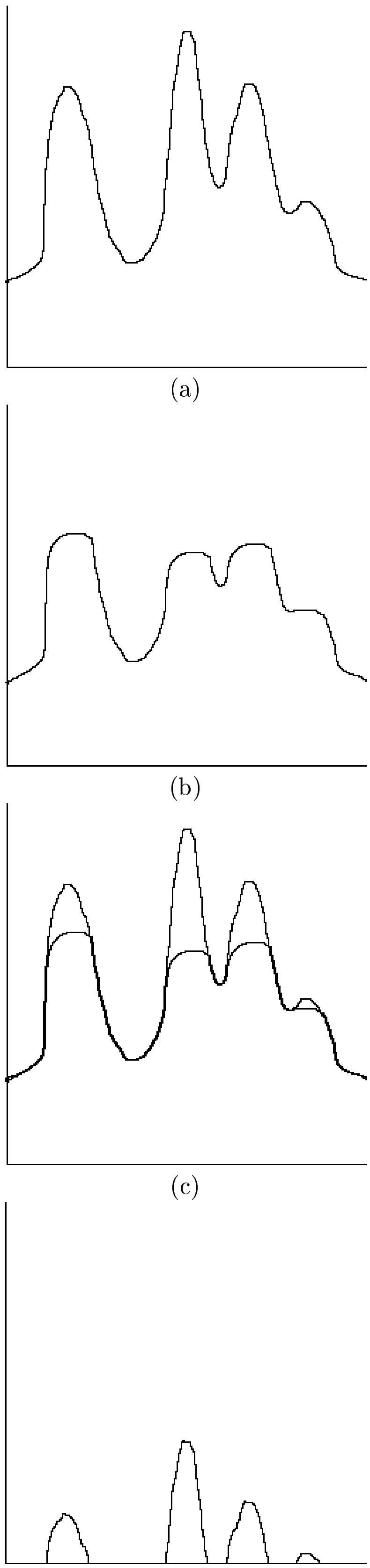

Figure 1: Illustrating bright tophat transformation through graylevel opening. (a) original function, (b) function opened with circular disk, (c) superposition of the previous two and (d) features after tophat transform.

$$g(r,c) = \underbrace{(g \circ B)(r,c)}_{\text{part 1}} + \underbrace{[g(r,c) - (g \circ B)(r,c)]}_{\text{part 2}} \quad (8)$$

where $(g \circ B)(r,c)$ is openning of graylevel image $g(r,c)$ by a graylevel structuring element $h(r,c)$ defined as

$$h(r,c) = \begin{cases} 0 & \text{if } (r,c) \in B \\ -\infty & \text{otherwise} \end{cases}$$

where B is a disk in discrete domain. Then the graylevel structuring element $h(r,c)$ at scale n, in this case, may be defined as

$$h(r,c) = \begin{cases} 0 & \text{if } (r,c) \in nB \\ -\infty & \text{otherwise} \end{cases}$$

The feature image [i.e., part 2 of equation 8] gives a measure of local contrast in the original image due to presence of bright features. Hence, combining equations (1) and (8) we suggest the following transformation for local contrast stretching

$$\tilde{g}(r,c) = (g \circ B)(r,c) + k[g(r,c) - (g \circ B)(r,c)] \quad (9)$$

where k is again a global amplification factor and is greater than one. So this transformation makes bright features brighter and, thus, improves the local contrast. Now suppose $k = 2$. The equation (9) becomes

$$\tilde{g}(r,c) = g(r,c) + [g(r,c) - (g \circ B)(r,c)] \quad (10)$$

Let us denote $[g(r,c) - (g \circ B)(r,c)]$ by $F_B^o(r,c)$, i.e, features at size less than that of B obtained by opening (more specifically, by bright tophat transformation). Accordingly, we can rewrite equation (10) as

$$\tilde{g}(r,c) = g(r,c) + F_B^o(r,c) \quad (11)$$

Thus, we have the bright-feature image $F_{nB}^o(r,c)$ at scale n as it contains all the features of $g(r,c)$ that are smaller than that of nB. Hence, the value $F_{nB}^o(r,c)$ of bright-feature image at (r,c) gives a measure of local contrast in the original image due to presence of bright features at scale n. Note that $F_{0B}^o(r,c)$ is an all-zero image. Now, let us define

$$\delta_n^o(r,c) = F_{nB}^o(r,c) - F_{(n-1)B}^o(r,c) \quad (12)$$

It is evident that $\delta_n^o(r,c)$ contains bright features of $g(r,c)$ that are larger than scale $(n-1)$, but smaller than scale n. Therefore, we obtain, using multiscale approach, local contrast stretching of bright features as

$$\tilde{g}(r,c) = g(r,c) + k_1\delta_1^o(r,c) + k_2\delta_2^o(r,c) + k_3\delta_3^o(r,c) + \cdots \quad (13)$$

where $k_1 > k_2 > k_3 > \ldots$, since we know that smaller the size of a bright feature, more should be its intensity for detectibilty. If features smaller than scale m are needed to be enhanced then

$$\tilde{g}(r,c) = g(r,c) + \sum_{i=1}^{m} k_i \delta_i^o(r,c) \quad (14)$$

Taking $k_{i+1} = k_i - 1$ for all i and neglecting $F_{mB}^o(r,c)$, we finally have local contrast stretching of bright features as

$$\tilde{g}(r,c) = g(r,c) + \sum_{i=1}^{m-1} F_{iB}^o(r,c) \quad (15)$$

Proceeding in a similar way based on multi-scale dark tophat transformations we achieve local contrast stretching of dark features as

$$\tilde{g}(r,c) = g(r,c) - \sum_{i=1}^{m-1} F_{iB}^c(r,c) \quad (16)$$

Hence, to obtain the modifed image, in which contrast of both (bright and dark) type of features are stretched locally, we combine equations (15) and (16) as follows

$$\tilde{g}(r,c) = g(r,c) + 0.5 \sum_{i=1}^{m-1} F_{iB}^o(r,c) - 0.5 \sum_{i=1}^{m-1} F_{iB}^c(r,c) \quad (17)$$

The constant multiplier 0.5 is used for avoiding clipping of graylevel of the pixels in the image as much as possible. For the sake of generalization, we slightly modify equation (17) as

$$\tilde{g}(r,c) = g(r,c) + 0.5 \sum_{i=n}^{m-1} F_{iB}^o(r,c) - 0.5 \sum_{i=n}^{m-1} F_{iB}^c(r,c) \quad (18)$$

where all the features, either dark of bright, smaller than scale n are assumed to be noise in the image.

4.2 One-dimensional case

For a better understanding of the local contrast enhancement scheme we first elaborate it in the context of one-dimensional function as shown in Fig. 2. Fig. 2 provides a simplified illustration of the proposed strategy. Here we intend to enhance a function $g(t)$ locally. The function $g(t)$ has salient features manifested as *crest*s and *trough*s of different height (*or depth*) and width located at different positions. We make use of the line segment L of unit length and its higher order dilates kL (where $k = 1, 2, 3..$) as structuring elements (SE) for extracting the salient features from the function as described below:

- The opening operation with the SE kL removes the crests which are narrower than the length k from the function while the closing operation fills up the troughs narrower than the length k .
- The function $top_k(t) = (g(t) - g(t) \circ kL)$ contains only the crests of width smaller than k of $g(t)$ and the function $bot_k(t) = (g(t) \bullet kL - g(t))$ contains only the troughs of width smaller than k of $g(t)$.
- The functions $S_{op}(t) = 0.5\Sigma top_k(t)$ and $S_{cl}(t) = 0.5\Sigma bot_k(t)$ are constructed by summing up the $top_k(t)$ and $bot_k(t)$ respectively followed by a scaling in magnitude to avoid probable clippings. In doing so the crests and troughs of narrower widths are made to have more contribution to $S_{op}(t)$ and $S_{cl}(t)$.
- The locally enhanced function is then formed by combining the functions $S_{op}(t)$ and $S_{cl}(t)$ with the original function as shown below:

$$g_{enh}(t) = g(t) + S_{op}(t) - S_{cl}(t) \quad (19)$$

comparing $g_{enh}(t)$ with $g(t)$ at each sample point t it can be found that height (depth) of the crests (troughs) have increased but disproportionately. The change in height (or depth) is more for crest (or trough) of narrower width which would have not been possible using simple linear stretching of the function. The scheme explained for this one-dimensional case may as well be extended in two-dimension. There we introduce the concepts of different *morphological towers* as discussed in the following sections.

4.3 Implementation

The implementation of equation (18) describing feature based local contrast enhancement involves construction of a number of morphological towers as elaborated below.

4.3.1 Construction of Morphological Towers

The image to be enhanced is made to undergo a sequence of grayscale morphological opening operations with a disc structuring element and its higher order homothetics. The resulting sequence of images are kept in a stack called the *opening tower* as shown in the fig 3.

An identical tower, called closing tower, is constructed with the sequence of the images resulting from multiscale closing of the input image. Therefore the i-th entry in the opening (closing) tower contains the image opened(closed) with the structuring element iB.

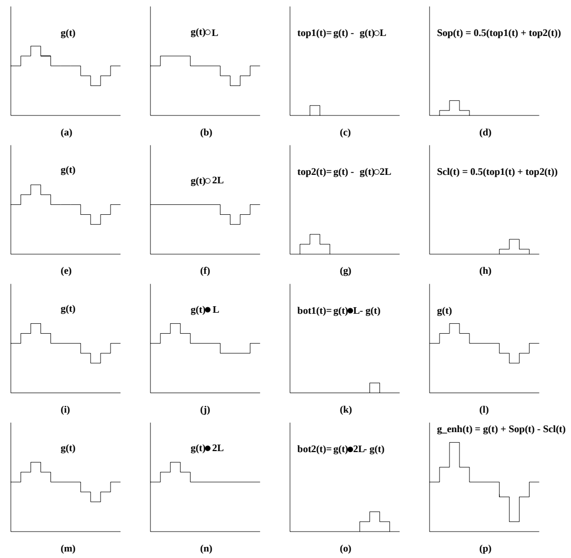

Figure 2: Local contrast enhancement of a function: (a),(e),(i),(m)and (l) original function, (b), (f) function opened with line SE, (j),(n) function closed with line SE (c),(g) tophat functions (k),(o) bottomhat functions (d) scaled sum of tophat functions (h) scaled sum of bottomhat functions and (p) the output function after local contrast enhancement.

4.3.2 Construction of difference Towers

The pixel values of opened image is less than or equal to that of the original image. Subtracting the opened image from the original one produces the feature image made up of bright features specific to the scale of the SE. Each entry of the opening tower is subtracted individually from the original image and the resulting bright feature images are then kept in corresponding entries in another tower called the *difference tower*. An identical difference tower is constructed for the dark feature images obtained by subtracting the original image from each entry of the closing tower. Therefore the i-th entry in the difference tower for opening (closing) contains an image consisting of bright (dark) features which are smaller than or equal to iB.

4.3.3 Construction of the enhanced image

For reconstructing the final image we carry out the following steps:

- We sum up all the entries in the difference tower corresponding to the opening operation. This results in an image consisting of bright features of all possible scales of interest that are present in the original image.

$$S^o(r,c) = \sum_{i=1}^{n} F^o_{iB}(r,c) \qquad (20)$$

The summation, here, denotes pixel-wise sum of n images.

- We perform the same operation on the difference tower corresponding to the closing operation. This results in an image consisting of dark features of all scales of interest that are present in the original image.

$$S^c(r,c) = \sum_{i=1}^{n} F^c_{iB}(r,c) \qquad (21)$$

Finally, the locally enhanced image is obtained by combining three images as given by

$$\tilde{g}(r,c) = g(r,c) + 0.5S^{op}(r,c) - 0.5S^{cl}(r,c) \qquad (22)$$

The '+' and '-' operations are applied between corresponding pixels of three different images.

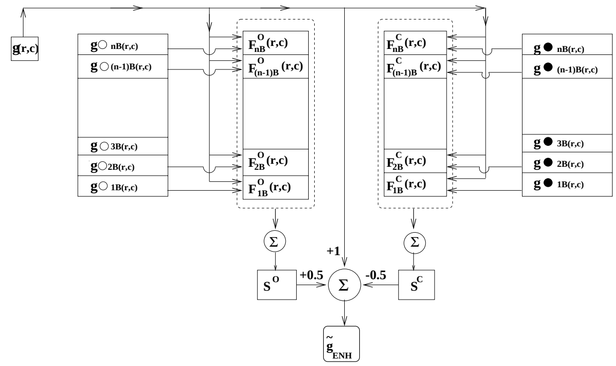

Figure 3: Schematic diagram of different morphological towers

5 Experimental results and discussion

The proposed algorithm has been tested on a pair of sample MR images of sagittal scan of human brain shown in fig. 4(a) and fig. 5(a). The results of global enhancement techniques like linear contrast stretching and histogram equalization performed on the original images are shown in fig 4(b-c) and fig 5(b-c) respectively. The examples reveal that the global techniques cannot improve the contrast satisfactorily. Results of the proposed algorithm are shown in fig 4(d) and fig 5(d) where we have used $n = 1$, $m = 6$ and

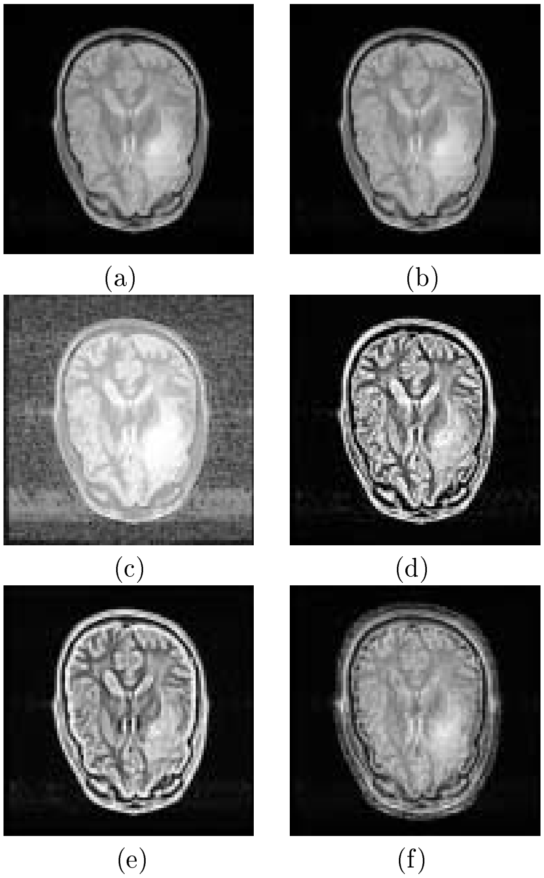

Figure 4: (a) Original image (b) linear global contrast stretching (c) global histogram equalization (d) local contrast enhancement using multiscale morphology (e) local contrast enhancement using Dorst's method (f) local histogram equalization

B as a 3×3 SE. The resulting image in each case is seen to have more contrast than its original version. Fig. 4(e-f) and 5(e-f) show the results of Dorst's method and local histogram equalization respectively. The mask size used in each case is 13×13 to produce visually optimum results. The amplification factor used in Dorst's method is set at 3 while the ratio of global mean to local variance in case of local histogram equalization is set at 0.5. The enhanced clarity of the features of relatively smaller size in the image resulting after equalization using the proposed method as compared to that in the image resulting after other methods proves the efficacy of the devised method.

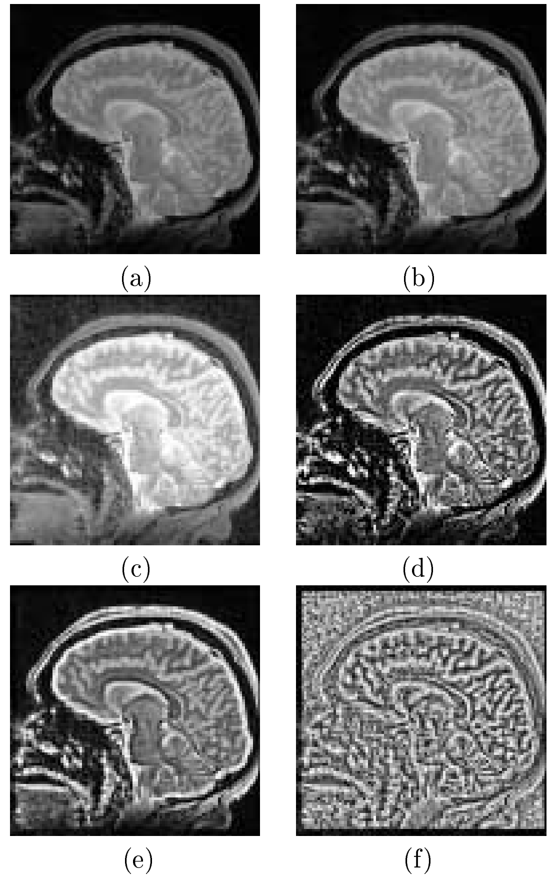

Figure 5: (a) Original image (b) linear global contrast stretching (c) global histogram equalization (d) local contrast enhancement using multiscale morphology (e) local contrast enhancement using Dorst's method (f) local histogram equalization

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